

## Dust Shells around OH/IR Stars

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### Abstract

We have made new models for mass-losing OH/IR stars to explain the properties of the dust shells around them using more accurate information about the material in the shell and the physical processes including pulsations. We have applied our dust opacity which has been deduced from observations and experimental data to various density distributions, calculated the model emergent spectra, and compared with observations. Contrary to previous suggestions, we could fit observations fairly well using density distribution  $\rho \propto r^{-2}$ , which is physically plausible, with proper choice of opacities. The time scales for dust formation, growth, and movement are calculated to be compared with pulsation periods. The change of the emergent spectrum depending on the phase of pulsation can be explained fairly well by changing dust condensation radius (for fixed dust condensation temperature) in step with the change in stellar luminosity. The effects of stellar wind models and pulsation models on dust shells with attention to emergent spectra are discussed.

### I. Introduction

OH/IR stars are losing their masses at rates of  $10^{-7} - 10^{-4} M_{\odot}/\text{yr}$  (e.g. Herman & Habing 1985). The mechanism for the mass loss seems to be some combination of pulsation-driven shock waves and radiation pressure on dust (e.g. Willson & Bowen 1985). In the atmospheres of OH/IR stars, the shock waves generated by large amplitude pulsations push the material outward. As the material flows outward, the temperature becomes cool enough for the grains to condense, and radiation pressure on them pushes the material further out.

OH/IR stars show strong OH line emission at  $\lambda = 18 \text{ cm}$  and the maximum of their spectral energy distribution lies in the infrared, typically around  $\lambda = 10 \mu\text{m}$ . OH/IR stars are considered to be an extension toward older age, longer periods, thicker dust shells and enhanced mass loss of the optically known Mira stars. The emission or absorption features around  $9.7 \mu\text{m}$  and  $18.6 \mu\text{m}$  are due to circumstellar silicate dust. They are generally weak emission features at early M (Miras),

progressively strengthening to strong emission features at around M6(dust thick Miras), and appear as strong absorption features for some late M type stars with thick dust shells(OH/IR stars). In this paper we shall develop a more coherent picture for the dust shells around OH/IR stars with proper attention to emergent spectra.

## II. Optical Properties of Dust

Dust grains around stars scatter and absorb the light from central stars. And they emit radiation at wavelengths much greater than those of the absorbed light. Knowledge of the optical properties of grains can be used to interpret the observed effects in terms of the nature(composition, sizes and shape) and spatial distribution of these particles.

Generally one has the following relationship for radiative transfer problems in circumstellar shells:

$$\mu \frac{\partial I_\nu(r, \mu)}{\partial r} + \frac{1 - \mu^2}{r^2} \cdot \frac{\partial I_\nu(r, \mu)}{\partial \mu} = -\kappa_\nu^a(r) \cdot \{ I_\nu(r, \mu) - B_\nu(r) \} \\ - \kappa_\nu^s(r) \cdot \{ I_\nu(r, \mu) - \frac{1}{2} \cdot \int_{-1}^{+1} I_\nu(r, \mu') p_\nu(\mu, \mu') d\mu' \} \dots\dots\dots [1]$$

where  $I_\nu(r, \mu)$  is specific intensity of radiation with frequency  $\nu$  at point  $r$  and in direction  $\mu$ ,  $B_\nu(r)$  is Planck function,  $\kappa_\nu^a(r)$  is absorption coefficient,  $\kappa_\nu^s(r)$  is scattering coefficient,  $p_\nu(\mu, \mu')$  is the phase function which determines the probability of scattering from direction  $\mu'$  into the direction  $\mu$  for photons with frequency  $\nu$  and  $\mu$  is  $\cos \theta$  ( $\theta$  is the angle between the incident wave and the scattered wave). The opacity  $\kappa_\nu$  is defined by:

$$\kappa_\nu = N_g \cdot Q_\nu \cdot \pi \cdot a^2 \dots\dots\dots [2]$$

where  $N_g$  is number density of dust grain,  $a$  is the radius of grain.  $Q_{\text{ext}, \nu}$  is the extinction efficiency factor which has two terms:

$$Q_{\text{ext}} = Q_{\text{sca}} + Q_{\text{abs}} \dots\dots\dots [3]$$

where  $Q_{\text{sca}}$  is the efficiency factor for scattering and  $Q_{\text{abs}}$  is the efficiency factor for absorp-

tion. We may define the efficiency for radiation pressure  $Q_{pr}$  by:

$$Q_{pr} = Q_{ext} - G \cdot Q_{sca} \dots\dots\dots [4]$$

where  $G = \langle \cos \theta \rangle$  is the anisotropy factor.

