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Search Design of Resolution Ⅲ.2 for 3^n, n=4,5,6+

Jung-Koog Um*

ABSTRACT

The basic conditions for a parallel-flats fraction to be a search design of Resolution Ⅲ.2 have been developed in Um(1980, 1981, 1983, 1984). In this paper, a series of resolution Ⅲ.2 search designs for 3^n, n=4,5,6, are presented.

1. Introduction

A parallel-flats fraction is defined as

$$T = \bigcup_{i=1}^{f} \{ \underline{t} \, ; A\underline{t} = \underline{c}_i \}$$

where A is rxn of rank r. Each equation $A\underline{t} = \underline{c}_i$ has 3^{n-r} points and is called a flat. The f flats have no points in common, hence are termed parallel, with $|T| = f \cdot 3^{n-r}$. All of the designs constructed in this paper contain r=n-2, that is, flats of size 9. The parallel-flats fractions will be denoted symbolically by At=C, Where $C = (\underline{c}_1 \, \underline{c}_2 \cdots \underline{c}_f)$.

The choice of A determines the alias sets for the fraction. The estimate of an effect of the jth alias set from the ith flat, denoted by $\hat{S}_{ij}$, is actually a linear combination of all the factorial effects in that alias set. The form of the linear combination depends on c_i, and is characterized by the permutation relation of levels of each effect in the set to the identified effect S_{ij}. These relations are given in the alias component permutation matrix (ACPM). The elements of the ACPM are from the permutation subgroup {e,(012), (021)} and express the way levels of each effect are related to the effect identified as S_j. The element of the

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* Dept. of Computer Science, Sogang University, Seoul, Korea

ACPM for an effect E in the jth alias set for the ith flat can be computed from a single linear function of the elements of c_i, say $\delta(c_i)$. The correspondence is $\delta(c_i)= 0 \rightarrow e$, $\delta(c_i) = 1 \rightarrow (012)$, and $\delta(c_i)= 2 \rightarrow (021)$. The construction of a fraction is completed by the specification of A and C. For each design presented the matrices A and C are given along with the alias sets, the functions $\delta(c_i)$, and the ACPM matrices. For further details, see Srivastava(1975, 1976) and Um(1980).

The differences

$$\hat{S}^{x}_{ij}-\hat{S}^{x}_{i'j} \quad i<i'=1,2,\cdots f;\ j=1,2,3,4;\ x=1,2,$$

are given the value 1 if the difference is nonzero and 0 if the difference is zero. This produces an observed vector called a (0,1) detection vector. Each configuration of interactions gives rise to an expected (0,1) detection vector which can be obtained from the ACPM matrices (see Um(1983)).

From the ACPM matrices a (0,1) detection vector is obtained for each combination of two or fewer two-factor interactions. If there are n main effects, then there are $\binom{n}{2}$ two-factor interactions, say m. Therefore, $\binom{m}{0}+\binom{m}{1}+\binom{m}{2}$ (0,1) detection vectors have been constructed for each design. For the various values of n the following number of (0,1) detection vectors are obtained:

n	: 4	5	6	7	8	9
number of (0,1) detection vectors	: 22	56	121	232	407	667

An element of a (0,1) detection vector is determined form the difference between the ith row and i' row of the submatrix composed of the columns corresponding to the effects of interest for each ACPM P_j, j=1,2,3,4. If the difference is zero, then the corresponding element of the (0,1) detection vector is zero. If not, then the corresponding element is one. Suppose that there are f flats. Then there are $\binom{f}{2}$ differences for each ACPM and hence $\binom{f}{2}\cdot 4$ elements in a (0,1) detection vector.

