Application of Linearization Method for Large-Scale Structure Optimizations

-구조물 최적화를 위한 선형화 기법-

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요 약

반복 비선형 계획법의 하나인 선형화 기법을 절대수렴의 전제아래 합성 구조물의 최적 설계에 응용한다. 선형화 기법은 설계문제의 제약조건을 선형화된 등가 제약조건으로 변형시키며 active-set 정책을 구사한다. 결과, 매 설계단계에서 풀어야 할 상태 및 수반 방정식의 수를 줄임으로써 실질적인 계산의 절감을 기한다. 기둥으로 받쳐진 판-보 구조물은 최적화 기법의 능력을 시험키 위한 합성구조물의 좋은 예로서, 설계결과 선형화 기법은 만족할만한 수렴치로써 최적해를 산출함을 알 수 있고 나아가 이 방법은 모든 종류의 최적화 문제에 적용될 수 있을 것으로 보인다.

Abstract

The linearization method as one of the recursive quadratic programming method is applied for the optimal design of a large-scale structure supported by Pshenichny's proof of global convergence of the algorithm and convergence rate estimates. The linearization method transforms all constraints of the design problem into an equivalent linearized constraint and employs the active-set strategy. This results in substantial computational savings by reducing the number of state and adjoint equations to be solved at every design iteration. The illustrative example of plates with beams supported by columns is the typical one of a large-scale structure to test the capability of the optimization algorithm. The linearization method among many is shown to give successful optimum solutions with satisfactory convergence criteria. Hopefully, the method may be applicable to all classes of optimization problems.

INTRODUCTION

The linearization method is one of the mathematical programming methods for solving extremal problem (constrained optimization problem) developed by Pshenichny[1) 2)] for engineering design optimization. There are many optimization algorithms in mathematical programming methods in which each

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algorithm has merits and demerits. In this respect, successful optimal design heavily depends on the choice of optimization algorithm. The strategy to select an appropriate optimization algorithm among many is based on accuracy of the result, rate of convergence, required computing time, computer storage required, and compatibility with structural analysis methods. Gradient method as one of the mathematical programming method is known to be suitable to a large-scale structure optimizaton especially when the exact gradients of the cost and constraint functions are available. The theoretical and numerical computation of the design derivatives of those functions, called design sensitivity analysis, is one of the difficult parts in structure optimization since most of the constraints are implicit functions of design variables.

The purpose of this paper is to introduce and apply the linearization method of Pshenichny together with design sensitivity analysis for optimal design of a large-scale structural system. Here a large-scale structure implies a relatively large and complex structure on which many design variables and constraints may be imposed. Pshenichny has proved convergence of the algorithm, using an active-set strategy that is essential in large-scale structure. Even though there are some other recursive quadratic programming methods[3) 4)] with proofs of global convergence, they require computing derivatives of all constraints of the problem at every iteration. This is prohibitively expensive for structural design problems since each design derivative evaluation requires multiple solutions of state and adjoint equations.[5)] The linearization method is shown to be particularly attractive in utilization of design sensitivity analysis technique and appears to be powerful for all classes of problems.

OPTIMAL DESIGN PROBLEM AND DESIGN SENSITIVITY ANALYSIS

To present ideas of the linearization method as it applies to the large-scale structural system optimization, a generalized mathematical formulation of the design problem is considered.

The optimal design problem is defined as follows:

Minimize $\psi_0(b, z)$ (1)

subject to

$$K(b)z = S(b) \tag{2}$$

$$\psi_i(b, z) \leq 0, \quad i=1, 2, \cdots, m \tag{3}$$

where b is a design variable vector, z is nodal displacement vector, K(b) is a structural stiffness matrix (symmetric and positive definite), and S(b) is an applied load vector. The functions $\psi_i(b,z)$ represent constraints(stress, displacement and others) and $\psi_0(b,z)$ represents cost function for the design problem.

