

◎Technical Paper

## Study on the Behavior of Tubular Member with Partial End Fixity<sup>+</sup>

K. N. Cho\*

(Received April 15, 1988)

부분 고정단을 가진 원통형 부재의 거동에 관한 연구

조 규 남

**Key Words :** Tubular Member(원통형 부재), Static Condensation Method(정적 압축법), Collision Energy(충돌 에너지), Plastic Deformation(소성변형), Semi-submersible(반잠수식 시추선)

### 초 록

본 논문은 반잠수식 시추선과 선박과의 충돌해석에의 정적 압축법의 응용에 대해 다루었다. 선박이 시추선의 취약 부재에 충돌하는 경우를 가정하였으며 이 취약한 부재의 충돌에너지 흡수능력을 상세 해석 없이 추출하는 방법으로, 관련된 구조물 전체 강성 매트릭스를 부재의 양단에 정적 압축을 시켜 양단 유연도를 추출한 뒤 이 유연도를 양단에 가진 원통형 부재를 해석함으로써 외력-변형 관계를 얻을 수 있었다. 충돌에너지 양은 외력-변형 선도를 적분함으로써 얻을 수 있다.

새로운 방법에 의한 결과를 3차원 수치해석 방법과 강제 프라스틱 방법에 의해서 얻어진 결과와 상호 비교하였으며, 이 새로운 방법이 해양구조물 충돌해석에 매우 효과적으로 응용될 수 있음을 알게 되었다.

### 1. Introduction

For the evaluation of the capability of a bracing member of an offshore structure to absorb the collision energy, 3-demensional numerical model approach is generally used. However in view of the inherent computational complexities of the 3-D approach, a simple method is employed for the collision analysis without performing the detailed analysis. The most common simple method is the rigid-plastic methods<sup>1)</sup>. In this method any characteristics for horizontal movement and rotation at the ends of the corresponding tubular member

are not included. In a real frame system of an offshore structure the tubular element sustains a certain degree of elastic support from the adjacent structure. In this investigation the flexibility at ends of a bracing element is extracted using the rational reduction of the modeling characteristics. The property reduction is based on the static condensation of the related global stiffness matrix of a model to end nodal points of the bracing element<sup>2,3)</sup>. The load-displacement relation at the collision point of the bracing member with the extracted end flexibility is obtained by extending Hodge's method<sup>4)</sup>.

<sup>+</sup> Presented at the 1988 KCORE Spring Conference

\* Member, Hyundai Maritime Research Inst, Hyundai Heavy Ind. Co., Ltd.

The theory and numerical model are applied to a cal semi-submersible and the results of the new typi approach are compared with 3-D numerical model approach and a rigid-plastic method approach.

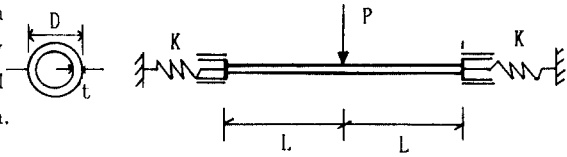


Fig. 1 Tubular beam with elastic horizontal restraints

### 2. End Flexibility Extraction

In the method developed here, the global stiffness of the related system is condensed to the nodal points of a bracing element of interest using a static condensation procedure<sup>2</sup>. The equilibrium equations of the model can be written in matrix partitional from as,

$$\begin{pmatrix} K_{nn} & K_{nm} \\ K_{mn} & K_{mm} \end{pmatrix} \begin{pmatrix} X_n \\ X_m \end{pmatrix} = \begin{pmatrix} F_n \\ F_m \end{pmatrix} \quad (1)$$

Here,  $X_m$  represents the degrees of freedom to be condensed to the corresponding bracing member nodal points.  $X_n$  represents the residual degrees of freedom of bracing member nodal points. Solving equation (1) for  $X_m$ , we obtain,

$$\{X_m\} = -[K_{mm}]^{-1}[K_{mn}]\{X_n\} + [K_{mm}]^{-1}\{F_m\} \quad (2)$$

Substituting this result into equation (1) and collecting terms, we can write the condensed equilibrium equations.

$$\begin{aligned} \{[K_{nn}] - [K_{nm}]^{-1}[K_{mn}]\}\{X_n\} &= \{F_n\} \\ -[K_{nm}][K_{mm}]^{-1}\{F_m\} & \end{aligned} \quad (3)$$

Where L.H.S. stiffness matrix is the effective stiffness matrix. The condensed stiffness matrix is used to calculate the stiffness of the "end spring" corresponding to a certain degree of elastic support from the adjacent structure.

This "end spring" restrictions can be included in the analysis of a bracing element subjected to lateral collision load.

### 3. Post-yield Behavior of a Tubular Element with Partial End Restraints

Here, the influence of partial end restraints in a bracing member subjected to lateral load is investigated by extending Hodge's theoretical study<sup>4</sup>. To apply the method to our tubular member,

we consider a tubular member which is supported as shown in Fig. 1.

