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−Research Paper−

MEASURING MORBIDITY : AN APPROACH USING POWER FUNCTIONS.

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ABSTARCT

Subjective scoring by different groups on different status of morbidity are compared to objective data obtained from legal awards. A power law is tested between subjective and objective scores. Regression analysis by means of a power function provides a measure of consistency in its regression coefficient. Power functions fitting also leads to a justified use of geometric averaging of individual scores into group scores.
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1. INTRODUCTION

In order to help decision - makers concerned with health systems to ameliorate planning and to cotntrol efficiency, hospital output should be measured. In a report on an Operations Research Workshop on 'Health and Welfare Systems' (ROSSER & WATTS, 1973) three possibilities have been proposed ; sanative output (the effect of the hospital on the immediate health of its patients), long term output (the effect of the hospital on the health of its patients one or more years after discharge) and mortality, including the morbidity state on adminission of those who died. A fundamental requirement for developing these measures seemed a classification of morbidity, which should be simple with relatively few dimensions.

In section 2 we present the Rosser and Watts model for classifying morbidity states.

In section 3 we estimate an empirical power function with Rosser and Watts' data.

In section 4 we justify the use of geometric averaging to obtain combined forecasts from individual estimates.

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2. THE ROSSER AND WATTS MODEL

In an attempt to estimate hospital output, Rosser and Watts use a classification of morbidity with two dimensions : disability and distress, defined as :

Disability = the extent to which a patient is unable to pursue the activities of a normal person at the time at which the classifcation is made ;

Distress = the patient's pain, mental suffering in relation to disablement, anxiety and depression

(ROSSER & WATTS, 1972, p. 364)

The use 8 states of disability and 4 states of distress.

The disability states are :

1. No disability
2. Slight social disability
3. Severe social disability and/or slight impairment of performance at work. Able to do all housework except very heavy tasks.
4. Choice of work or performance at work very severely limited. Housewives and old people able to do light housework only but able to go out shopping.
5. Unable to undertake any paid employment. Unable to continue any education. Old people confined to home except for escorted outings and short walks and unable to do shopping. Housewives able only to perform a few simple tasks.
6. Confined to chair or to wheelchair or able to move around in the house only with support from an assistant.
7. Confined bo bed.
8. Unconscious

The distress states are :

1. No distress
2. Mild
3. Moderate
4. Severe

Combination of states (8,2), (8,3), (8,4) were not considered for obvious reasons, so 29 morbidity states, result.

Subjects wers interviewed to compare different morbidity states. The subjects were taken from 6 populations : medical patients, psychiatric patients, medical nurses, psychiatric nurses, healthy volunteers and doctors. A scale was constructed from the subjects' answers to questions like 'how many times more ill is a person described as being in state 2 as compared with state 1 ?' (KIND, ROSSER & WILLIAMS, 1981). This aspect has been criticised for the interviewees were told that implications of their estimates were in terms of resource allocations and not of legal awards (LEU, 1981). A scale C_1, used by ROSSER & KIND (1978), is defined between $(0, +\infty)$.

A second scale C_2, used by KIND, ROSSER & WILLIAMS (1981), is defined between $(-\infty, 1)$, using the transformation $C_2 = 1 - \frac{C_1}{C_1(7,3)}$, where the morbidity state (7,3) was equated with 'dead', so that the second scale has values 1 for 'fit' and 0 for 'dead'. In a comparative study (KIND, ROSSER & WILLIAMS, 1981) it was shown that a morbidity state scale constructed in the basis of legal awards, is significantly correlated with all psychometric scales, but most with the doctors' scales and least with those from nurses.

3. INTRODUCING AN EMPIRICAL POWER FUNCTION

The Rosser and Watts experiments generate data of the ratio type ;

$$\left(\frac{I_i}{I_j}\right)_{sk}$$

where I_i, I_j are measures of illness in morbidity state i and j.
s_k is a subscript indicating a subjective estimate of group k.
Besides this, the legal ratio is taken to be the objective estimate :

$$\left(\frac{I_i}{I_j}\right)_{0}$$

where 0 is a subscript indicating the objective estimate.

These ratios where shown to be positively correlated by KIND, ROSSER & WILLIAMS (1981). But correlation measures a linear relationship. A nonlinear specification may be more appropriate, as is suggested by STEVENS' psychophysical law, stating that a perceived stimulus can be plotted against the originating physical stimulus following a power function (STEVENS, 1966). This psychophysical law has been applied successfully in an environment similar to the one under study : opinions on the seriousness concerning a number of offenses (see e. g. EKMAN (1962) and SELLIN and WOLFGANG (1964)).

