Probabilistic Analysis of Slope Stability for Progressive Failure

進行性 破壞에 대한 斜面安定의 確率論的 解析

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要 旨

균질토 사면에서 진행성 파괴에 대한 확률론적 모델이 제시되었다. 파괴면 위의 어떤 절편에 대한 국부적인 Safety Margin은 정규분포라 가정하였다. 파괴면을 따라 존재하는 전단강도의 불확실성은 1차원 Random Field Models로 표현되었다. 이 연구에서는 파괴가 Toe에서 시작되어 사면정상까지 진행되는 경우만을 고려하였다. 파괴면위의 어느 두 인접 절편의 Safety Margin의 Joint Distribution은 Bivariate Normal로 가정하였다. 활동파괴의 전체적인 파괴확률은 일련의 Conditional events의 곱으로 표현되었다. 최종적으로 개발된 절차가 절취사면의 신뢰도를 얻기 위하여 한 예에 적용되었다.

Abstract

A probabilistic model for the progressive failure in a homogeneous soil slope consisting of strain-softening material is presented. The local safety margin of any slice above failure surface is assumed to follow a normal distribution. Uncertainties of the shear strength along potential failure surface are expressed by one-dimensional random field models. In this paper, only the case where failure initiates at toe and propagates up to the crest is considerd. The joint distribution of the safety margin of any two adjacent slices above the failure surface is assumed to be bivariate normal. The overall probability of the sliding failure is expressed as a product of probabilities of a series of conditional events. Finally, the developed procedure has been applied in a case study to yield the reliability of a cut slope.

1. Introduction

The Probabilistic study of soil slopes has been most widely performed in the field of geotechnical reliability in recent years[4,10,12,14]. Most of the previous study is based on the limit equiliblium method[1,8], and the assumption that peak shear strength operates over the whole of slip surface at the instant of complete failure along a potential failure surface. However, due

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to stress concentration at some location of slip surface, deformation will not be uniform along the surface. Failure always starts at one location and then extends progressively to other areas [8,10]. Even when there are no external signs of failure, certain regions within a soil mass may already be overstressed, and propagation of failure may have begun. Hence, the peak strength will not be reached at the all points of the potential failure surface at the same times, At some points, the strength will be governed by the residual strength due to excessive deformation that had occured[6,7,9]. Failure may start at the one extremity of a slip surface and progress towards the other extremity; it may start somewhere within the slip surface and propagate in either direction, i.e., towards the crest or the toe of the slope[6,7]. It has been suggested that long term failures are likely to propagate from the ends of a slip surface (crest or toe) while short term, undrained failures of clay slopes may develop in the interior and then propagate outwards[11].

In earlier reliability analysis of soil slopes, the shape of the failure surface has been assumed to be a circle and a $\phi=0$ condition has been considered in a probabilistic model of failure propagation based on a bivariate normal distribution of the safety margin of the overlapping segments of a potential failure surface[9,11].

In this paper, only one mode is treated, namely the case where failure initiates at the lower extremity and propagates up wards along a slip surface. The shape of the failure is assumed to be a log-spiral curve, and a drained condition is adopted to explain the notion of the present model.

2. Random Field Model of Soil Properties

On the base of measured point properties, it is necessary to infer the statistics of the spatial average property. The discrepancy in soil property between two points in a stratum is expected to increase as the two points become further apart. Random field modeling of the soil properties in a homogeneous stratum provides a mathematically tractable tool to bridge this gap[13]. Using a one-dimensional random field model, the mean and variance of the spatial average soil property over length L can be expressed as Fig. 1.

$$\bar{U}_L=\bar{U}$$

$$\hat{U}_L^2=\left[\frac{2}{L}\int_0^L\left(1-\frac{\tau}{L}\right)\rho_u(\tau)\,d\tau\right]\hat{U}^2=\Gamma_u^2(L)\,\hat{U}^2 \qquad (1)$$

where the model parameters $\bar{U}$, $\hat{U}$, and $\rho_u(\tau)$ are respectively the mean, standard deviation and

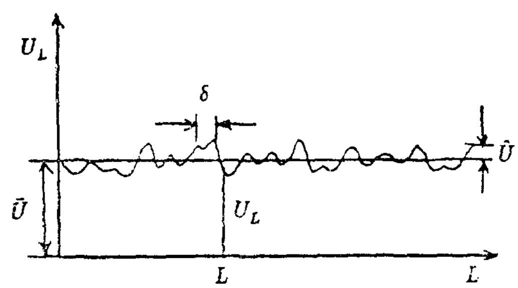

Fig. 1. One-dimensional random field model

correlation coeffcient between soil properities at distance τ apart and $\Gamma_u^2(L)$ is the variance function whose value range between 0 and 1.