<u>Design Sensitivity Analysis of Finite Dimensional Structures</u>

Representing the cost and constraint functions as ψ in general, the total derivative of ψ with respect to b is written as

$$\frac{d\psi}{db} = \frac{\partial \psi}{\partial b} + \frac{\partial \psi}{\partial z}\frac{dz}{db} \tag{4}$$

Differentiating both sides of Eq. 2 with respect to b yields

$$K(b)\frac{dz}{db} = -\frac{\partial}{\partial b}(K(b)\tilde{z}) + \frac{\partial S(b)}{\partial b} \tag{5}$$

where ~ indicates a variable that is to be held constant for the process of partial differentiation. Instead of solving Eq. 5 for dz/db and substituting the result into Eq. 4 to obtain the desired result, one may solve an adjoint equation

$$K(b)\lambda = \frac{\partial \psi^T}{\partial z} \tag{6}$$

for λ to obtain

$$\frac{d\psi}{db} = \frac{\partial \psi}{\partial b} + \frac{\partial}{\partial b}[\tilde{\lambda}^T S(b) - \tilde{\lambda}^T K(b)\tilde{z}] \tag{7}$$

Consider the eigenvalue formulation for natural frequency described by

$$K(b)y = \zeta M(b)y \quad (8)$$

where ζ is the eigenvalue, M(b) is the positive definite mass matrix, and y is the normalized eigenvector of

$$y^T M(b) y = 1 \quad (9)$$

Here, K(b) and M(b) are presumed to be differentiable with respect to design. Following the design sensitivity analysis of a simple(non-repeated) eigenvalue[6], the desired result is

$$\frac{d\zeta}{db} = \frac{\partial}{\partial b}[\bar{y}^T K(b)\bar{y}] - \zeta \frac{\partial}{\partial b}[\bar{y}^T M(b)\bar{y}] \quad (10)$$

Note here that no adjoint equation is necessary.

Design Sensitivity Analysis of Distributed Parameter Structures

Recently developed design sensitivity analysis method[6] for distributed parameter structures such as beams, plates and shells employs the variational approach where the infinite dimensional function spaces of displacements z and designs u are associated.

The general variational formulation is written as

$$a(z, \bar{z}) = \ell(\bar{z}), \quad \text{for all } \bar{z} \in Z \quad (11)$$

where $a(z, \bar{z})$ is the energy bilinear form, $\ell(\bar{z})$ is the load linear form, and Z is the set of kinematically admissible displacements. Each structural type has its own $a(z, \bar{z})$ and $\ell(\bar{z})$.

Consider a measure of structural performance written in integral form as

$$\psi = \int_{\Omega} g(z, \nabla z, \nabla^2 z, u)\, d\Omega \quad (12)$$

where ∇z and $\nabla^2 z$ represent the first and second derivatives of displacements z, respectively, and the function g is continuously differentiable with respect to its arguments. Following the procedure of design sensitivity analysis for static response [6], the explicit design sensitivity of Eq. 12 becomes

$$\psi' = \int_{\Omega} g_u\, \delta u\, d\Omega + \ell'(\lambda) - a'(z, \lambda) \quad (13)$$

where primes(′) denote the explicit design derivatives. Here the adjoint variables λ is obtained by solving the following adjoint equation

$$a(\lambda, \bar{\lambda}) = \int_{\Omega} [g_z \bar{\lambda} + g_{\nabla z} \nabla \bar{\lambda} + g_{\nabla^2 z} \nabla^2 \bar{\lambda}]\, d\Omega \quad \text{for all } \bar{\lambda} \in Z \quad (14)$$

Similarly, the eigenvalue design sensitivity may be obtained as[6]

$$\zeta' = a'(y, y) - \zeta d'(y, y) \quad (15)$$

where the bilinear form a(.,.) is the same as occurred in static response and the bilinear form d(.,.) represents mass effects in vibration(kinetic energy bilinear form).

LINEARIZATION METHOD OF PSHENICHNY

Together with the basic assumptions, theoretical and numerical algorithms of the linearization method are presented without proofs. Pshenichny has shown that the algorithm converges after a finite number of iterations for linear problems and that in the general nonlinear case, the algorithm converges at a geometric rate and quadratic rate.

General mathematical programming is to find $b \in R^n$ to minimize $f_0(b)$, with constraints

$$\left.\begin{array}{ll} f_i(b) \le 0, & i = 1, 2, \cdots, m' \\ f_i(b) = 0, & i = m'+1, \cdots, m \end{array}\right\} \quad (16)$$

where f_i, i=0, 1, ⋯, m, are continuously differentiable functions.