Optical efficiency factors are calculated by using the Mie theory. We used a computer subroutine program which has been developed by Bohren & Huffman(1983) and modified by us to include the anisotropy factor( $G$ ) which is useful for calculating radiation pressure efficiency factors. For given characteristics of the dust material, namely optical constants at given wavelength and the size distribution of the grains, optical efficiencies and opacities are calculated. The opacities for various types of size distribution are calculated. We used grain material density  $\rho_g = 3.3 \text{ g}\cdot\text{cm}^{-3}$  for silicate.

*\* The choice of the opacity pattern for astronomical silicate*

One way to find an accurate opacity is to solve the radiative transfer problem and compare with observations when we know the appropriate stellar and dust shell characteristics. After many trials, we have found a good opacity pattern for the dust shells around OH/IR stars which fits better with observations, but does not stray too far from the known characteristics of silicate materials.

Figure 1 shows the opacity patterns of our choices(type 1, type 2) and Draine's(1985) for

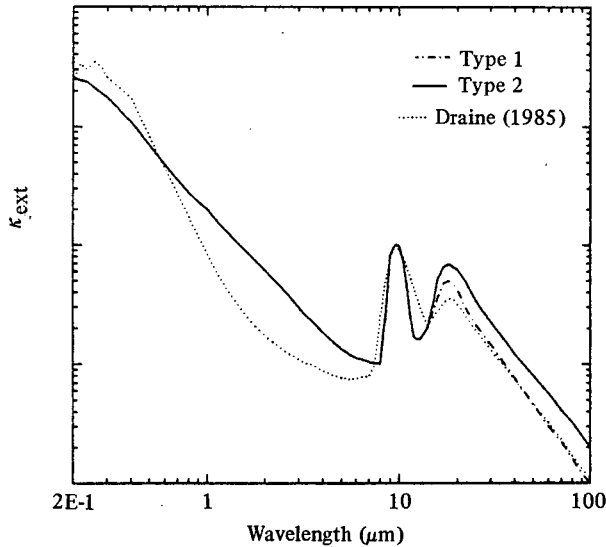


Fig. 1. Relative Opacity patterns

comparison. In modelling Miras and OH/IR stars, we found that the opacity pattern for Miras (type 1) should be different from that for OH/IR stars (type 2). Table 1 summarizes the procedure. Figure 2 shows the optical constants that we adopted (a: Type 1, b: Type 2).

Table 1. The procedure for obtaining the optical constants

$\lambda(\mu\text{m})$	n	k
0.2-8	1.55	
8-30	Day(1979) for $\text{Mg}_2\text{SiO}_4$	Deduced from fitting
30-100	Day(1981) for $\text{Mg}_2\text{SiO}_4$	