2. Search Designs of Resolution Ⅲ. 2 for 3^4

The A-matrix selected for the 3^4 factorial is

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}.$$

The alias sets partitioned by the A-matrix are given by

$$S_0=\{\mu\},$$
$$S_1=\{F_1,\ F_2F_3,\ F_2F_4^2,\ F_3F_4\},$$
$$S_2=\{F_2,\ F_1F_3,\ F_1F_4,\ F_3F_4^2\},$$
$$S_3=\{F_3,\ F_1F_2,\ F_1F_4^2,\ F_2F_4\},$$
$$S_4=\{F_4,\ F_1F_2^2,\ F_1F_3^2,\ F_2F_3^2\}.$$

With the single flat $c=(c_1\ c_2)$ the defining vectors of ACPM where columns are associated with effects in the same ordering as listed in the alias sets are given by

$$C_1^*=(0\ 2c_1\ 2c_2\ c_1+c_2),\ C_2^*=(0\ 2c_1\ c_2\ c_1+2c_2),$$
$$C_3^*=(0\ 2c_1\ 2(c_1+c_2)\ 2c_1+c_2),\ C_4^*=(0\ 2c_2\ 2(c_1+c_2)\ c_1+2c_2).$$

There are only eight equivalence classes of C-matrix with two rows and three columns. The number of those classes are enumerated in Um(1981, 1984). In order to find a search design it is enough to consider only one element in each equivalence class. Table 2.1 shows a representative matrix from each equivalence class.

TABLE 2.1. Representative Matrix from Each Equivalence Class

Class 1	Class 2	Class 3	Class 4	Class 5	Class 6	Class 7	Class 8
$\begin{bmatrix}0&0&1\\0&1&2\end{bmatrix}$	$\begin{bmatrix}0&1&2\\0&0&1\end{bmatrix}$	$\begin{bmatrix}0&0&1\\0&1&0\end{bmatrix}$	$\begin{bmatrix}0&0&1\\0&1&1\end{bmatrix}$	$\begin{bmatrix}0&0&0\\0&1&2\end{bmatrix}$	$\begin{bmatrix}0&1&2\\0&0&0\end{bmatrix}$	$\begin{bmatrix}0&1&2\\0&1&2\end{bmatrix}$	$\begin{bmatrix}0&1&2\\0&2&1\end{bmatrix}$

Consider the representative element in each class from Table 2.1. Using C_1^*, the following alias component permutation matrices (ACPM) P_1 for the class 5, 6, 7, and 8 are obtained respectively:

$$P_1=\begin{bmatrix}e&e&e&e\\e&e&(021)&(012)\\e&e&(012)&(021)\end{bmatrix},\qquad P_1=\begin{bmatrix}e&e&e&e\\e&(021)&e&(012)\\e&(012)&e&(021)\end{bmatrix},$$

$$P_1=\begin{bmatrix}e&e&e&e\\e&(021)&(021)&(021)\\e&(012)&(012)&(012)\end{bmatrix},\qquad P_1=\begin{bmatrix}e&e&e&e\\e&(021)&(012)&e\\e&(012)&(021)&e\end{bmatrix},$$

Since the above matrices do not have a full rank, the C-matrices for classes 5, 6, 7, and 8 do not produce resolution Ⅲ. 2 search designs.

Consider $C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ in class 1, which produces the following ACPM.

$$P_1 = \begin{bmatrix} e & e & e & e \\ e & e & (021) & (012) \\ e & (021) & (012) & e \end{bmatrix}, \quad P_2 = \begin{bmatrix} e & e & e & e \\ e & e & (012) & (021) \\ e & (021) & (021) & (021) \end{bmatrix},$$

$$P_3 = \begin{bmatrix} e & e & e & e \\ e & e & (021) & (012) \\ e & (021) & e & (0(012) \end{bmatrix}, \quad P_4 = \begin{bmatrix} e & e & e & e \\ e & (021) & (021) & (021) \\ e & (012) & e & (021) \end{bmatrix}.$$

These matrices have a full rank and these ACPM produce the distinct(0,1) detection vectors for every combination of two or fewer two-factor interactions. Table 2.2 shows the (0,1) detection matrix produced by the ACPM. In table 2.2, the first row denotes the difference of ith row and i' row, and the second row represents the subscripts of ACPM. The column 4-8 denotes the subscripts of two-factor interactions. Therefore, the C-matrices in class 1 produce a resolution Ⅲ.2 search design. Similary, it can be shown that classes 2, 3, and 4 also produce search designs of resolution Ⅲ.2.