Assumptions: Let

$$F(b) = \max\{0, f_1(b), \cdots, f_m(b)\} \quad (17)$$

Note that $F(b) \ge 0$ for all $b \in R^n$

Given $\delta \ge 0$, define the active constraint set

$$A(b,\delta)=\{i : f_i(b)\geq F(b)-\delta, i=1,2,\cdots, m\} \quad (18)$$

(a) Suppose there is $N>0$ such that the set

$$\Omega_N=\{x : \Phi_N(b)\leq\Phi_N(b^\circ)\} \quad (19)$$

is bounded, where b° is an initial design and $\Phi_N(b)=f_0(b)+NF(b)$.

(b) Suppose gradients of functions $f_i(b)$, $i=0, 1, 2, \cdots, m$, satisfy Lipschitz conditions in Ω_N ;i.e., there exists $L>0$ such that

$$\| f_i'(b^1)-f_i'(b^2)\|\leq L\| b^1-b^2\|,\quad b^1, b^2\in\Omega_N \quad (20)$$

where $f_i'=[\partial f_i/\partial b_1, \cdots, \partial f_i/\partial b_n]^T$ is the design sensitivity vector and $\|b\|=(\sum_{i=1}^{n} b_i^2)^{1/2}$.

(c) Suppose the problem of quadratic programming ; find $p\in R^n$ to minimize

$$(f_0'(b),\ p)+1/2\|p\|^2 \quad (21)$$

subject to the linearized constraints

$$(f_i'(b),\ p)+f_i(b)\leq 0,\quad i\in A(b,\delta) \quad (22)$$

is solved with any $b\in\Omega_N$ and there are Lagrange multipliers $u_i(b)$, $i=A(b,\delta)$, such that

$$\sum_{i\in A(b,\delta)} u_i(b)\leq N \quad (23)$$

Theoretical Algorithm: Given $0<\varepsilon<1$, for the kth iteration,

(1) Solve the quadratic programming problem of Eqs. 21 and 22 with $b=b^k$ and solution $p^k=p(b^k)$.

(2) Find the smallest integer i such that

$$\Phi_N(b^k+\frac{1}{2^i}p^k)\leq\Phi(b^k)-\frac{1}{2^i}\varepsilon\|p^k\|^2 \quad (24)$$

If this inequality is satisfied with $i=i_0$, let $\alpha_k=2^{-i_0}$ and $b^{k+1}=b^k+\alpha_k p^k$.

It is advantageous[1] to solve the dual of the quadratic programming problem, which has the form

$$\phi(u)=\min[(f_0'(b),\ p)+1/2\|p\|^2+\sum_{i\in A(b,\delta)}u_i(f_i'(b),p)+f_i(b))] \quad (25)$$

Equating the derivatives with respect to p of the right side of Eq. 25 to zero, the minimum is attained with

$$p=-f_0'(b)-\sum_{i\in A(b,\delta)}u_i f_i'(b) \quad (26)$$

Hence p is uniquely determined by u_i, $i\in A(b,\delta)$, substituting Eq. 26 into Eq. 25 yields

$$\phi(u)=\frac{1}{2}\| f_0'(b)+\sum_{i\in A(b,\delta)}u_i f_i'(b)\|^2+\sum_{i\in A(b,\delta)}u_i f_i(b) \quad (27)$$

The dual problem is now to maximize $\phi(u)$ with constraints $u_i\geq 0$, $i\in A(b,\delta)$. The value of the maximum of the objective functions in the dual problem is the minimum of the objective function of the quadratic programming problem of Eqs. 21 and 22.

In basic assumptions, it was supposed that the quadratic programming problem of Eqs. 21 and 22 is solvable with any $b\in\Omega_N$. When one solves the dual of the quadratic programming problem, the weak duality theorem[7] says that if $\sup\phi(u)=\infty$, then the quadratic programming problem is insolvable.

Numerical Algorithm: The following algorithm is intended for solving the problem of minimizing $f_0(b)$ subject to condition of Eq. 16.

Define

$F(b)=\max\{0, f_1(b), \cdots, f_{m'}(b), |f_{m'+1}(b)|, \cdots, |f_m(b)|\}$

$A(b,\delta)=\{i: f_i(b)\geq F(b)-\delta,\ i=1, 2, \cdots, m'\}$

$B(b,\delta)=\{i: |f_i(b)|\geq F(b)-\delta,\ i=m'+1, \cdots, m\}$

$\Phi_N(b)=f_0(b)+NF(b)$

Select the initial aproximation b^0, N_0 sufficiently large, $\delta_0>0$, and $0<\varepsilon<1$.

