⊙ **Technical Paper**

A Note on the Asymptotic Structure of the 90°-Corner Flow+

Yong Kweon Suh*
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90°－모서리 유동의 점근적 구조에 관한 少考

徐　龍　權

Key Words : Separation Point (박리점), Schwarz-Christoffel Transformation (슈바르쯔-크리스토펠 변환), Breakaway Point (이탈점), Triple Deck Theory (트리플덱 이론), Free-Streamline Theory (자유유선 이론)

초　록

90°-모서리 주위의 유동에 관한 수치해석결과를 바탕으로 그 유동구조를 점근적으로 해석하였다. 전체유동구조는 고전적 자유유선 이론에 의한 모델과 일치하였으며 경계층 박리섬 주위의 유동구조는 최근의 트리플뎁 이론에 의한 모델과 일치함이 發見되었다. 後者의 發見은 박리점의 위치를 구하는 점근적 식이 트리플뎁 이론에 근거하였으며 그 결과가 수치해석의 결과와 아주 잘 맞는데 따른 결과이다.

1. Introduction

In estimating the wave forces acting on the marine structures, knowledge on the mechanism of the separated flow, the position of the separation point, and structure of the wake flow etc., is essential especially for high Keulegan-Carpenter numbers. Up to now there has not been a rational theory covering all these features. The most reliable model for the separated flow field is recently proposed by Smith[1]. The key property in his model is that near the separation point the so called triple-deck theory can be applied. However, the model is composed of lots of local structures which makes the computation almost impossible. The essential reason for this inaccessiblity comes from the fact that the given obstacle is isolated in the uniform flow. As a matter of consequence we reasonably build a substitutive flow problem simpler than that of the isolated abstacle but still preserving the local intrinsic nature for instance near the separation point.

We consider a finite flat plate attached normally to an infinite flat wall. The given geometry is subject to the stagnation flow. Due to the corner point, the boundary layer flow will separate ahead of the corner. Numerical solutions of the full Navier-Stokes equations to this problem have been given by Suh[2] up to *Re* (Reynolds number) of 2800. *Re* here is based on the plate length, the reference velocity (the velocity of the potential flow evaluated at the leading edge of the plate) and the kinematic viscosity of the fluid. His numerical results show that on the wall of the recirculated region, the

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* Member, Mechanical Engineering Dep't, Dong-A University

pressure is almost constant, and the distance between the leading edge and the separation point is represented by $O\ (Re^{-1/3})$. Upon these numerical results, extended study is made in this paper concerning the possibility of the joint of the free streamline theory and the triple deck theory for the bondary layer separation (the so called "reconciliation" problem[3]).

2. Inviscid Solution to the Problem Based on the Free Streamline Theory

Fig. 1(a) shows the geometry in the z-plane concerned in this problem. Note that the dimensions are scaled by the lenght of the plate C-B. The boundary of the domain of the problem is D_1-A-A'-D_2. The governing equation

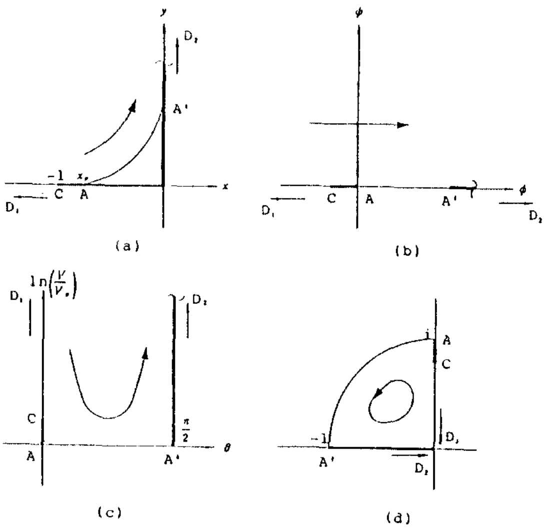

Fig. 1 Physical and transform planes: (a) z-plane, (b) w-plane, (c) ω-plane, and (d) ζ-plane

for this domain subject to inviscid and irrotational flow, and the appropriate boundary conditions based on the free streamline theory are

$$\nabla^2 \psi = 0 \tag{1}$$

$$\psi = 0 \text{ along } D_1\text{-A-A'-}D_2 \tag{2}$$

$$\frac{\partial \psi}{\partial \bar{s}} = V_o \text{ along } A\text{-}A' \tag{3}$$

where ψ is the stream function and $\bar{s}$ is coordingat along the breakaway streamline (or free streamline ; streamline of $\psi = 0$).

