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An Optimal Design of Automated Storage/Retrieval Systems

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ABSTRACT

This paper deals with design problem of unit load automated storage/retrieval systems (AS/RS). We propose an optimal design model in which the investment and maintenance costs of AS/RS, operating under dual command mode,|is minimized over a time horizon satisfying the warehouse dimensional constraints. The model is formulated as an integer nonlinear program and an algorithm is proposed to find an optimum solution. The validity of the solution algorithm is illustrated through an example.

1. Introduction

Compared with traditional warehouse, AS/RS has many benefits such as savings in labor cost, improved material flow and inventory control, high flow-space utilization, and so on. Due to these benefits, AS/RS has been the subject of many research works. However, only a limited number of articles have been published on the area of AS/RS design. Zollinger [9] and Koenig [6] presented simulation approaches where the search for the optimum configuration is confined to certain values of the design variables specified by the user. A design model based on analytical techniques was developed by Karasawa, Nakayama and Dohi [5] whose total cost function includes terms for storage racks, stacker cranes, building and land. However, They considered only single command mode for crane operation with no site and height restrictons for construction. For dual command mode, Ashayeri, Gelders and Wassenhove [1] found the number of S/R machine based on the empirical data assuming that height of building is already fixed.

This paper presents an analytical model of AS/RS for design process which minimizes the installation cost and the operating cost over a time period. Given throughput, it calculates a minimum number of crane and optimal dimensions(height, width and length) of the AS/RS operationg under dual command mode. All these decision variables are subject to the capacity of the S/R machine, dimension restrictions due to the building site and the storage volume requirement.

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2. Design concept of AS/RS

In this section, we briefly describe principal components of AS/RS and terminologies used for the development of the model.

Figure 1 shows the principal components of AS/RS.

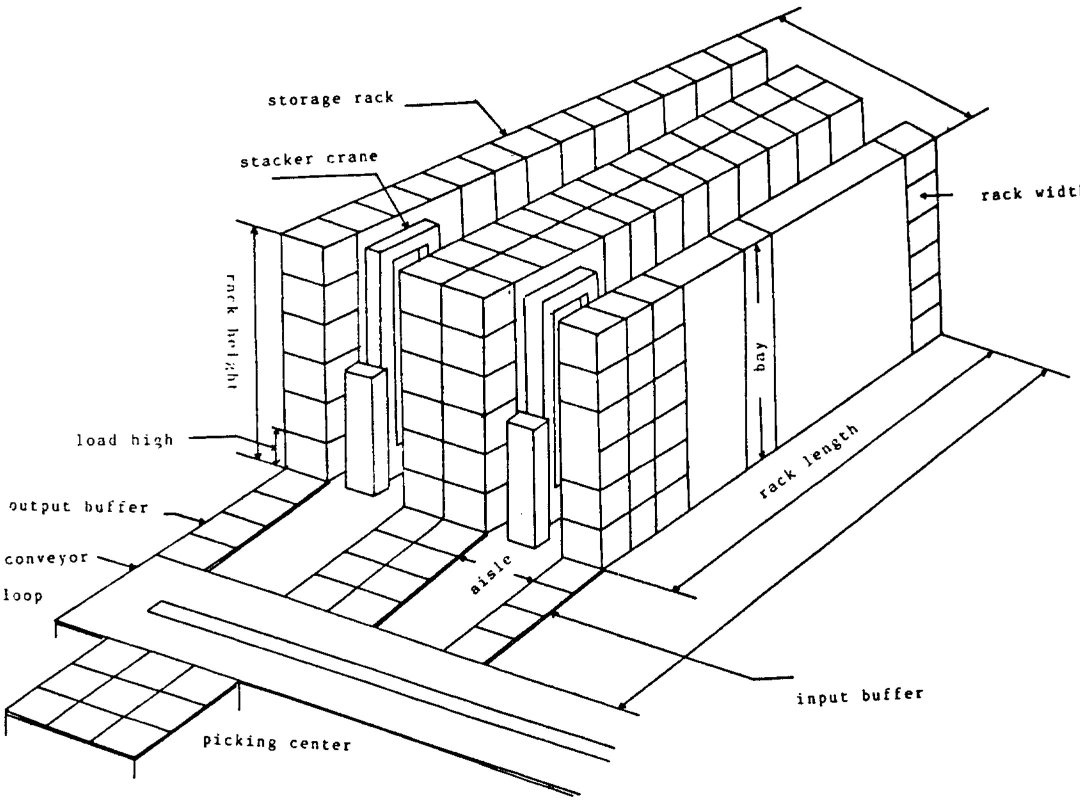

Figure 1

Load high : it is equal to the height of a rack opening and is the principal decision variable to determine the dimensions of automated warehouse.