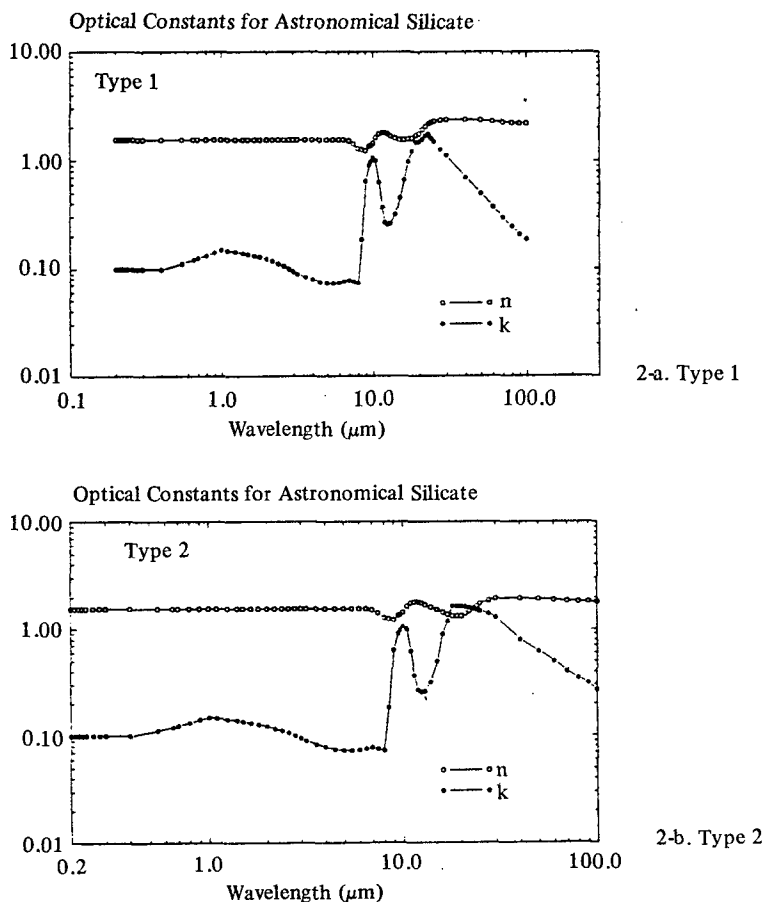


Fig. 2. Optical Constants for Silicate

The reason why we had to use a different opacity pattern for the stars with thicker dust shells may be found from experiments. Day(1976a, b) found that extinction efficiency is dependent on temperature. We summarize the results as follows:

Temperature (K)	100	200	400
$Q_{\text{ext}}(20\mu\text{m})/Q_{\text{ext}}(10\mu\text{m})$	0.72	0.63	0.5

The reason for these changes should be due to the change of optical constants depending on temperature(different grain sizes can not be a possibility). OH/IR stars have more cold material than Miras, and that could be the reason for having characteristics at cold temperature.

### III. Dust Formation in Stellar Winds

Most of the heavy elements in interstellar medium with only a few exceptions(e.g. noble gases, sulphur) are almost completely condensed into small solid state particles. The main sources of interstellar grains are considered to be cool, luminous mass-losing stars like OH/IR stars. The nuclei of grains are formed in such hotter and denser environments associated with continuous or sporadic mass loss events during stellar evolution. These stable condensation nuclei, once mixed into the interstellar medium(ISM), collect the available material in the ISM not yet condensed into grains.

#### *\* Silicate Features*

The 10  $\mu\text{m}$  feature is interpreted as stretching vibrations of the Si-O bond(Woolf and Ney 1969) while the 18  $\mu\text{m}$  feature is interpreted as bending vibration of the bond angle in a O-Si-O group(Treffers and Cohen 1974). The material is believed not to be in a crystalline but in a highly disordered amorphous state. But the structure and chemical composition of the condensate are not well known because the two features only refer to a special active group with a broadly smeared out resonance which is present in many different materials.

#### *\* Silicate Condensation*

In a typical orthosilicate the silicon atom is fourfold singly bounded to oxygen atoms. In a solid state the  $(\text{SiO}_4)^{--}$  and  $\text{Mg}^{++}$  and  $\text{Fe}^{++}$  ions are held together by the Coulomb interaction. The  $\text{Mg}^{++}$  and  $\text{Fe}^{++}$  ions arbitrarily can be replaced or interchanged within the lattice, forming a continuous series of materials  $\text{Fe}_x\text{Mg}_{2-x}\text{SiO}_4$  (Olivine) with the two extreme compositions  $\text{Mg}_2\text{SiO}_4$  (Forsterite) and  $\text{Fe}_2\text{SiO}_4$  (Fayalite). We used the optical constant  $n$  for  $\text{Mg}_2\text{SiO}_4$ .

Silicate can be formed as a result of a chemical modification of a different condensate by reac-

tions with molecules from gas phase(SiO, H<sub>2</sub>O, Mg, MgS, Fe) or by growth processes on the surface of nuclei of a different nature. Gail and Sedlmayr(1987, 1986) conclude that dust formation in early type M-star is initiated by SiO clustering while in late type M-stars it is initiated by MgS clustering.

*\* Formation and growth of dust grains in OH/IR stars*

Classical nucleation theory describes the formation of critical nuclei in a supersaturated vapor by means of thermodynamic quantities. There exists a critical cluster size  $N_*$  separating the regions of small unstable clusters from the region of stable supercritical clusters which grow to macroscopic sizes. The rate of formation of grains is determined by the transition rate  $J$  between both regions.