As the problem is two-dimensional, we utilize the complex function theory. For the complex potential

$$w = \phi + i\psi \tag{4}$$

we introduce as is usual the complex function ω such that

$$\omega = i \ \ln \frac{1}{V_0} \frac{dw}{dz} = \theta + i \ \ln \frac{V}{V_0} \tag{5}$$

where $V = \left| \frac{dw}{dz} \right|$ is the absolute velocity and $\theta = \tan^{-1}\left(\frac{-\partial\psi/\partial x}{\partial\psi/\partial y}\right)$ is the angle that the velocity vector makeswith the x-axis. Then the mapping $\omega \leftrightarrow w$ can be obtained by the Schwarz -Christoffel transformation :

$$\omega = C_1 \int \frac{dw}{\sqrt{w}\ \sqrt{w-\phi_0}} + C_2 \tag{6}$$

By introducing ζ defined as

$$w = \frac{\phi_0}{4}\left(\zeta + \frac{1}{\zeta}\right)^2 \tag{7}$$

(6) reduces to

$$\omega = -i \ \ln\ (-i\zeta\ ) \tag{8}$$

Thus with $\frac{dw}{dz} = \frac{dw}{d\zeta}\frac{d\zeta}{dz}$ and (8), z can be written from (5) as

$$z = \frac{-\phi_0}{2V_o}\ i\ \left(\frac{1}{3}\zeta^3 + \frac{1}{\zeta}\right) \tag{9}$$

To attain $w \rightarrow -z^2$ as $|z| \rightarrow \infty$ (or $|\zeta| \rightarrow 0$) (this was the far field condition in the numerics of Suh[2]), we must require that

$$\phi_0 = V_o^2 \tag{10}$$

Evaluating at $\zeta = i$ gives a ;

$$a = -\frac{2}{3} V_o$$

Note that as $a \rightarrow -1$ (breakaway starting from the leading edge),

$$V_0 \rightarrow \frac{3}{2}$$

Fig. 1 (a) to (d) show the four planes associated with the transformations.

3. Prediction of the Position of the Separation Point

We next find the asymptotic representation for the shape of the free streamline near the point A as is necessary in locating the separation point using the triple deck theory later. We set $\zeta = \exp[i(\sigma + \frac{\pi}{2})]$ so that, near A, $\sigma << 1$. Then (9) becomes

$$z = (a + V_0\sigma^2 + \cdots) + i\left(\frac{2}{3}V_0\sigma^3 + \cdots\right) \tag{11}$$

Thus asymptotically for small σ, the shape of the free streamline takes the form

$$y=\frac{2}{3}\ V_o^{-1/2}(x-a)^{3/2}+\cdots \qquad (12)$$

Now we change the variable x as $x=-1+s$ and the constant a as $a=-1+s_o$ so that s starts from the leading edge and s_o is the distance of the breakway point from the leading edge. Then by taking only the highest-order term, (12) becomes

$$y=\frac{2}{3}V_o^{-1/2}(s-s_o)^{3/2} \qquad (13)$$

Upon Smith[4], for the local coordinates (X, Y), the shape of the displacement far downstream of the separation point required for the self-consistent boundary layer separation should be

$$Y=\frac{2}{3}\epsilon^{1/2}\lambda^{9/8}\alpha\,(X-X_s)^{3/2} \text{ as } X\rightarrow X_s \qquad (14)$$

where λ is the skin friction coefficient to be given by the boundary layer calculation for the upstream region ($\lambda=0.332$ for the present case), and ϵ is $-\frac{1}{8}$ power of the appropriate Reynolds number (Res), and α is 0.44 critical for the sake of the existence of the solution.

For the function (13) to be matched with (14), (x,y) and u must be scaled by s_o and V_o respectively. Thus by $x=s_0X$ and $y=s_0Y$, (12) becomes

$$Y=\frac{2}{3}V_0^{-1/2}s_0^{1/2}(X-1)^{3/2} \qquad (15)$$

Now by this scale

$$Res=\frac{s_0 V_0}{\nu}=\epsilon^{-8}$$

which is related to Re used by Suh by the following equation.