The treatment combinations from class 1 are obtained in flats of size nine by solution to

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}.$$

The treatment combinations are displayed below:

Flat 1	Flat 2	Flat 3
0 0 0 0	0 0 0 1	0 0 1 2
0 1 2 1	0 1 2 2	0 1 0 0
0 2 1 2	0 2 1 0	0 2 2 1

1 0 2 2	1 0 2 0	1 0 0 1
1 1 1 0	1 1 1 1	1 1 2 2
1 2 0 1	1 2 0 2	1 2 1 0
2 0 1 1	2 0 1 2	2 0 2 0
2 1 0 2	2 1 0 0	2 1 1 1
2 2 2 0	2 2 2 1	2 2 0 2

TABLE 2.2 The(0,1) Detection Matrix for the 3^4 Factorial

			1 - 2	1 - 3	2 - 3
			1 2 3 4	1 2 3 4	1 2 3 4
1	MAIN		0 0 0 0	0 0 0 0	0 0 0 0
2		12	0 0 0 1	0 0 1 1	0 0 1 1
3		13	0 0 0 1	0 1 0 0	0 1 0 1
4		14	0 1 1 0	0 1 0 0	0 1 1 0
5		23	0 0 0 1	1 0 0 1	1 0 0 0
6		24	1 0 1 0	1 0 1 0	1 0 0 0
7		34	1 1 0 0	0 1 0 0	1 0 0 0
8	12	13	0 0 0 1	0 1 1 1	0 1 1 1
9	12	14	0 1 1 1	0 1 1 1	0 1 1 1
10	12	23	0 0 0 1	1 0 1 1	1 0 1 1
11	12	24	1 0 1 1	1 0 1 1	1 0 1 1
12	12	34	1 1 0 1	0 1 1 1	1 0 1 1
13	13	14	0 1 1 1	0 1 0 0	0 1 1 1
14	13	23	0 0 0 1	1 1 0 1	1 1 0 1
15	13	23	1 0 1 1	1 1 1 0	1 1 0 1
16	13	34	1 1 0 1	0 1 0 0	1 1 0 1
17	14	23	0 1 1 1	1 1 0 1	1 1 1 0
18	14	24	1 1 1 0	1 1 1 0	1 1 1 0
19	14	34	1 1 1 0	0 1 0 0	1 1 1 0
20	23	24	1 0 1 1	1 0 1 1	1 0 0 0
21	23	34	1 1 0 1	1 1 0 1	1 0 0 0
22	24	34	1 1 1 0	1 1 1 0	1 0 0 0

3. Search Designs of Resolution Ⅲ.2 for 3^5 and 3^6

3.1. The 3^5 Factorial

The A-matrix selected for the 3^5 factorial is

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 1 \end{bmatrix}.$$

The alias sets partitioned by the A-matrix are given by

$$\begin{aligned} S_0 &= \{\mu,\ F_1F_3\}, \\ S_1 &= \{F_1,\ F_3,\ F_1F_3^2,\ F_2F_4,\ F_2F_5^2,\ F_4F_5\}, \\ S_2 &= \{F_2,\ F_1F_4,\ F_1F_5,\ F_3F_4^2,\ F_3F_5^2,\ F_4F_5^2\}, \\ S_3 &= \{F_4,\ F_1F_2,\ F_1F_5^2,\ F_2F_3^2,\ F_2F_5,\ F_3F_5\}, \\ S_4 &= \{F_5,\ F_1F_2^2,\ F_1F_4^2,\ F_2F_3,\ F_2F_4^2,\ F_3F_4\}. \end{aligned}$$

The single flat $C = (c_1\ c_2\ c_3)$ produces the following defining vectors of ACPM.