Grain formation process is determined by the equation of grain growth and the equation of consumption of vapor. We take the time  $t=0$  when the shell reaches the equilibrium point. Adopting Yamamoto and Hasegawa's treatment(1977), the equations of grain growth and the vapor consumption are given by:

$$\frac{da}{dt} = \alpha_s \Omega \langle v \rangle n(t), \dots\dots\dots [5]$$

the monomer concentration is described by:

$$C_1(t) = C_1(0) - \int_0^t J(t') \frac{4\pi}{3\Omega} a^3(t, t') dt', \dots\dots\dots [6]$$

where  $\alpha_s$  is sticking probability,  $\Omega$  is the volume of vapor molecule,  $\langle v \rangle$  is mean speed at which the vapor molecules collide with a grain,  $n(t)$  is the concentration of vapor molecules,  $J(t)$  is steady state nucleation rate,  $a(t, t')$  is the grain radius measured at  $t$  which nucleated at  $t'$ . The nucleation rate  $J(t)$  is defined by:

$$J(t) = \alpha_s \Omega \left(\frac{2\sigma}{\pi m}\right)^{1/2} C_1(t)^3 \exp\left\{-\frac{4\mu^2}{27(\ln S(t))^2}\right\} \dots\dots\dots [7]$$

where  $S(t)$  is supersaturation ratio and  $\sigma$  is the surface tension of condensate. And  $\mu$  is defined by:

$$\mu = \frac{4\pi\sigma a_0^2}{kT} \approx \frac{4\pi\sigma a_0^2}{kT_e} \dots\dots\dots [8]$$

where  $a_0$  is the radius of vapor molecule,  $T_e$  is the equilibrium temperature of condensate.

From these basic assumptions, we may derive various useful equations describing the formation and growth processes.

We used the relevant parameter for silicate:  $T_e = 1000$  K,  $\mu = 20$  (Yamamoto & Hasegawa 1977). The scale parameter which characterizes the grain formation process  $\Lambda (= \tau_{\text{sat}} / \tau_{\text{coll}}$ ; where  $\tau_{\text{sat}}$  is e-folding time for supersaturation ratio and  $\tau_{\text{coll}}$  is the mean collision interval of monomers) at dust condensation point is calculated by Kozasa *et al.* (1984) for various stellar luminosity and mass loss rate. The value is about  $\Lambda = 1 \times 10^4$  for  $L_* = 1 \times 10^4 L_\odot$  and  $\dot{M} = 5 \times 10^{-5} M_\odot/\text{yr}$  (typical for luminous OH/IR stars).

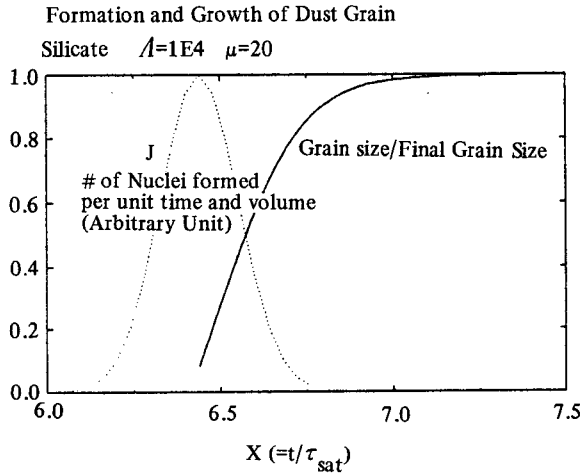


Fig. 3. Dust Formation and Growth Time Scales

Figure 3 shows the results of calculations. The saturation time ( $\tau_{\text{sat}}$ ) is calculated to be about 1 day for dust condensation radius  $R_c \simeq 7000 R_\odot$  and gas velocity at dust condensation radius  $V_c \simeq 1$  km/sec (typical for luminous OH/IR stars: Deguchi 1980, Kozasa *et al.* 1984). The final grain radius is calculated to be about  $0.1 \mu\text{m}$  for the same parameters. That was the reason why we used  $0.1 \mu\text{m}$  for the average grain radius when we attempted to find the opacity pattern by comparing the results of model calculations with observations. We see that the formation takes about a week and the grain size reaches its final value within a very short time. We summarize the results as follows:

- $t = 0$  :  $T = 1000$  K (equilibrium temperature for silicate),
- $t = 6 \tau_{\text{sat}}$  : Formation begins (after supercooling  $\Delta T \simeq 100$  K),
- $t = 7.2 \tau_{\text{sat}}$  : Formation and Growth completed.