3. Probability of Local Failure

The shape of the failure surface is assumed to be a log-spairal (Fig. 2) the exact location of which depends on the ϕ-parameter of strength and two polar coordinates of its center (h_0, θ_0)[2,3]. The statistacal values of h_0 and θ_0 are functions of the height h and angle β of the slope. In this paper, both h_0 and θ_0 are taken to be random variables following a general beta distribution. The mean values, variances and lower and upper limits of h_0 and θ_0 depend on the boundary geometry of the slope. The total length of the failure surface is expressed as a fuction of three factors (h_0, θ_0, t) and is equal to

$$L=\frac{h_0}{\cos\theta_0}\left(1+\frac{1}{t^2}\right)^{1/2}(e^{\theta_H t}-1) \tag{2}$$

where $t=\tan\phi$. In Fig. 3 are shown the static forces which act on a differential element dL of the failure surface. If dx is the width of a slice, the weight dW is equal to

$$dW=\gamma_m(z-w)dx+\gamma'_m w\,dx \tag{3}$$

where γ_m is the unit weight of the soil and γ'_m is its submerged unit weight, w is the height of the water above the failure surface, and z is the vertical distance of the failure surface from the slope boundary. The elevation of the ground water table is specified by the parameter R_u defined as

$$R_u=\frac{u}{\gamma_m z} \tag{4}$$

where u is the pore water pressure at the failure surface. The normal and tangent components of the weight dW are

$$\begin{aligned} dN&=dW\cos\epsilon \\ dT&=dW\sin\epsilon \end{aligned} \tag{5}$$

where the angle which the failure surface makes with horizontal ϵ is equal to

$$\epsilon=\frac{\tan(\theta-\theta_0)-\tan\phi_p}{\tan(\theta-\theta_0)\tan\phi_p+1} \tag{6}$$

The driving force dS and the resistance force dR acting along the element dL of the failure surface are equal to

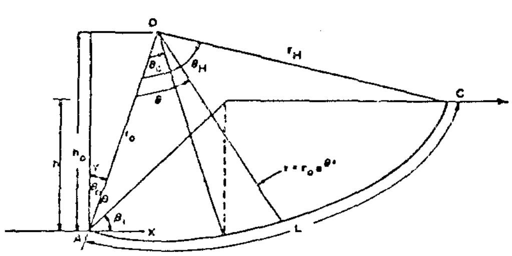

Fig. 2. Shape of rupture surface

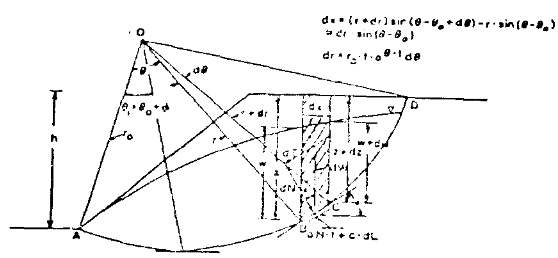

Fig. 3. Forces on a differential element along the rupture surface

$$dS = dT$$
$$dR = dNt + cdL \tag{7}$$

where t and c are the strength parameters of the soil. The overall driving force S and resisting force R are found through a numerical integration of Equation 7. The numerical values of the probability of failure of the soil slope are found by using program MDNOR. A flow chart that leads to the critical failure surface is shown in Fig. 4. It is assumed that the soil mass above the critical slip surface is subdivided into a number of vertical slices(Fig. 3). The base of each slices is thus a segment of potential slip surface. The normal and tangential components of the weight of the ith slice W_i are

$$N_i = W_i \cos \epsilon_i$$
$$T_i = W_i \sin \epsilon_i \tag{8}$$

The capacity, C_i (shearing resistance), and the demand D_i (shear force), along ith slice of the slip surface is givened by

$$C_i = N_i t + cL_i$$
$$D_i = W_i \sin \epsilon_i \tag{9}$$

The safety margin SM_i of the ith slice is

$$SM_i = C_i - D_i \tag{10}$$

The probability of local failure is equal to

$$P_{fi} = P[SM_i \leq 0] = P[(C_i - D_i) \leq 0] \tag{11}$$

In order to dertermine this probability, the probability density function of the safety margin is assumed to be a normal distribution.