$$Res=\frac{V_0}{2}s_0 Re \qquad (16)$$

where $Re=\frac{1\times 2}{\nu}$. Then equality of (14) and (15) makes

$$s_0=2^{1/9}\alpha^{16/9}\lambda^2 V_0^{7/9}Re^{-1/9}=0.0379Re^{-1/9} \qquad (17)$$

where the asymptotic value 1.5 for V_0 is used as an approximation. However for the range of Re used in Suh's numerics, s_0 is as a result too small compared with s_s the calculated distance of the separation point from the leading edge. At $Re=2800$, for instance, 0.0157 is obtained for s_0 while 0.13 for s_s. Thus, in the following, we shall try higher approximation by including into (17) (s_s-s_0) in the triple deck scale $O\ (Res^{-3/8})$. If we let S be that amount in the triple deck scale, then[3]

$$s_s-s_0=\epsilon^3\lambda^{-5/4}S=15.081S\ Re^{-1/3} \qquad (18)$$

By addition of (17) and (18),

$$s_s=0.0379Re^{-1/9}+C\ Re^{-1/3} \qquad (19)$$

where $C=15.081S$.

4. Comparison of the Present Results with the Numerical Results of the Navier-Stokes Equations

Fig. 2 shows the streamlines obtained by the present calculation in comparison with those obtained by the finite difference calculation for the Navier-Stokes equations[2]. It is seen, as a whole, that as Re is increased the numerical results tend to fit the free-steamline results. The discrepency shown at low Re especially in the downstream region is due to the effect of the boundary layer developed in that region near the wall. The separating streamline ($\psi=0$) surely tends to approach the breakaway streamline ($\psi=0$) as Re is increased. However, the streamlines for $\psi=0.1$ obtained by the numerics tend to fall apart toward the wall from the corresponding ones by the theory ; this is due to the tendency of the shear-layer developed around the separating streamline to be thinned as Re is increased causing the displacement decreased. However, after all they shall come close again pushed by the separating streamline when Re is large enough. In an attempt to compare qualitatively the shapes of the shreamlines of $\psi=0$, V_0 is adjusted such that the free streamline fits the separating streamline as shown in Fig. 3. Also shown is the one calculated by Chernyshenko[5] via the Batchelor's model. It is seen that the separating streamline exactly fits the free streamline except in the small region near the reattachment point. On the other hand, the line predicted by the use of the Batchelor's model anyhow never seems to be able to fit the separating streamline. Moreover, the Batchelor's model permits only one vortex in the domain, while the numerics results in the two vortices[2]. Further, noteworthy is that the numerics reveals and almost constant pressure along the wall of the recirculated region which is the basis of the free streamline theory, while the Batchelor's model has significant variation in the wall pressure distribution. Upon the above argument, it can be stated that the free streamlinemodel is closer to the nature than the Batchelor's model as far as the numerical solutions for this problem of the Navier-Stokes equations are concerned.

Finally, the validity of (19) for the location of the separation point is pursued. In the work of

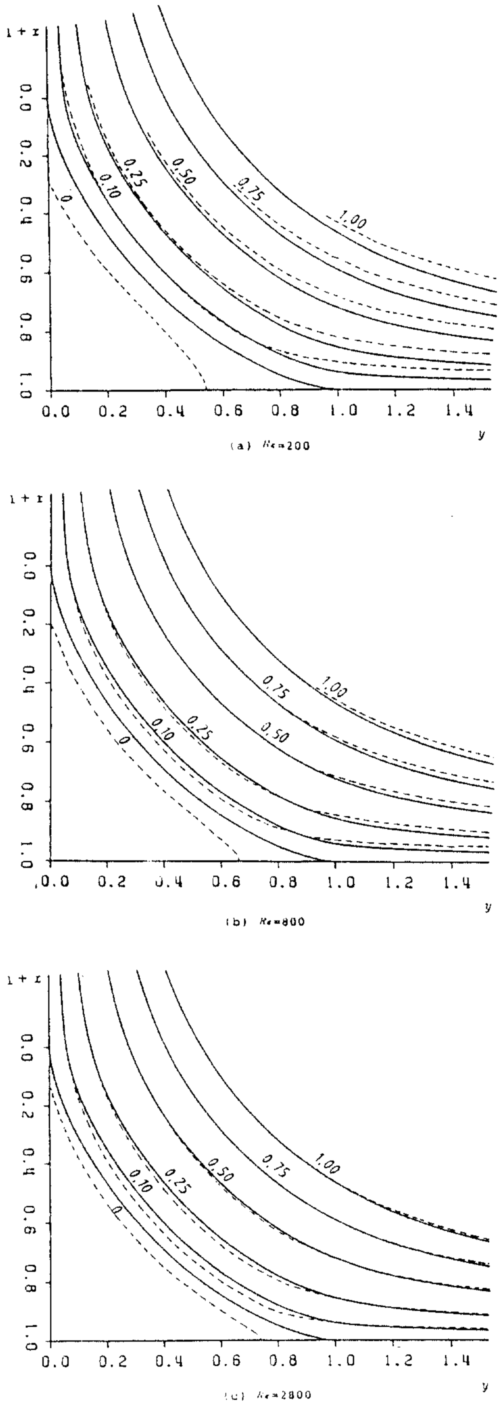

Fig. 2 Streamlimes obtained by the free-streamline model with the breakaway point at the leading edge (—), and those by the Navier-Stokes equations for (a) $Re=200$, (b) $Re=800$, and (c) $Re=2800$(---)