Step 1. In the kth iteration, solve the problem of minimizing

$$\phi(u)=\frac{1}{2}\| f_0'(b^k)+\sum_{i\in A(b^k,\delta)\cup B(b^k,\delta)} u_i f_i'(b^k)\|^2+\sum_{i\in A(b^k,\delta)} u_i f_i(b^k)$$

subject to $u_i \geqq 0$, $i\in A(b^k,\delta)$, and u_i arbitrary for $i\in B(b^k,\delta)$.

If the solution u^k is such that $\phi(u^k)=-\infty$, then set $b^{k+1}=b^k$, $\delta_{k+1}=1/2\,\delta_k$, and $N_{k+1}=N_k$ and return to step 1. Otherwise, let

$$p^k=-f_0'(b^k)-\sum_{i\in A(b^k,\delta)\cup B(b^k,\delta)} u_i^k f_i'(b^k) \tag{28}$$

and go to step 2.

Step 2. Set

$$b^{k+1}=b^k+\alpha_k p^k$$
$$\delta_{k+1}=\delta_k$$

where α_k is chosen equal to $\frac{1}{2^{q_0}}$ and q_0 is the smallest integer for which

$$\Phi_{N_k}(b^k+\frac{1}{2^q}p^k)\leqq\Phi_{N_k}(b^k)-\frac{1}{2^q}\varepsilon\|p^k\|^2$$

Step 3. If

$$2\Big(\sum_{i\in A(b^k,\delta)} u_i^k+\sum_{i\in B(b^k,\delta)}|u_i^k|\Big)\geq N_k>\sum_{i\in A(b^k,\delta)} u_i^k+\sum_{i\in B(b^k,\delta)}|u_i^k|$$

then let $N_{k+1}=N_K$. Otherwise, let

$$N_{k+1}=2\Big(\sum_{i\in A(b^k,\delta)} u_i^k+\sum_{i\in B(b^k,\delta)}|u_i^k|\Big)$$

Step 4. If $\|p^k\|$ is sufficiently small, terminate. Otherwise return to Step 1.

NUMERICAL EXAMPLE

Optimal design of a complex structure that consists of beams and plates supported by columns as shown in Fig. 1 is considered using the classical small-deflection theory.

Define Ω_1^{ij}, Ω_2^{ij} and Ω_3^{ij} as domains of plate, longitudinal beam, and transverse beam conponents, respectively as shown in Fig. 1. The superscripts i and j are also used to identify the design and state variables defined in the corresponding domains.

The design variables (Fig. 1), in vector form, are

$$u\equiv(t^{ij},\tilde{d}^{ij},\tilde{h}^{ij},\hat{d}^{ij},\hat{h}^{ij},A_k) \tag{29}$$

In vector form, the state variables (displacements) are

$$z\equiv(w^{ij},\tilde{v}^{ij},\tilde{\theta}^{ij},\hat{v}^{ij},\hat{\theta}^{ij},q_k) \tag{30}$$

where w is the deflection of plate, $\tilde{v}(\hat{v})$ is the deflection of longitudinal (transverse) beam, $\tilde{\theta}(\hat{\theta})$ is the slope of longitudinal (transverse) beam, and q is the deflection of column.

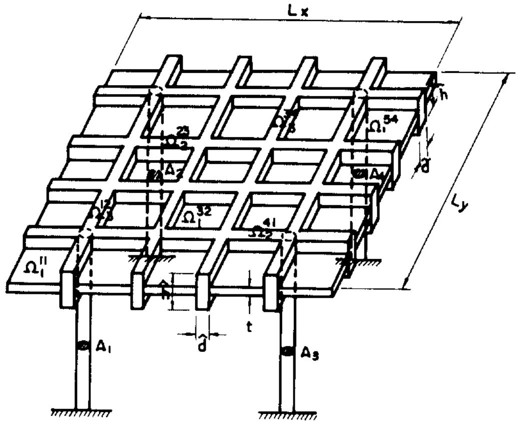

Fig. 1. Structural System and Design Variables.

The displacements z of Eq. 30 satisfy the kinematic boundary conditions such as same displacements and slopes of plates & beams at the interfaces, and zero displacements of columns at the ground supports.