Bay : it is one column of the rack which is composed of the connected rack openings and is also the principal decision variable.

Unit load : it refers to a basic treatment unit for storage items in the event of storing or retrieving.

Load activity : it refers to the number of unit loads to be stored and retrieved per time unit(e.g. hour).

Storage volume : it refers to the total amount of average inventory which has been converted to unit loads.

The number of total rack openings is determined by storage volume.

cycle time : it refers to the expected travel time for dual command plus pick - up or deposit time into or out of the rack.

i) Calculation of the required number of cranes : the required number of cranes (CRN) is obtained by dividing the load activity by the number of cycles which a crane can perform during the same period of time :

$$CRN = \frac{\text{Load activity/hour}}{(\text{Number of cycles/hour}) \times r}$$

where r is a factor reflecting average utilization of cranes (e.g. 0.75).

Obviously CRN has to be rounded to the next highest integer value.

ii) Calculation of number of loads high : the required number of loads high(NLH) is obtained by dividing the height of the rack by the height required for one unit load. The height required for one unit load is equal to the height of one unit load plus clearance :

$$NLH = \frac{\text{Height of rack}}{\text{Height of a load high} + \text{Clearance}}$$

iii) Calculation of the number of bays : the required number of rack openings depends upon storage volume. To obtain the required number of bays(NB), storage volume is divided by the product of the number of loads high and twice the number of cranes in case of two - side racks :

$$NB = \frac{\text{Storage volume}}{2 \times \text{Number of cranes} \times \text{Number of loads high}}$$

Note that given storage volume, only two among CRN, NLH and NB are decision variables. The reminder is automatically determined.

iv) dimension of rack height, length and width

Rack height = (height of a unit load + clearance) × NLH

Rack length = (length of a unit load + clearance) × NB

Rack width = (3 × width of a unit load + clearance) × CRN

Once various dimensions of rack are determined, the height, length and width of AS/RS are found considering the clearance for S/R machine operation and building construction.

3. Development of the optimal design model

the following assumptions are made ;

1) The AS/RS consists of a single crane serving two racks by the both side of the aisle.
2) All rack openings are the same size and a rack opening contains one unit load.
3) Each unit load consists of only one storage item and all unit loads are the same size.
4) I/O point is located at the lower left - hand corner.
5) Randomized storage is used. That is, any rack opening is equally likely to be selected for storage or retrieval.

6) The demand of each storage item is known and constant during some time period(e.g. month, year). That is, total storage volume and load activity are known and constant.

3. 1 Cost elements of the model

In this section, each cost element which consists of the component of the objective function is represented.

i) Cost of cranes (CRNCOST) : C_1N where C_1 represents the individual crane cost, and N number of cranes (CRN).

ii) Cost of rack structure (RCOST) : C_2V where V is the storage volume expressed as the number of unit loads and C_2 is the cost estimate per a rack opening. This estimate is provided by Zollinger [9] as follows;

$$\text{Cost/rack opening}(C_2) = \$25(0.92484 + 0.025x + 0.0004424y - y^2/82500000 + 0.23328z - 0.00476z^2)$$

where x = number of cubic feet of a unit load

y = weight of a unit load

z = number of loads high(NLH)

Using this estimate, rack structure cost is represented by the following form ;

$$RCOST = C_2V$$
$$= a_1V + a_2VN_1 + a_3VN_1^2$$

where N_1 = number of loads high (NLH)

$a_1 = \$25(0.92484 + 0.025x + 0.0004424y - y^2/82500000)$

$a_2 = \$25(0.23328)$

$a_3 = \$25(-0.00476)$

iii) Cost of foundation and roof (FDCOST) : $(C_3 + C_4)x_1x_2$ where x_1 and x_2 represent the building width and length respectively. C_3 and C_4 are appropriate unit costs for foundation and roof respectively.

iv) Cost of walls (WALLCOST) : $C_5(x_1 + 2x_2)x_3$ where x_3 is the buiding height and C_5 the wall cost per square meter. This formula includes the cost of the back wall and the two - side walls of building.

v) Cost of conveyor loop(CONVCOST) : $2C_6x_1$ where C_6 is the conveyor cost per meter. This conveyor circulates unit loads to be stored or retrieved from the rack for order picking.