For typical Miras and OH/IR stars, the dust condensation radius ( $R_c$ ) is about  $3000 \sim 10000 R_\odot$  and terminal gas velocity  $V_{ter} \simeq 10 \sim 20$  km/sec, therefore it takes a few years to travel  $1 R_c$ ! Pulsation periods are about 1-5 years, therefore dust formation and growth processes can be considered to be instantaneous (time scale  $\simeq$  a week).

*\* The Results of stellar wind models and the effect on emergent spectra*

Kwok(1975) found that once dust condensation takes place, radiation pressure on grains can drive the gas to the observed expansion velocities under the condition that the gas density at the condensation point is high enough. The gas density at condensation point can be significantly increased by the effect of pulsation-driven shock waves (Jones *et al.* 1981, Wilson & Bowen 1985). The response of a stellar atmosphere to repeated injection of energy and momentum by periodic shocks is to expand, decreasing the density gradient in close to the star.

The results of stellar wind models which are important for the emergent spectra from the stars with thick dust shells is that dust grains are accelerated relatively fast and approach and maintain terminal velocity within 3-5  $R_c$  (Kozasa *et al.* 1984). Therefore they predict overall constant out flow velocity, namely, overall density distribution should be  $\rho(r) \propto r^{-2}$ .

As we will discuss later, dust density distribution makes significant differences in emergent spectra. But the change of dust density distribution for small inner portion ( $r \leq 5 R_c$ ) does not make much differences in emergent spectra. Therefore we may assume that density distribution is  $\rho(r) \propto r^{-2}$ , but will show the effect of a narrow inner acceleration zone as well.

#### IV. Models for OH/IR Stars

For given characteristics of the central star and the dust shell around it, we may find the emergent spectra by solving the radiative transport problem and compare the results with observations. We may check whether our modeling is accurate or not and find ways to improve our input parameters. Dust grains in the shell absorb and scatter the stellar radiation and reemit it at longer wavelengths. We have used Leung's radiative transfer code (CSDUST3: Egan, Leung & Spagna 1987) which is considered to be one of the most accurate and efficient one so far available for the task.

The basic idea of Leung's (1975, 1976a,b) work is to treat the radiative transfer problem as a two-point boundary value problem. The equation of radiation transport is cast into a quasi-diffusion form by introducing two auxiliary functions – the anisotropy factor ( $f_p$ ) and the configuration function ( $\xi_p$ ) which depends on the geometry of the medium. By first assuming the radiation



field to be isotropic(diffusion approximation), the system of nonlinear moment and energy balance equations are solved to determine the mean intensity and the grain temperature distribution, i.e. the source function. From the source function, one then solves for the angular distribution of the radiation field by ray tracing methods. The anisotropy factor and the configuration function are then updated. The coupled nonlinear moment and energy equations are solved by the Newton-Raphson method. The form of the linearized equations allows the use of an efficient elimination scheme proposed by Rybicki(1971).

Model spectra for Miras and OH/IR stars have been made by many authors(e.g. Jones & Merrill 1976, Rowan-Robinson & Harris 1983, Volk & Kwok 1988). We have modeled OH/IR stars in detail because previous works had following problems:

- 1) The results for OH/IR stars do not fit observations well,
- 2) The results do not clarify the density distribution,
- 3) They have not modeled OH/IR stars depending on the phase of pulsation.

*\* The Procedure of Model Calculations*

For input parameters, we have to specify stellar parameters(luminosity and temperature) and dust parameters(scattering and absorption efficiencies, anisotropy factor, grain size, dust density distribution, and optical depth at  $\lambda = 9.7 \mu\text{m}$ ). We have used 125 radial grid points(logarithmic scale from 1 to  $1000 \times R_c$  with extra points in outer boundary to improve the convergence speed) and 70 wavelength grid points( $0.2 \mu\text{m} - 100 \mu\text{m}$ ).