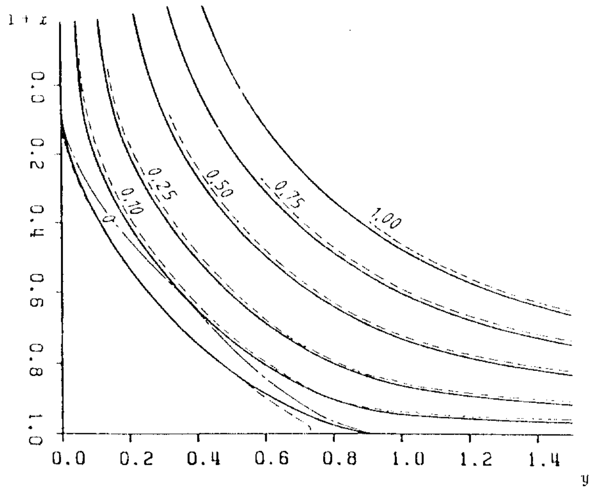

Fig. 3 Streamlines obtained by the free-streamline model with the breakaway point $s_0=0.08$ (—), and those by the Navier-Stokes equations for $Re=2800$ (---). Also shown is the breakaway streamline obtained by Chernyshenko (taken by Chernyshenko 1984) based on the Batchelor's single-eddy model (— · —)

Suh[2], the curve fitting was made in the log-log plot of s_s versus Re with the result

$$s_s = 1.7445\,Re^{-1/3} \tag{20}$$

as reproduced in Fig. 4. Actually the term of $Re^{-1/9}$ on the right hand side of (19) is less significant than that of $Re^{-1/3}$ for the given range of Re ($100 \leqq Re \leqq 2800$) ; at $Re=2800$ the first term gives 0.0157 while the second gives 0.124 with $C=1.7445$, the first being 12.7% of the second. Thus the second term dominant in (19) is agreed to by the numerical results as far as the exponent of Re is concerned. Since, the present analysis casts the high possibility of successful prediction of the flow structure near the separation point by use of the interactive boundary layer theory ; the key to the nethod will be coupling of the boundary layer calculation with the inviscid calculation based on the free streamline theory in the body scale specific for each problem concerned.

5. Conclusions

The free streamline theory is applied to obtain the asymptotic flow structure at high

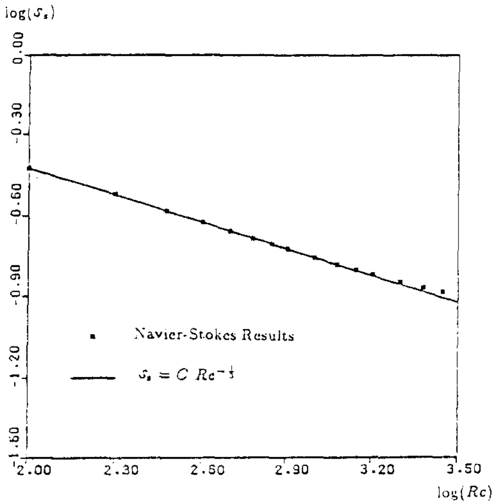

Fig. 4 Numerical result for the distance of the separation point from the leading edge of the plate obtained by Suh[2] for the Navier-Stokes equations. The constant C used in plotting the line fitting the data points is 1.7445

Reynolds number for two-dimensional corner flow of 90° angle which was studied numerically by Suh. The streamlines by this theory agree well with those of the numerical works of him. The asymptotic shape of the free stream-line near the breakaway point turns out to satisfy the requirement of the triple deck theory. The present analysis predicted the asymptotic position of the separation point with the aid of the result of the triple deck calculation. The second order term in the equation, however, not the first one fits with unknown constant the numerical result for the given range of Re. This study, thus, recommends the use of the streamline model as the basis of the inviscid calculation in the interative boundary layer theory for the geometry treated here.

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