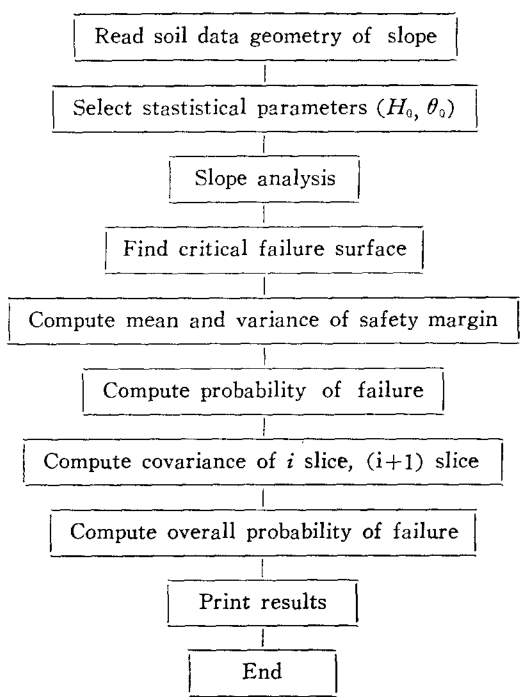

Fig. 4. Risk analysis of slopes simplified flow chart.

using one dimensional random field model, the mean and variance of safety margin of slice i is

$$\overline{SM_i}=\bar{c}_p L_i+N_i(\tan\bar{\phi}_p)\,T_i \qquad (12)$$

$$\mathrm{Var}(SM_i)=L_i^2[\Gamma_p^2(L_i)\delta_p^2+\Delta_p^2]\bar{c}_p^2 + N_i^2(\tan\bar{\phi}_p)^2[\Gamma_{p1}^2(L_i)\delta_{p1}^2+\Delta_{p1}^2]\left(\frac{\partial\tan^2\phi_p}{\partial\phi_p}\right)^2 \qquad (13)$$

where $\bar{c}_p$, δ_p and Δ_p are respectively point mean, point coefficient of variation and systematic modeling error of cohesion (peak shear strength). $\bar{\phi}_{p1}$ δ_{p1} and Δ_{p1} are respectively those of friction angle (peak shear strength). Γ's is one-dimensional variance function. In this paper, a triangular correlation function is used.

The triangular correlation function that decreases linearly from 1 to 0 as $|\tau|$ goes from 0 to δ;

$$\rho_u(\tau)=\begin{cases}1-\dfrac{|\tau|}{\delta}, & |\tau|\leq\delta,\\ 0 & |\tau|\geq\delta.\end{cases}$$

Its variance function is

$$\Gamma_j^2=\begin{cases}1-\dfrac{L_j}{3\delta} & L_j\leq\delta\\ \dfrac{\delta}{L_j}\left(1-\dfrac{L_j}{3\delta}\right) & L_j>\delta\end{cases} \qquad (14)$$

where j stands for p and $p1$.

$$P[SM_i\leq 0]=\int_{-\infty}^{0} f_{SM_i}(SM)\,dSM=F_{SM_i}(0) \qquad (15)$$

where f_{SM_i} and F_{SM_i} are the normal and cumulative normal distribution respectively.

4. Probabilistic Model of Progressive Failure

The resistance of the first i slice after failure is assumed to be governed by the average residual strength over the entire length from slice 1 to i, whereas the shear resistance of the $(i+1)$th slice is assumed to be governed by the average peak strength over the length L_{i+1}[11]. The total shear stress applied to the $(i+1)$th slice is the summation of the component shear stress T from slice to$(i+1)$. Using one-dimensional random field model, the mean and variance of safety margin of slice i is

$$\overline{SM_i}=\bar{c}_p L_i+N_i(\tan\bar{\phi}_p)+\bar{c}_r L_{1,i-1}+N_{1,i-1}(\tan\bar{\phi}_r)-T_{1,i} \qquad (16)$$

$$\begin{aligned}\mathrm{Var}(SM_i)=&L_i^2[\Gamma_p^2(L_i)\delta_p^2+\Delta_p^2]\bar{c}_p^2 \\ &+N_i^2(\tan\bar{\phi}_p)^2[\Gamma_{p1}^2(L_i)\delta_{p1}^2+\Delta_{p1}^2]\left(\frac{\partial\tan^2\phi_p}{\partial\phi_p}\right)^2 \\ &+L_{1,i-1}^2[\Gamma_r^2(L_{1,i-1})\delta_r^2+\Delta_r^2]\bar{c}_r^2 \\ &+N_{1,i-1}^2(\tan\bar{\phi}_r)^2[\Gamma_{r1}^2(L_{1,i-1})\delta_{r1}^2+\Delta_{r1}^2]\left(\frac{\partial\tan^2\phi_r}{\partial\phi_r}\right)^2\end{aligned} \qquad (17)$$