*\* Dust Component*

The emergent spectra are highly dependent on the opacity pattern. So we did our best to use more reliable data. After some considerable experimentation with various grain opacities and size distributions, we have chosen the type 2 silicate opacity pattern with Gaussian grain size distribution which peaks at  $0.1 \mu\text{m}$  with rms deviation of  $0.05 \mu\text{m}$  for OH/IR stars.

For dust density distributions, we have used a simple power law( $\rho \propto r^{-\beta}$ ) with exponents of 1 to 3. To avoid singularities at small  $r$ , an inner gaussian density profile is matched smoothly onto the power law profile at a point such that 90% of the total optical depth occurs within the power law region. The dust condensation temperature( $T_c$ ) is assumed to be 1000 K. The dust condensation radius( $R_c$ ) is obtained after a few trials.

*\* Results of Model Calculations*

OH/IR stars have thick dust shells and their central stars are considered to be cooler than those of Miras. To investigate the effect of changing density distribution, we made model calculations for various  $\beta$ 's. The parameters are as follows:

stellar parameters:  $T_* = 2500$ ,  $L_* = 1 \times 10^4 L_\odot \rightarrow R_* = 541 R_\odot$

$\tau(\text{at } \lambda = 9.7 \mu\text{m}) = 8$

$\beta$	$R_c(R_*)$	$T_c(K)$
1	8.2	994
2	8.2	1057
3	8.2	1061

Figure 4 shows the results. The result with  $\beta = 2$  closely resembles to OH 39.7+1.5(Herman *et al.* 1984). The results with other  $\beta$ 's do not fit any observed objects.

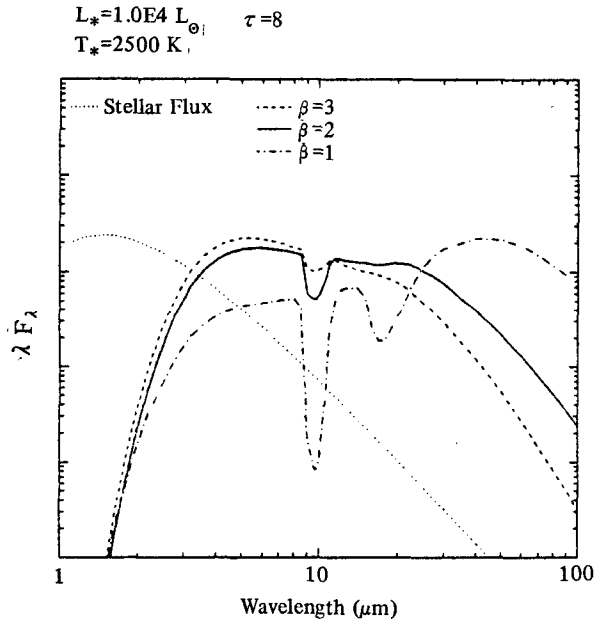


Fig. 4. Model Emergent Spectra for OH/IR stars  
 $\beta = 1, 2, 3$

We modelled dust thick OH/IR stars using the parameters as follows:

stellar parameters:  $T_* = 2200$ ,  $L_* = 1 \times 10^4 L_\odot \rightarrow R_* = 699 R_\odot$

$\beta = 2$

$\tau(9.7\mu\text{m})$	$R_c(R_*)$	$T_c(\text{K})$
15	7.0	1012
27	7.6	1011
35	8.2	997

The results for modelling OH/IR stars are shown in Figure 5. We find that our results closely resemble to typical observations(Herman *et al.* 1984, Engels *et al.* 1983).