Optimal design problem is formulated as to minimize the cost function (volume of entire structure)

$$\psi_0=\sum_i^5\sum_j^5\iint_{\Omega_1^{ij}} t^{ij}\,d\Omega+\sum_i^5\sum_j^4\int_{\Omega_2^{ij}}\tilde{d}^{ij}\tilde{h}^{ij}\,d\Omega$$
$$+\sum_i^4\sum_j^5\int_{\Omega_3^{ij}}\hat{d}^{ij}\hat{h}^{ij}\,d\Omega_3+\sum_k^4(A_k\ell_k) \tag{31}$$

subject to the following constraints.

Displacement Constraint;

$$\psi_1=\Big|\iint_{\Omega_1^{ab}}\hat{\delta}(x-\hat{x})w\,d\Omega\Big|-z^a\leq 0 \tag{32}$$

where $\hat{x}\in\Omega_1^{ab}$, is a fixed point, $\hat{\delta}(x)$ is the dirac measure in the plane acting at the origin, and z^a is the maximum allowable value.

Stress Constraint on Plate Element:

$$\psi_2 = \iint_{\Omega_1^{ab}} \phi^{ab}(\sigma) M_p d\Omega - \sigma_p^a \leq 0 \quad (33)$$

where M_p is the characteristic function defined on plate element and σ_p^a is a given allowable yield stress and

$$\phi(\sigma) = (\sigma_{xx}^2 + \sigma_{yy}^2 + 3\tau_{xy}^2 - \sigma_{xx}\sigma_{yy})^{1/2} \quad (34)$$

is Von-Mises yield stress[8].

Stress Constraint on Beam Element:

$$\psi_3 = \int_{\Omega_{2(3)}^{ab}} \sigma_b M_b d\Omega - \sigma_b^a \leq 0 \quad (35)$$

where σ_b is the bending stress, M_b is the characteristic function defined on beam element, and σ_b^a is given allowable stress.

Eigenvalue Constraint:

$$\psi_4 = \zeta_0 - \zeta \leq 0 \quad (36)$$

where $\zeta = \omega^2$ is the computed smallest eigenvalue, ω is the natural frequency, and ζ_0 is the lower bound.

Design Variable Bounds:

$$u^\ell \leq u \leq u^u \quad (37)$$

where the superscripts ℓ and u denote lower and upper bounds, respectively.

Results and Discussion:

A finite dimensional optimization method[5] is employed for the present problem to match the accuracy of the finite element structural analysis, where the non-conforming method for plate bending is used[9]. To solve the static and eigenvalue equations, symbolic factorization technique[10] is used to take advantage of sparsity of the global stiffness and mass matrices of the structure. The subspace iteration method[11] is employed for solving the eigenvalue problem. The design sensitivity analysis method introduced early in this paper is applied for the computation of design derivative of the constraints formulated using the adjoint variable method.

The input data used are: elastic modulus $E=3\times10^7$ psi⟨2.07×10^5 MPa⟩, Poisson's ratio $\nu=0.3$, the overall dimension=15 in×15 in ⟨38.1cm×38.1 cm⟩, uniform thickness of plate element t=0.1 in ⟨0.254 cm⟩, uniform height(width) of beam element h=0.5 in ⟨1.27cm⟩ (d=0.15 in ⟨0.38cm⟩), cross-sectional area (length) of column element A=0.4 in² ⟨2.58cm²⟩ (ℓ=4.93 in ⟨12.5cm⟩), equal spacing of beams=3 in ⟨7.62 cm⟩, uniformly distributed load f=0.1 lb/in²⟨689.5Pa⟩, mass density $\rho=0.1$ lb/in³ ⟨0.0271N/m³⟩, the lower and upper bounds $u^\ell=0.8u_0$ and $u^u=1.2u_0$, respectively, and allowable bounds $z^a=0.0006$ in ⟨1.524×10^{-3}cm⟩,