The yield condition for positive moment for the tubular member without any local indentation and buckling or crumpling phase is

$$m - \cos \frac{\pi}{2} n = 0 \quad (4)$$

where  $m = M/M_0 = M/D^2 t \sigma_0$

$$n = N/N_0 = N/\pi D t \sigma_0 \quad (5)$$

$M$ : Bending moment

$M_0$ : Plastic capacity for bending moment

$N$ : Axial force

$N_0$ : Plastic capacity for axial force

$D$ : Diameter of the member

$t$ : Thickness of the member

$\sigma_0$ : Yield stress

It is also assumed that the ends of the member is supported by springs so that

$$F = KU \quad (6)$$

The tubular member is assumed to reach the yield-point value of  $4M_0/L$  without any motion. The hinge is assumed to form in the center of the member. The extension  $\Lambda$  and the rotation  $\theta$  of the hinge are related as shown in Fig. 2.

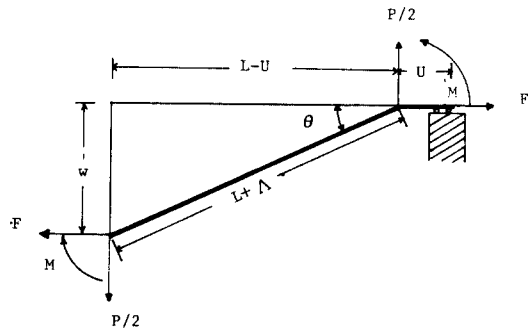


Fig. 2 Configuration of the tubular member

The small parameter  $d$  and dimensionless quantities are defined as in Hodge's method.

$$d = D/L, \quad w = W/d, \quad u = UL/d^2, \quad \lambda = \Lambda N_0/M_0 \quad (7)$$

The kinematic quantities can be written in the form

$$\lambda = \frac{\pi}{2} d \left[ (w^2 - 2u) + d^2 w^2 \left( u - \frac{w^2}{4} \right) + \text{higher order terms} \right] \quad (8)$$

$$\theta = dw \left[ 1 + d^2 (u - w^2/3) + \text{higher order terms} \right]$$

The equilibrium condition gives

$$m = p - \frac{\pi}{2} w f - d^2 p u \quad (9)$$

$$n = \cos \theta \left( f + \frac{2}{\pi} d p \tan \theta \right) \quad (10)$$

where  $f = F/N_0$ ,  $p = PL/4M_0$

$$\text{From equation (6)} \quad u = f/2c \quad (11)$$

where we defined the dimensionless constant

$$c = K \frac{d}{2\pi L t \sigma_0} \quad (12)$$

For the case  $n < 1$ , the slope of the strain vector is perpendicular to the tangent to the interaction curve.<sup>5)</sup>

This condition gives.

$$\dot{\lambda} = \frac{\pi}{2} \sin \left( \frac{\pi}{2} n \right) \dot{\theta} \quad (13)$$

For simplicity we keep only leading terms. The equilibrium equation becomes,

$$m = p - \frac{\pi}{2} w f \quad (14)$$

$$n = f$$

Using equations (8), (11), (13), we obtain

$$\frac{dn}{dw} + c \sin \left( \frac{\pi}{2} n \right) = 2cw \quad (15)$$

For  $n < 1$ , we get load-displacement relation by solving equation (15) after combining equations (4), (14).

## 4. Evaluation

### 4.1 Description of Numerical Model

A semi-submersible drilling rig consists of two pontoons, four columns, four internal columns and additional bracings is chosen for a numerical analysis. The ship is assumed to collide head to the weakest brace of the semi-submersible. The weakest brace element dimensions from the model are,

$$D = 1.8m, \quad t = 28mm, \quad L = 19m$$

The semi-submersible modeled using equivalent

tubular structural units is shown in Fig.3.

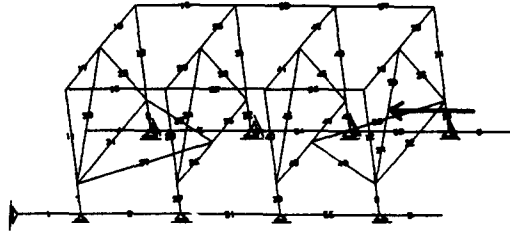


Fig. 3 Modeling of semi-submersible

### 4.2 Numerical Results

The static condensation method described earlier, equation (3) is applied and the resulting adjacent spring constant  $K$ , representing the end flexibility is obtained.

The equations (4), (14), (15) are employed to get post-yielding load-displacement relations with the calculated end flexibility. The load-displacement relation with the corresponding end flexibility is shown in Fig.4. This relation is based on the computation by solving the equations with corresponding section properties. The value of  $c$  used here is 1.2602. This figure shows the load-displacement relation of the member after the load exceeds the plastic collapse load  $P_0$ .