where $\bar{c}_r$, δ_r and Δ_r are respectively point mean, point coefficient of variation and systematic modeling error of cohesion (residual shear strength). $\bar{\phi}_r$, δ_{r1} and Δ_{r1} are respectively those of friction angle (residual shear strength), and $\Gamma_j^2(L)$ is one-dimensional variance function (a triangular correlation function). where j stands for p and r, respectively. The correlation coefficient between SM_i and SM_{i+1} may be calculated as follows

$$r_{SM_i, SM_i+1} = \frac{B1+B2}{S_{SM_i} S_{SMi+1}} \tag{18}$$

where $B1$ and $B2$ can be expressed:

$$\begin{aligned} B1 &= [r_{cp_i, cp_i+1} \Gamma_p(L_i) \Gamma_p(L_{i+1}) \delta_p^2 (1+\Delta_p^2) + \Delta_p^2] \bar{c}_p^2 L_i L_{i+1} \\ &\quad + [r_{(\tan\phi p)_i, (\tan\phi p)_{i+1}} \Gamma_{p1} L_i) \Gamma_{p1}(L_{i+1}) \delta_{p2}^1 (1+\Delta_{p1}^2) + \Delta_{p1}^2] \\ &\quad (\tan\bar{\phi}_p)^2 N_i N_{i+1} \\ B2 &= [r_{c_{1,i}, c_{r1,i-1}} \Gamma_r(L_{1,i}) \Gamma_r(L_{1,i-1}) \delta_r^2 (1+\Delta_r^2) + \Delta_r^2] \bar{c}_r^2 L_{1,i} L_{1,i-1} \\ &\quad + [r^)_{\tan\phi_r)_{1,i}, (\tan\phi_r)_{1,i-1}} \Gamma_{r1}(L_{1,i}) \Gamma_{r1}(L_{1,i-1}) \delta_{r1}^2 (1+\Delta_{r1}^2) + \Delta_{r1}^2] \\ &\quad (\tan\bar{\phi}_r)^2 N_{1,i} N_{1,i-1} \end{aligned} \tag{19}$$

It is assumed that there is no modeling error in c_p or c_r and ϕ_p or ϕ_r. By defining T_i, $i=1$ to n, as the event that failure progresses up to and including the ith slice and F_i, $i=1$ to n, as the event of failure of the ith slice, and using the definition of the conditional probability[5], the probability of failure along the failure surface is given by

$$\begin{aligned} P(T_n) &= P(T_{n-1} F_n) \\ &= P(F_n | T_{n-1}) P(T_{n-1}) \\ &= P(F_n | T_{n-1}) P(F_{n-1} | T_{n-2}) \cdots P(F_2 | T_1) P(F_1) \end{aligned} \tag{20}$$

A concise form of Equation 20 can be expressed as

$$P(T_n) = \begin{cases} \prod_{i=1}^{n-1} P(T_i F_{i+1}) / \prod_{i=2}^{n-1} P(T_i) & n>2 \\ P(T_1 F_2) & n=2 \end{cases}$$

Equation 20 may be rewritten as

$$P(T_n) = \prod_{i=1}^{n-1} P(SM_i \leq 0 \cap SM_{i+1} \leq 0) / \prod_{i=2}^{n-1} P(SM_i \leq 0) \tag{21}$$

for the case $n>2$.

The safety margin of different slices is a function of the same random variables. Therefore, the safety margin of adjacent slices are not independent of each other and their joint distribution must be considered. (Fig. 5, 6)[5,9]. As SM_i, $i=1, 2, \cdots, n$, is normally distributed and the joint distribution of SM_i and SM_{i+1} is bivariate normal.