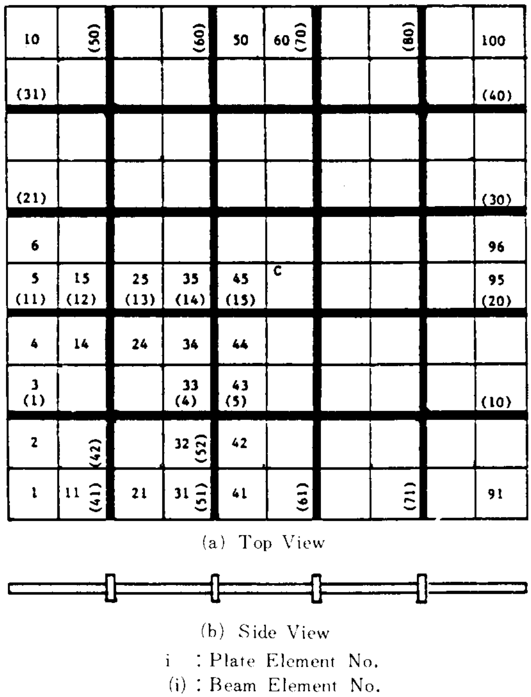

Fig.2. Finite Element Model of a Plate-Beam Structure.

σ_p^a=100 psi⟨0.689 MPa⟩, σ_b^a=400 psi ⟨2.76 MPa⟩, and ζ_0=800(rad/sec)².

Finite element structural analysis is carried out with finite element models of Fig. 2 in which a total of 184 finite elements and 363 degrees of freedom are used to model the structure, including 100 rectangular plate elements, 80 beam elements, and 4 column elements. Analysis results are checked by the finite element package program SPAR and show good agreements up to 3 significant digits.

The numerical accuracy of the design sensitivity terms is critical in the gradient optimization algorithm for successful optimal design. Hence, it is checked

Table 1. Design Sensitivity Agreements

Censtraint o		ψ_1	$\Delta\psi$	$\delta\psi$	$\delta\psi/\Delta\psi$ (%)
Diaplace-ment	C	0.4775E-03	-0.8052E-04	-0.9071E-04	112.7
Stress on plate element	1	0.1484E 02	-0.7100E 00	-0.6750E 00	95.1
	2	0.5829E 02	-0.5980E 01	-0.6780E 01	113.4
	3	0.5263E 02	-0.5220E 01	-0.5810E 01	111.3
	4	0.5256E 02	-0.5760E 01	-0.6320E 01	109.7
	5	0.8497E 02	-0.1028E 02	-0.1126E 02	109.5
	11	0.5829E 02	-0.5980E 01	-0.6780E 01	113.4
	12	0.6780E 02	-0.7870E 01	-0.8630E 01	109.7
	13	0.5827E 02	-0.6720E 01	-0.7580E 01	112.8
	14	0.5269E 02	-0.6240E 01	-0.6830E 01	109.5
	15	0.7658E 02	-0.9360E 01	-0.1034E 02	110.5
	21	0.5263E 02	-0.5220E 01	-0.5810E 01	111.3
	22	0.5827E 02	-0.6720E 01	-0.7580E 01	112.8
	23	0.5450E 02	-0.6690E 01	-0.7300E 01	109.1
	24	0.5850E 02	-0.6990E 01	-0.8060E 01	115.3
	25	0.6155E 02	-0.7740E 01	-0.8500E 01	109.8
	31	0.5256E 02	-0.5760E 01	-0.6320E 01	109.7
	32	0.5269E 02	-0.6240E 01	-0.6830E 01	109.5
	33	0.5850E 02	-0.6990E 01	-0.8060E 01	115.3
	34	0.4697E 02	-0.6030E 01	0.6340E 01	105.1
	35	0.4621E 02	-0.5880E 01	-0.6770E 01	115.1
	41	0.8497E 02	-0.1028E 02	-0.1126E 02	109.5
	42	0.7658E 02	-0.9360E 01	-0.1034E 02	110.5
	43	0.6155E 02	-0.7740E 01	-0.8500E 01	109.8
	44	0.4621E 02	-0.5880E 01	-0.6770E 01	115.1
	45	0.3975E 02	-0.5250E 01	-0.5980E 01	113.9
Stress on beam element	1	0.2956E 02	-0.3640E 01	-0.3960E 01	108.8
	2	0.1850E 03	-0.2428E 02	-0.2672E 02	110.0
	3	0.1200E 03	-0.1608E 02	-0.1764E 02	109.7
	4	0.2041E 03	-0.2552E 02	-0.2792E 02	109.4
	5	0.3549E 03	-0.4444E 02	-0.4872E 02	109.6
	11	0.1656E 02	-0.2360E 01	-0.2520E 01	106.8
	12	0.6312E 02	0.7920E 01	-0.8680E 01	109.6
	13	0.2192E 02	-0.2400E 01	-0.2640E 01	110.0
	14	0.7964E 02	-0.1088E 02	-0.1192E 02	109.6
	15	0.1454E 03	-0.1960E 02	-0.2140E 02	109.2
Eigenvalue		0.1242E 04	0.2408E 03	0.2199E 03	91.3