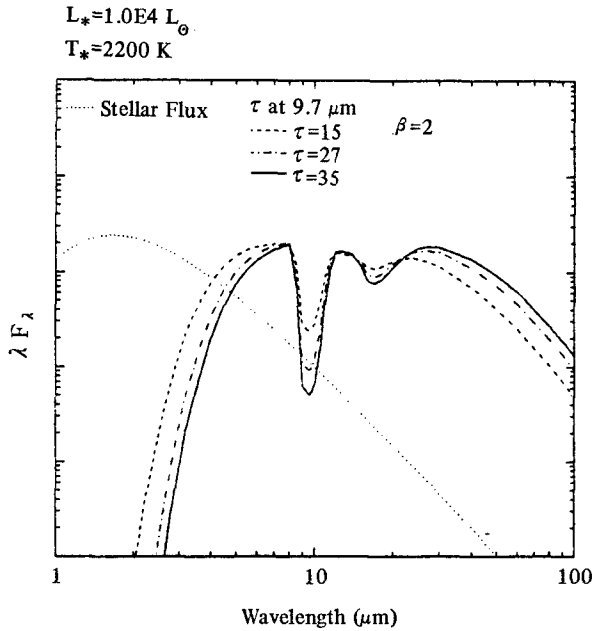


Fig. 5. Model Emergent Spectra for Dust-Thick OH/IR stars

We find that the shape of model emergent spectra depends mainly on dust shell parameters (dust density distribution & total optical depth). Stellar parameters and dust condensation radius (if deviation from dust condensation temperature is less than about 200 K) have relatively little effect on it.

## V. Model Calculations Depending on the Phase of Pulsations

– A model for OH 26.5+0.6 –

In Miras and OH/IR stars which show large amplitude pulsations, pulsation-driven shock waves change the structure of the atmosphere greatly. The size, temperature and luminosity of the photosphere change depending on the phase of the pulsation. Those changes affect the structure of the dust envelopes around the stars. For instance, if grains condense at a fixed temperature, the radius at which the grains condense varies throughout the cycle because of the changes of the luminosity of photosphere. If there is a steady mass loss from the star at constant terminal velocity, then the optical depth of the shell should vary throughout the cycle. It would reach a maximum when the star is at minimum light when grain condensation radius is minimum and vice versa. Such an effect has been observed in silicate absorption at  $9.7 \mu\text{m}$  by Forrest *et al.* (1978).

### \* Observations

OH 26.5+0.6(AFGL 2205) has been monitored for a long time, Forrest *et al.* (1978)'s observations provide wide range spectra of the object at two phases. Light curves at many wavelengths are available (Kleinmann *et al.* 1981, Jones 1987). Therefore, we have fairly accurate observational data. The star is one of the OH/IR stars which show the variations due to pulsations. The period is about 1550 days and bolometric luminosity change ( $\Delta m_B$ ) is about 1.2 magnitudes which is obtained from our model calculations (as we will discuss later).

### \* The change of parameters and physical implications

Our explanation for the change of the depth of  $9.7 \mu\text{m}$  feature is as follows. Depending on the phase of central star's pulsation, the radius, temperature, and luminosity of the photosphere changes. So the dust condensation radius changes. With the density distribution we have used ( $\rho \propto r^{-2}$ ), the results of the modelling are not dependent on the outer boundary of the dust shell because it is set to be  $1000 R_c$ . So the optical depth is given by:

$$\begin{aligned} \tau_\lambda &= \pi a^2 Q_\lambda \int_{R_c}^{\infty} N_g(r) dr \\ &\simeq \pi a^2 Q_\lambda N_g(R_c) R_c \dots\dots\dots [9] \end{aligned}$$

Assuming that material is outflowing at constant rate ( $N_g \propto r^{-2}$ );

$$\tau R_c \simeq \text{constant} \dots\dots\dots [10]$$

for given wavelength at all phases.

Table 2 shows the model parameters. After fitting the minimum phase using  $\tau(9.7 \mu\text{m}) = 22$ , we used the equation [10] to find optical depths at other phases.

Table 2. Adopted Parameters for OH 26.5+0.6 depending on the Phase

$L_B$	Minimum	Medium	Maximum	
$L_*$	$1 \times 10^4$	$1.8 \times 10^4$	$3 \times 10^4$	
$T_*$	2000 K	2200 K	2400 K	
$R_*$	846	938	1017	
$R_c (R_\odot)$	5098(6 $R_*$ )	6650(7 $R_*$ )	8645(9 $R_*$ )	
( $\times 10^{-4}$ pc)	1.15	1.55	1.95	
$\tau(9.7 \mu\text{m})$	22	16.3	13	from eq. ... [10]
$T_c$	1003 K	993 K	1008 K	

$L_B$  : Bolometric Luminosity

Figure 6 shows the results of the model calculations superimposed on observational data. Observational data for minimum and intermediate phases (open circles and closed circles) are from Forrest *et al.* (1978) and for maximum phase (open triangles) from the light curves of Suh (1988) and Kleinmann *et al.* (1981).