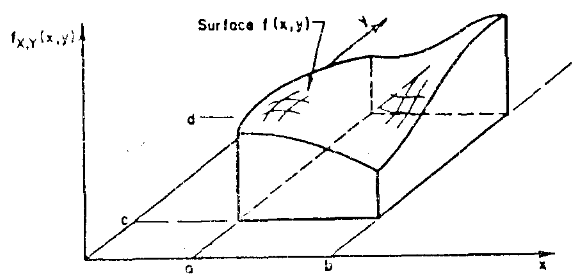

Fig. 5. Joint PDF of X and Y

The numerator of Equation 21 is equal to the cumulative bivariate normal distribution evaluated at zero, i.e.,

$$\begin{aligned} P[SM_i \leq 0 \text{ and } SM_{i+1} \leq 0] &= \int_{-\infty}^{0} \int_{-\infty}^{0} f_{SM_i, SM_{i+1}}(SM_i, SM_{i+1}) dSM_i dSM_{i+1} \\ &= F_{SM_i, SM_{i+1}}(0, 0) \end{aligned} \tag{22}$$

where

$$f_{SM_i,SM_{i+1}}(SM_i,\ SM_{i+1})=\frac{1}{2\pi S_{SM_i}S_{SM_{i+1}}\sqrt{1-r^2}}$$

$$\exp\left\{\frac{1}{-2(1-r^2)}\left[\left(\frac{SM_i-\overline{SM}_i}{S_{SM_i}}\right)-2r\left(\frac{SM_i-\overline{SM}_i}{S_{SM_i}}\right)\left(\frac{SM_{i+1}-\overline{SM}_{i+1}}{S_{SM_{i+1}}}\right)+\left(\frac{SM_{i+1}-\overline{SM}_{i+1}}{S_{SM_{i+1}}}\right)^2\right]\right\} \tag{23}$$

in which r denotes $r_{SM_i,SM_{i+1}}$ and $-\infty<SM_i<\infty<SM_i<\infty$; $-\infty<SM_{i+1}<\infty$; $S_{SM_i}>0$; $S_{SM_{i+1}}>0$; $-1\leq r\leq 1$; $-\infty<\overline{SM}_i<\infty$; $-\infty<\overline{SM}_{i+1}<\infty$

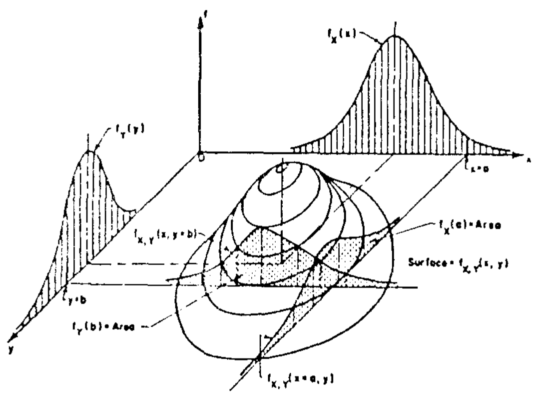

Fig. 6. Joint and marginal PDF of continuous random variables X and Y

5. Illustrative Example

An analysis with respect to probability of failure is performed on a statistically homogeneous soil slope having a H=15m and an angle $\beta=40°$. The center of the surface is expressed in polar coodinates by means of two random valiables, h_0 and θ_0 (Fig. 2).

The unit weight of soil (γ_m) is 19KN/m^3 and its submerged unit weight (γ_m') is 9.2KN/m^3.

The pore water pressure ratio (R_u) is 0.27.

The mean values, variances and lower and upper limits of h_0 and θ_0 are givened in Table 1. The statistical values of peak and residual strength parameters are given in Table 2. First, the critical failure surface is determined using a one-dimensional homogeneous random field model of soil shear resistances and program MDNOR The analysis with respeet to probability of failure is performed on the critical failure surface. Probabilities of local failure path initiated at the toe are calculated respectively for the case where the failure surface is subdivided into 13 slices and for δ=1, 3, 5, 7, 9, and 11 respectively. It is clear from Table 3 that as δ increase probabilities of local failure increase.

As shown Table 4, the coefficient of correlation of any two adjacent slices decreases as δ increases.

As shown in Table 5, the overall probability of failure highly increases in case of progressive failure than in case of nonprogressive failure as the δ increases.

Table 1. Location of center.

	h_0(m)	θ_0(°)
Mean	15	−1
Standard deviation	5	9
Maximum	30	−29
Minimum	0	0

Table 2. Statistical values of strength parameters.