by comparing the predictions (sensitivity evaluations) with the actual changes of constraints after design modification. Table 1 shows the design sensitivity agreements for 5% uniform changes of all design variables, where ψ_1 is the constraint values at initial design, $\Delta\psi$ is the actual changes of constraint values after design modification, $\delta\psi$ is the predictions, and $\delta\psi/\Delta\psi$ is the sensitivity agreements. Results in Table 1 show good agreements of 91-115% for all constraints considered. This accuracy looks more than adequate for iterative design.

The gradient projection method, which is known to be the most powerful one among gradient

Table 2. Optimal Design Results Under Displacement, Stress, Eigenvalue Constraints

		Initial	Final			Initial	Final
Plate thick-ness (×0.1)	(1)	1.0000	0.80008	Beam width	(1)	0.1500	0.12000
	(2)	–	0.80009		(2)	–	0.12000
	(3)	–	0.80010		(3)	–	0.12000
	(4)	–	0.80010		(4)		0.12001
	(5)	–	0.80010		(5)		0.12001
	(11)		0.80009		(11)		0.12000
	(12)		0.80009		(12)		0.12000
	(13)		0.80009		(13)	–	0.12000
	(14)		0.80009		(14)	–	0.12000
	(15)		0.80009		(15)	0.1500	0.12000
	(21)		0.80010	Beam height	(1)	0.5000	0.40005
	(22)		0.80009		(2)	–	0.43199
	(23)		0.80009		(3)	–	0.40005
	(24)		0.80009		(4)		0.50015
	(25)		0.80008		(5)		0.59996
	(31)		0.80010		(11)		0.40005
	(32)		0.80009		(12)		0.40004
	(33)		0.80009		(13)	–	0.40004
	(34)		0.80010		(14)	–	0.40008
	(35)		0.80005		(15)	0.5000	0.40986
	(41)	–	0.80010	Column	(1)	0.4000	0.32004
	(42)	–	0.80009		(2)	–	–
	(43)	–	0.80008		(3)	–	–
	(44)	–	0.80005		(4)	0.4000	0.32004
	(45)	1.0000	0.80009				

*Quarter of whole design variables due to symmetry

methods, and other methods (even non-gradient methods) have been applied to this problem resulting in lots of computing time and failures of convergences.

The solution in Table 2 shows the successful optimal design obtained by the application of the linearization method. The cost is reduced from 41.68 to 32.40, which is 22.3% reduction, while L-2 norm of the direction vector as a convergence criteria is reduced from 34.69 to 0.791×10^{-3} after 17 iterations. The plate thickness, cross-sectional area of column, and the beam width tend to approach the lower bound, while the optimum design is obtained by controlling the beam height. In this case, the outer beams that are close to the free edges have the characteristics of building up the beam height, particularly conspicuous around the center of the beam. Additionally, when the optimum solution in Table 2 is obtained, the stress constraints on plate elements 5 and 6, the stress constraints on beam elements (5) and (45) (from quarter of entire structure in Fig. 2) become tight as implied, and they play crucial roles in determining the optimum distribution of design variables. Initially, if the beam height is not too large compared with the plate thickness, different characteristics of smoother distribution of both plate thickness and beam height as optimum design are expected.

CONCLUSIONS

Most of large-scale structure optimization problems involve many constraints written implicitly in terms of design variables, and hence system state and adjoint equation must be solved in design sensitivity analysis at every iteration. The linearization method essentially denotes all constraints of the design problem into an equivalent linearized constraint. The beauty of the method is in its active set strategy and fast convergence rate in which substantial computational savings are achieved by minimizing the number of state and adjoint equations to be solved in each design iteration.

The large-scale structure illustrated consists of the finite dimensional and distributed parameter structures where the design sensitivity analysis methods are different. The example problem treated requires use of adjoint variable design sensitivity analysis methods, illustrating compatibility of the linearization method with this approach to structural system design. The inherent approximation error caused by using distributed parameter structural theory can be mitigated by applying the linearization method. Potential of the linearization method has been proved to be applicable to a variety of structural optimization problems.

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