$$\begin{aligned} C_0^* &= (1\ \ c_1) \\ C_1^* &= (0\ \ 2c_1\ \ c_1\ \ 2c_2\ \ 2c_3\ \ c_1+c_3) \\ C_2^* &= (0\ \ 2c_2\ \ c_3\ \ c_1+2c_2\ \ 2c_1+c_3\ \ c_2+2c_3) \\ C_3^* &= (0\ \ 2c_2\ \ 2(c_2+c_3)\ \ c_1+2c_2\ \ 2c_2+c_3\ \ 2(c_1+c_2+c_3)\ ) \\ C_4^* &= (0\ \ 2c_3\ \ 2(c_2+c_3)\ \ c_1+2c_3\ \ c_2+2c_3\ \ 2(c_1+c_2+c_3)). \end{aligned}$$

With the C-Matrix

$$C = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix},$$

the Alias Component Permutation Matrices are given by

$$P_0 = \begin{array}{c} \\ \end{array}\begin{array}{ccc} \mu & F_1F_3 & (F_1F_3)^2 \\ \end{array}$$

$$P_0 = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \quad (\text{columns: } \mu,\ F_1F_3,\ (F_1F_3)^2)$$

$$P_1 = \begin{bmatrix} e & e & e & e & e & e \\ e & (021) & (012) & e & (021) & (012) \\ e & (012) & (021) & (021) & (021) & (021) \\ e & e & e & (012) & (012) & (012) \end{bmatrix} \quad (\text{columns: } F_1,\ F_3,\ F_1F_3^2,\ F_2F_4,\ F_2F_5^2,\ F_4F_5)$$

$$P_2 = \begin{bmatrix} e & e & e & e & e & e \\ e & e & (012) & (012) & e & (021) \\ e & (021) & (012) & (012) & (021) & e \\ e & (012) & (021) & (012) & (021) & e \end{bmatrix} \quad (\text{columns: } F_2,\ F_1F_4,\ F_1F_5,\ F_3F_4^2,\ F_3F_5^2,\ F_4F_5^2)$$

$$P_3 = \begin{bmatrix} e & e & e & e & e & e \\ e & e & (012) & (012) & (012) & (012) \\ e & (021) & (012) & (012) & e & (021) \end{bmatrix} \quad (\text{columns: } F_4,\ F_1F_2,\ F_1F_5^2,\ F_2F_3^2,\ F_2F_5,\ F_3F_5)$$

$$P_4 = \begin{bmatrix} e & e & e & e & e & e \\ e & (021) & (021) & e & (021) & (012) \\ e & (021) & (012) & (012) & e & (021) \\ e & (012) & (021) & (012) & e & (021) \end{bmatrix} \quad (\text{columns: } F_5,\ F_1F_2^2,\ F_1F_4^2,\ F_2F_3,\ F_3F_4^2,\ F_3F_4)$$

The following treatment combinations are obtained in flats of size 9.

Flat 1	Flat 2	Flat 3	Flat 4
0 0 0 0 0	0 0 1 0 1	0 0 2 1 1	0 0 0 2 2
0 1 2 2 1	0 1 0 2 2	0 1 1 0 2	0 1 2 1 0
0 2 1 1 2	0 2 2 1 0	0 2 0 2 0	0 2 1 0 1
1 0 2 2 2	1 0 0 2 0	1 0 1 0 0	1 0 2 1 1
1 1 1 1 0	1 1 2 1 1	1 1 0 2 1	1 1 1 0 2
1 2 0 0 1	1 2 1 0 2	1 2 2 1 2	1 2 0 2 0
2 0 1 1 1	2 0 2 1 2	2 0 0 2 2	2 0 1 0 0
2 1 0 0 2	2 1 1 0 0	2 1 2 1 0	2 1 0 2 1
2 2 2 2 0	2 2 0 2 1	2 2 1 0 1	2 2 2 1 2

Comments. Since F_1 and F_3 are aliased with each other in the alias set S_1, there are ten cases which produce identical (0,1) detection vectors. Those ten cases are main effects and F_1F_3, and all $\{F_i, F_j, (F_iF_j, F_1F_3)\}$ where $F_iF_j \neq F_1F_3$. The (0,1) detection matrix has all distinct vectors except for these ten cases.