Data analysis, using a power function of the form $Y=ax^b$ with slope b and scaling constant a, where Y is the perceived incidence and X is the official incidence, generated quite reasonable results with explained variances between 81% and 88%.
Introducing the idea of the power function in our case leads to the following formula :

$$\left(\frac{I_i}{I_j}\right)_{sk} = a_k\left(\frac{I_i}{I_j}\right)_0^{b_k}$$

where a_k and b_k are constant for a specific group of interviewees k.
The interviewees' answers were all transformed so that all states are compared to the state 'fit'. This transformation implies that in our model $a_k=1$ for all k. The objective scores, obtained from legal awards, were rescaled so that the state 'fit' carries the value 1 and 'dead' the value 0, allowing states to have negative values.
By transforming the scores ;

$$\left(\frac{I_i}{I_{fit}}\right)_{sk} \quad \text{into} \quad \left(\frac{I_i}{I_{fit}}\right)_{sk} \Big/ \left(\frac{I_{dead}}{I_{fit}}\right)_{sk}$$

legal score into 1 − legal score

we obtain comparable scales, having only positive values (in order to avoid mathematical problems). This leads to the following equality :

$$\left(\frac{I_i}{I_{fit}}\right)_k / \left(\frac{I_{dead}}{I_{fit}}\right)_k = \left[\left(\frac{I_i}{I_{fit}}\right)_0 / \left(\frac{I_{dead}}{I_{fit}}\right)_0\right]^{b_k}$$

$$= (1 - \text{legal score})^{b_k}$$

Results from the regression analysis are shown in Table 1 and compared with explained variance from a model of the type $Y = aX + b$ as previously published in KIND, ROSSER and WILLIAMS(1981). (see columm 5)

Tabel 1.

(1) Subject identification(k)	(2) Estimated coefficient b_k	(3) Standard error on b_k	(4) % variance explained in power model	(5) % variance explained in linear model
Medical patients	2.73	0.22	88	63
Psychiatric patients	3.13	0.28	86	63
Medical nurses	2.46	0.33	77	59
Psychiatric nurses	3.19	0.20	90	59
Healthy volunteers	3.73	0.09	98	64
Doctors	3.65	0.34	79	68

All estimated coefficients b_k, being significantly greater than 1, refer to curves, which lay in the interval (0,1) below the legal curve. This is a statistical confirmation of the observation by KIND, ROSSER and WILLIAMS(1981, p.165) that legal data assign more balanced lower valuations between the severe states judged to be somewhat less or more serious than death.

Concerning the form of the relationship between objective (legal) data and subjective estimates it can be said that the power function is for all subject groups superior to the linear function, in terms of the variance explained. While the linear regression coefficient is not a valid for the consistency of both scales(R.E.LEU, 1981, p.292), we propose the following :

- both scales called consistent if in our model $b_k = 1$
- assuming a continuous scale, the surface between the lines x and x^{bk} are a measure of consistency, written as :

$$\int_0^1 (x - x^{bk})dx + \int_1^A (x^{bk} - x)dx = 1 - \frac{2}{b_k + 1} + \frac{A^{bk+1}}{b_k + 1} - \frac{A^2}{2}$$

where A is an upper bound which should be finite.

Holding A constant it can be seen that this surface is increasing with increasing b_k. This allows b_k to be a measure of consistency itself.
Ranking the subject groups' estimates from more to less consistent with the legal scale, is thus the same as ranking b_k from low to high. This places the Medical Nurses at top of consistency and Healthy Volunteers at the bottom. These results are quite defferent from using the linear regression coefficient as a measure of consistency, placing Doctors at top of consistency and Medical Nurses at the bottom (KIND, ROSSER & WILLIAMS, 1981,p.165).

4. A PROPOSAL FOR USING GEOMETRIC AVERAGING.

As the subject groups are different only by their skill of measuring illness, it can be realistic to assign the same estimating power function to each individual g belonging to group k.

$$\left(\frac{I_i}{I_j}\right)_{s_k^g} = \left(\frac{I_i}{I_j}\right)_o^{b_k}$$

where s_k^g is a subscript indicating the subjective estimate of individual g belonging to group k.
When geometric averaging is used to obtain the group estimate, we would proceed as follows :

$$\left(\frac{I_i}{I_j}\right)_{s_k} = \left[\prod_{g \in k}\left(\frac{I_i}{I_j}\right)_{s_k^g}\right]^{1/N_k}$$

where N_k=the numer of subjects in group k.

$$\left(\frac{I_i}{I_j}\right)_{s_k} = \left[\prod_{g \in k}\left(\frac{I_i}{I_j}\right)_o^{b_k}\right]^{1/N_k} = \left[\left(\left(\frac{I_i}{I_j}\right)_o^{b_k}\right)^{N_k}\right]^{1/N_k} = \left(\frac{I_i}{I_j}\right)_o^{b_k}$$

which is the power function relation between suject groups' estimates and the legal scale.

5. CONCLUSION

An empirical power function relation used in relating public beliefs about crime to the actual figures can also be used profitably in the estimation of morbidity.

The power function coefficient can be used as a measure of consistency between scales obtained from different subject groups.

The power function leads to a justified use of geometric averging of individual estimates into a group estimate.

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