Statistical parameter	Soil strength			
	Peak		Residual	
	ϕ	C	ϕ	C
Mean	20°	23KN/m²	10°	13KN/m²
C.O.V.	0.1	0.4	0.1	0.4
Systematic error	0.08	0 3	0.08	0.3
Correlation parameter (δ (m))	1	1	1	1
	3	3	3	3
	5	5	5	5
	7	7	7	7
	9	9	9	9
	11	11	11	11

Table 3. Probability of local failure

Slice number	Probability of local failure					
	$\delta=1$	3	5	7	9	11
1	2.67×10^{-3}	6.98×10^{-3}	8.13×10^{-3}	8.65×10^{-3}	8.94×10^{-3}	9.13×10^{-3}
2	1.16×10^{-3}	2.91×10^{-3}	3.37×10^{-3}	3.58×10^{-3}	3.70×10^{-3}	3.78×10^{-3}
3	1.24×10^{-3}	3.20×10^{-3}	3.72×10^{-3}	3.95×10^{-3}	4.09×10^{-3}	4.17×10^{-3}
4	3.19×10^{-3}	7.18×10^{-3}	8.17×10^{-3}	8.61×10^{-3}	8.85×10^{-3}	9.01×10^{-3}
5	1.29×10^{-2}	2.32×10^{-2}	2.54×10^{-2}	2.64×10^{-2}	2.70×10^{-2}	2.73×10^{-2}
6	5.22×10^{-2}	7.43×10^{-2}	7.86×10^{-2}	8.04×10^{-2}	8.14×10^{-2}	8.20×10^{-2}
7	1.55×10^{-1}	1.84×10^{-1}	1.89×10^{-1}	1.92×10^{-1}	1.93×10^{-1}	1.94×10^{-1}
8	3.07×10^{-1}	3.28×10^{-1}	3.31×10^{-1}	3.33×10^{-1}	3.34×10^{-1}	3.34×10^{-1}
9	4.26×10^{-1}	4.35×10^{-1}	4.37×10^{-1}	4.37×10^{-1}	4.38×10^{-1}	4.38×10^{-1}
10	4.52×10^{-1}	4.58×10^{-1}	4.59×10^{-1}	4.60×10^{-1}	4.60×10^{-1}	4.60×10^{-1}
11	3.01×10^{-1}	3.25×10^{-1}	3.29×10^{-1}	3.30×10^{-1}	3.31×10^{-1}	3.32×10^{-1}
12	9.67×10^{-2}	1.29×10^{-1}	1.36×10^{-1}	1.38×10^{-1}	1.40×10^{-1}	1.41×10^{-1}
13	1.75×10^{-2}	3.40×10^{-2}	3.79×10^{-2}	3.96×10^{-2}	4.06×10^{-2}	4.12×10^{-2}

Table 4. Correleation coefficient of two adjacent slice.

Slice number	Correlation coefficient					
	$\delta=1$	3	5	7	9	11
1						
	0.68	0.65	0.64	0.64	0.64	0.64
2						
	0.81	0.77	0.76	0.76	0.76	0.76
3						
	0.85	0.80	0.79	0.78	0.78	0.78
4						
	0.88	0.82	0.80	0.80	0.79	0.79
5						
	0.90	0.84	0.81	0.81	0.80	0.80
6						
	0.92	0.86	0.83	0.81	0.81	0.80
7						
	0.94	0.87	0.84	0.82	0.81	0.81
8						
	0.95	0.89	0.85	0.83	0.82	0.81
9						
	0.95	0.90	0.86	0.84	0.83	0.82
10						
	0.96	0.90	0.87	0.85	0.84	0.83
11						
	0.96	0.91	0.88	0.86	0.84	0.83
12						
	0.97	0.92	0.88	0.87	0.85	0.84
13						

Table 5. Overall probability of failure

	$\delta=1$	3	5	7	9	11
Progressive failure	1.16×10^{-4}	5.17×10^{-3}	4.38×10^{-2}	1.20×10^{-1}	2.04×10^{-1}	2.81×10^{-1}
Non-progressive failure	1.83×10^{-2}	2.62×10^{-2}	3.38×10^{-2}	4.10×10^{-2}	4.75×10^{-2}	5.34×10^{-2}

6. Conclusion

Progression model of soil slope consisting of strain-softening material has been developed using a probabilistic segment-by-segment failure model, in which the uncertainties of peak and residual soil shear strength along potential failure surface are expressed by one-dimensional random field model. This study concludes the following:

1. In practice, there is always a probability of local failure.
2. Coefficients of correlation of any two adjacent slices in cut slope were all high, within a relatively narrow range and decrease as δ increases.
3. The probability of failure of the entire failure surface is shown to increase as the correlation parameter increases.

Acknowledgement

Thanks are due to the Korea Science and Engineering Foundation for its financial support in the preparation of this paper.

Additional thanks are also due to Dr. Achintya Haldar in G.I.T. who have assisted in the completion of this paper.

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(접수일자 1988. 4. 21)