For each case we have to check whether the submatrix obtained from ACPM is full rank or not. For example, consider F_1F_2 and (F_1F_2, F_1F_3). From P_1, P_3, and P_4 the following submatrices are obtained respectively.

$$
\begin{array}{ccc} F_1 & F_3 & F_1F_3^2 \end{array}
\qquad
\begin{array}{cc} F_4 & F_1F_2 \end{array}
\qquad
\begin{array}{cc} F_5 & F_1F_2^2 \end{array}
$$

$$
\begin{bmatrix} e & e & e \\ e & (021) & (012) \\ e & (012) & (021) \\ e & e & e \end{bmatrix}
\qquad
\begin{bmatrix} e & e \\ e & e \\ e & (021) \\ e & (012) \end{bmatrix},
\qquad
\begin{bmatrix} e & e \\ e & (021) \\ e & (021) \\ e & (012) \end{bmatrix}
$$

It is clear that these matrics are full rank. Similarly, it can be shown that for each case the submatrices obtained from ACPM are full rank.

3.2. The 3^6 Factorial

The A-matrix selected for the 3^6 factorial is

$$A=\begin{bmatrix}1&1&1&0&0&0\\1&2&0&1&0&0\\1&2&0&0&1&0\\1&1&0&0&0&1\end{bmatrix}.$$

The alias sets partitioned by the A-matrix are given by

$$\begin{aligned}
S_0 &= \{\mu,\ F_3F_6^2,\ F_4F_5^2\}\ ,\\
S_1 &= \{F_1,\ F_2F_3,\ F_2F_4^2,\ F_2F_5^2,\ F_2F_6,\ F_3F_4,\ F_3F_5,\ F_4F_6,\ F_5F_6\}\ ,\\
S_2 &= \{F_2,\ F_1F_3,\ F_1F_4,\ F_1F_5,\ F_1F_6,\ F_3F_4^2,\ F_3F_5^2,\ F_4F_6^2,\ F_5F_6^2\}\ ,\\
S_3 &= \{F_3,\ F_6,\ F_1F_2,\ F_1F_4^2,\ F_1F_5^2,\ F_2F_4,\ F_2F_5,\ F_3F_6\}\ ,\\
S_4 &= \{F_4,\ F_5,\ F_1F_2^2,\ F_1F_3^2,\ F_1F_6^2,\ F_2F_3^2,\ F_2F_6^2,\ F_4F_5\}\ .
\end{aligned}$$

The single flat $C=(c_1\ c_2\ c_3\ c_4)$ produce the following defining vectors of ACPM.

$$\begin{aligned}
C_0^* &= (1\ \ c_1+2c_4\ \ c_2+2c_3)\\
C_1^* &= (0\ \ 2c_1\ 2c_2\ 2c_3\ 2c_4\ c_1+c_2\ c_1+c_3\ c_2+c_4\ c_3+c_4)\\
C_2^* &= (0\ \ 2c_1\ c_2\ c_3\ 2c_4\ c_1+2c_2\ c_1+2c_3\ 2c_2+c_4\ 2c_3+c_4)\\
C_3^* &= (0\ \ 2c_1+c_4\ 2c_1\ 2(c_1+c_2)\ 2(c_1+c_3)\ 2c_1+c_2\ 2c_1+c_3\ c_1+2c_4)\\
C_4^* &= (0\ \ 2c_2+c_3\ 2c_2\ 2(c_1+c_2)\ 2(c_2+c_4)\ c_1+2c_2\ 2c_2+c_4\ c_2+2c_3)
\end{aligned}$$