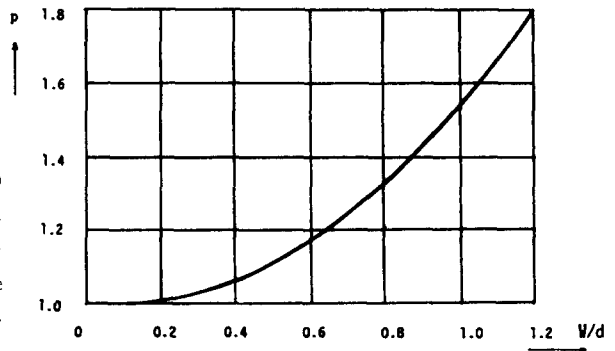


Fig. 4 Load-displacement curve for tubular member with elastic horizontal restraints

#### 4.3 Comparison among 3-D Model, Rigid-plastic Model and New Method

The semi-submersible is modeled as shown in Fig.3. This 3-dimensional model is used for the analysis. The collision force is applied at the middle of the weakest brace perpendicularly with respect to the element span. Total number of 27 load increments are imposed which are thought to be sufficient increments for this analysis. To prevent rigid body motions constraints in the Y and Z directions are imposed through the corresponding nodal points. The collision loads are increased step by step up to the point at which the element is supposed to collapse. The load-displacement relation at the collision point is obtained<sup>6)</sup>.

Also the rigid-plastic methods<sup>1)</sup> is employed. The method provide relatively simple analytical results, often with acceptable accuracy. Evidently, the load-displacement curve by this rigid-plastic method should coincide with the load displacement curve by the new approach when we set the value of  $c$  to be infinite value, *i.e.*, very large value of the stiffness.

Fig.5 is the load-displacement curves from the 3-dimensional modeling approach and the rigid plastic approach and the new approach employed here. Because of the end restraints of the element, the membrane forces are activated during global deformation resulting in the strengthening of the member. The present method lies between the rigid-plastic method and the 3-dimensional analysis as expected. It is self-evident that the rigid-plastic method gives high strengthening membrane effect of the member during global deformation resulting in the steeper slope than the presents method. On the while full 3-dimensional analysis gives less strengthening membrane effect of the member since there are deformations of the adjacent members, resulting in the slowgoing load-displacement curves.

Comparison of the curve by the new approach developed here with those by conventional methods shows that the subject methods provides efficient and precise results.

#### 4.4 Collision Energy Calculation

Conservation of the energy requires that the kinetic energy of the impacting ship is transferred into the elastic deformation energy and the plastic dissipation of energy in ship [and platform. The total kinetic energy can be assumed to be dissipated in the platform itself. The absorbed energy is calculated by integrating the area under the curve in the load-displacement relations obtained. The resulting curve which provides the collision energy at various displacement steps were to be satisfactory in general.

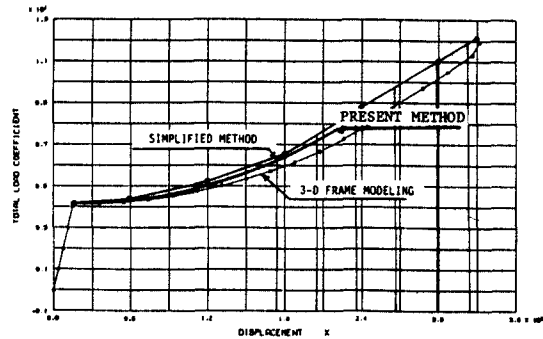


Fig.5 Load-displacement curves with various approaches

### 5. Conclusions

The main contribution of this investigation is the development of a analytical rational procedure to solve the post-yielding behavior of a tubular member. The approach determines the end flexibility more efficiently and precisely. The method provides the tool for practical collision analysis of the typical offshore structure without performing detailed computations.

### Acknowledgement

The author would like to thank Mr. W. S. Yi and Ms. S. A. Cho for their valuable and friendly supports.

### References

- 1) Soreide, T., "Ultimate Load Analysis of Ma-

- rine Structures", Tapir Publishing Co., Trondheim, Norway, 1981
- 2) Wilson, E.L., "The Static Condensation Algorithm", International Journal for Numerical Methods in Engineering, Vol.8., 1974
- 3) Cho, K.N., "An Improved 2-dimensional Grillage method for Analysing Orthogonally Stiffened Plated Grillage Structures", Ph.D. Thesis, The University of Michigan, Ann Arbor, U.S.A., 1985
- 4) Hodge, P.G. "Post-Yield Behavior of a Beam with Partial End Fixity", Int. J. Mech. Sci., Vol.16., 1974
- 5) Hodge, P.G., "Plastic Analysis of Structures", McGRAW-Hill, New York, U.S.A., 1959
- 6) Cho, K.N., "Practical Collision Analysis of a Semi-submersible", The Third Int. Sym. on PRADS' 87, Trondheim, Norway, 1987



☆ 뉴

☆ 스

● 국제 학술대회 개최 안내 ●

제1차 실험 열전달, 유체역학 및 열역학 국제회의

—First World Conference on Experimental Heat Transfer,  
Fluid Mechanics and Thermodynamics—

주 관 : 미국 기계학회, 미국 화학공학회, 소련 열 및 물질전달학회, 일본 열전달학회, 일본화학공학회, 아시아 태평양 에너지, 열 및 물질전달 지역센터, 유고슬라비아 공학회

분 야 : 열전달, 유체역학 및 열역학의 모든 부분에 대한 실험, 이론, 해석 및 수치적 연구

일 시 : 1988년 9월 4~9일

장 소 : 유고슬라비아 두브로브릭