We find a fairly good fitting. Thus, a simple change in the dust condensation radius with change in stellar luminosity is adequate to explain the spectrum of OH 26.5+0.6 at all phases. But a little deviation especially at around  $20 \mu\text{m}$  was not avoidable even by using different types of opacity pattern or density distribution. Werner *et al.* (1980) argued that a flatter density distribution fits better for this object. But there seems to be no physical reasons why it should be flatter and we could fit better with  $\rho \propto r^{-2}$ .

The amplitude of the light curve at  $3.5 \mu\text{m}$  is observed to be 2.1 magnitudes but our model calculations show that the bolometric change needs be only about 1.2 magnitudes, which is much smaller than expected. The time scale (from minimum to maximum) is about 6 years for dust movement, 2 years for pulsation, and about a week for dust formation and growth. So about 2/3 of dust material will be evaporated while the other portion will be moved out. And formation and growth processes may be considered to be instantaneous.

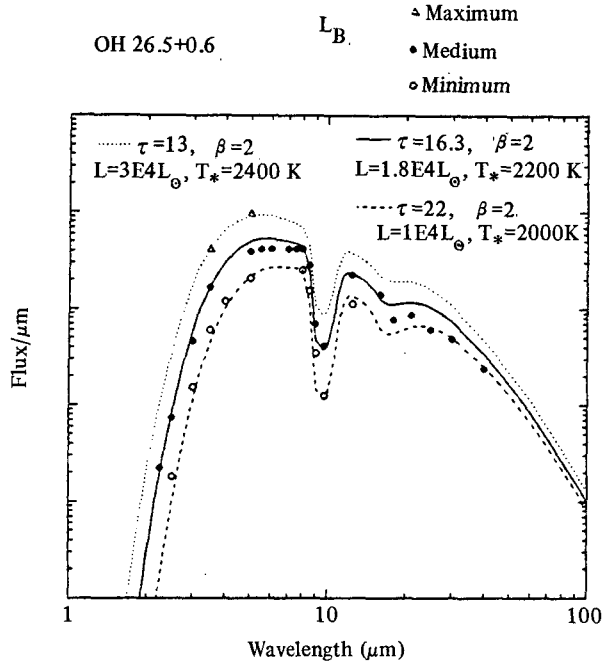


Fig. 6. Results of Model Calculations for OH 26.5+0.6

*\* The model calculations using the results of pulsation models*

In Bowen(1988)'s model, the density, temperature, and velocity structures of the dynamic atmosphere are determined by stellar parameters and by complex interactions of the driving amplitude, dust opacity, and thermal relaxation processes. But his treatment of dust was greatly oversimplified.

Bowen(1988; Private communication) has made a pulsation-shock wave model for the object including dust opacity. The results can be summarized as follows:

- 1) The gas velocity approaches terminal one(14 km/sec) very soon (within  $3 R_c$ ) which is similar to the results from a pure radiation pressure model.
- 2) The density distribution does not change very much depending on the phase of pulsation.

To investigate the effects of pulsations, we modeled the object using two power law indexes depending on the location of the shell. For small radii ( $< 3 R_c$ ) where dust grains are being accelerated, we used a steeper index( $\beta = 3$ ). Outside that radius, we used  $\beta = 2$  which is for steady, constant velocity outflow.

We have found that the change of density distribution affects the shape of the emergent spectra

only very little, given our freedom to adjust the total optical depth through the shell. Therefore either pulsation or radiation pressure model does not produce the emergent spectrum with density distribution which is largely deviated from  $\rho(r) \propto r^{-2}$ .

## VI. Conclusions

The change of the spectral shape of OH/IR stars depending on the phase of pulsation can be explained fairly well by using a simple pulsation model with changing dust condensation radii (for fixed dust condensation temperature).

Dust density distribution does not look to be very different from  $\rho(r) \propto r^{-2}$ . We could fit observations fairly well with properly chosen opacity pattern and other parameters. Either flatter or steeper density distribution does not fit observations well and is not physically plausible. Either radiation pressure model or pulsation model predicts overall constant outflow (i.e.  $\rho(r) \propto r^{-2}$ ). Change of distribution for a small portion of the shell does not make much differences in emergent spectra.

Further work for measuring more detailed optical constants of the candidate materials and clarifying the change of those depending on temperature is necessary to improve our models.

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