With the C-Matrix

$$C=\begin{bmatrix}0&1&2&0&2\\0&1&1&2&0\\0&2&0&1&1\\0&0&1&1&2\end{bmatrix},$$

the Alias Component Permutation Matrices are given by

$$P_0=\begin{array}{c}\begin{matrix}\mu & F_3F_6^2 & (F_3F_6^2)^2 & F_4F_5^2 & (F_4F_5^2)^2\end{matrix}\\ \begin{bmatrix}1&-1&1&-1&1\\1&0&-2&0&1\\1&0&-2&0&-2\\1&1&1&0&-2\\1&-1&1&1&1\end{bmatrix}\end{array}$$

$$P_1=\begin{array}{ccccccccc} F_1 & F_2F_3 & F_2F_4^2 & F_2F_5^2 & F_2F_6 & F_3F_4 & F_3F_5 & F_4F_6 & F_5F_6 \\ \hline e & e & e & e & e & e & e & e & e \\ e & (021) & (021) & (012) & e & (021) & e & (012) & (021) \\ e & (012) & (021) & e & (021) & e & (021) & (021) & (012) \\ e & e & (012) & (021) & (021) & (021) & (012) & e & (021) \\ e & (012) & e & (021) & (012) & (021) & e & (021) & e \end{array}$$

$$P_2=\begin{array}{ccccccccc} F_2 & F_1F_3 & F_1F_4 & F_1F_5 & F_1F_6 & F_3F_4^2 & F_3F_5^2 & F_4F_6^2 & F_5F_6^2 \\ \hline e & e & e & e & e & e & e & e & e \\ e & (021) & (012) & (021) & e & e & (021) & (021) & (012) \\ e & (012) & (012) & e & (021) & (012) & (021) & e & (012) \\ e & e & (021) & (012) & (021) & (012) & (021) & (021) & e \\ e & (012) & e & (012) & (012) & (021) & (012) & (021) & (012) \end{array}$$

$$P_3=\begin{array}{cccccccc} F_3 & F_6 & F_1F_2 & F_1F_4^2 & F_1F_5^2 & F_2F_4 & F_2F_5 & F_3F_6 \\ \hline e & e & e & e & e & e & e & e \\ e & (021) & (021) & (012) & e & e & (012) & (012) \\ e & (021) & (012) & e & (012) & (021) & (012) & (012) \\ e & (012) & e & (012) & (021) & (021) & (012) & (021) \\ e & e & (012) & (012) & e & (012) & (021) & e \end{array}$$

$$P_4=\begin{array}{cccccccc} F_4 & F_4 & F_1F_2^2 & F_1F_3^2 & F_1F_6^2 & F_2F_3^2 & F_2F_6^2 & F_4F_5 \\ \hline e & e & e & e & e & e & e & e \\ e & (012) & (021) & (012) & (021) & e & (021) & (021) \\ e & (021) & (021) & e & (012) & (012) & e & (012) \\ e & (021) & (012) & (012) & e & (012) & (021) & (012) \\ e & (012) & e & (012) & (012) & (021) & (021) & (021) \end{array}.$$

The following treatment combinations are obtained in flats of size 9.

Flat 1	Flat 2	FLat 3	Flat 4	Flat 5
000000	001120	002101	000211	002012
012112	010202	911219	912929	911121
021221	022011	020022	021102	020200
102222	100012	010120	102100	101201
111001	112121	110102	111212	110010
120110	121200	122211	120021	122122
201111	202201	200212	201022	200120
210220	211010	212021	210101	212202
222002	220122	221100	222210	221011

Comments. The (0,1) detection vector for main effects is identical to the (0,1) detection vectors for F_3F_6 and for F_4F_5. This implies that the (0,1) detection for main effects is also identical to the (0,1) detection vector for (F_3F_6, F_4F_5). The (0,1) detection vector for F_iF_j where $F_iF_j \neq F_3F_6$ and for $F_iF_j \neq F_4F_5$, is identical to the (0,1) detection vecotrs for $(F_iF_j F_3F_6)$ and for (F_iF_j, F_4F_5). Therefore, there are 29 cases partitioned into 14 sets which produce the identical (0,1) detection vectors. The (0,1) detection matrix has all distinct vectors except for these 29 cases. It can be verified that for each partition the submatrices obtained from ACPM is full rank.

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