vi) Cost of I/O buffers (BFCOST) : fC_7N where C_7 is the cost of one buffer position and (f/2) is the number of input or output positions that are provided between each stacker crane and conveyor loop. It is assumed that f is equal to 5% of hourly load activity [1].

vii) Cost of land (LDCOST) : $C_8x_1x_2$ where C_8 is the land price per square meter.

viii) Maintenance cost : PC_1N, which means that the total maintenance cost of the system per year is assumed to be approximately P% of the cost of cranes.

xi) Order picking cost : S is the yearly salary cost for the workers who collect items from the unit loads to satisfy client orders.

x) The discounted operating cost (DOPCOST), assuming a lifetime of T years and a discount rate i, is then;

$$DOPCOST=\sum_{t=1}^{T}(PC_1N+S)/(1+i)^t$$

3. 2 The optimal design model

In this section an analytical model which includes all relevant cost elements and appropriate operating constraints is presented. This model formalizes the optimal design process and solving it will yield the most economical design.

The objective function of the model includes all cost elements previously stated with the constraints derived on the basis of relationships of principal components. These constraints are as follows ;

i) The constraints on storage volume :

$$2\times N\times N_1\times N_2\geq V$$

where N_1 is the number of loads high (NLH), N_2 number of bays (NB) and V storage volume. The total number of rack openings is no small than the given storage volume.

ii) Required number of cranes :

$$N\geq LD\times T/r$$

where LD is load activity, T cycle time for crane, and r the average crane utilization as defined before (Note that T has the same time unit as LD).

iii) The capability of AS/RS being dependent upon the number of cranes should be able to manage load activity.

The mathematical form for T is derived on the basis of the work of Bozer and White [3] and

$$T=Tc+4b_1$$

where b_1 is the operation time for one pick-up or deposit (four such operations are required in one cycle) and is assumed to be constant.

Therefore, the constraint on the number of cranes is formulated in two ways ;

on the condition that

$$\frac{x_k}{v_x} \geq \frac{x_h}{v_y}$$

$$N \geq \frac{LD}{r}\left(\frac{4x_k}{3v_x} + \frac{x_{1k}^2 v_x}{2x_k v_y^2} - \frac{x_h^3 v_x^2}{30x_k^2 v_y^3} + 4b_1\right)$$

on the condition that

$$\frac{x_k}{v_x} \geq \frac{x_h}{v_y}$$

$$N \geq \frac{LD}{r}\left(\frac{4x_h}{3v_y} + \frac{x_k^2 v_y}{2x_h v_x^2} - \frac{x_k^3 v_y^2}{30x_h^2 v_x^3} + 4b_1\right)$$

where

v_x = horizontal crane speed

v_y = vertical crane speed

x_k = rack length

x_h = rack height

iv) Constructional restrictions on the width, length, and height of the building.

Building width : $x_1 = (3b_2 + b_3) \times N + b_4$

where

b_2 = width of a unit load

b_3 = clearance for one aisle unit

b_7 = building allowance between rack and wall

Building length : $x_2 = (b_5 + b_6) \times N_2 + b_7$

where

b_5 = length of a unit load

b_6 = clearance for the length of a unit load

b_7 = building allowance for crane operation

Building height : $x_3 = (b_8 + b_9) \times N_1 + b_{10}$

where

b_8 = height of a unit load

b_9 = clearance for the height of a unit load

b_{10} = building allowance between rack and roof

Therefore, constructional restrictions are as follows ;

$$x_1 \le e_1,\ x_2 \le e_2,\ x_3 \le e_3$$

where e_1, e_2, and e_3 are the permitted width, length, and height of building respectively

Now the optimization model for design of automated warehouse can be stated as follsws ;

$$\text{Min. } TC = C_1N + C_2V + (C_3+C_4)x_1x_2 + C_5(x_1+2x_2)x_3 + 2C_6x_1 + fC_7N + C_8x_1x_2 + \sum_{t=1}^{T}(PC_1N+S)/(1+i)^t$$

subject to

$$x_1 = (3b_2+b_3)N + b_4$$
$$x_2 = (b_5+b_6)N_2 + b_7$$
$$x_3 = (b_8+b_9)N_1 + b_{10}$$
$$x_1 \le e_1,\ x_2 \le e_2,\ x_3 \le e_3$$

$$N \ge \frac{V}{2 \times N_1 \times N_2}$$
$$N > LD\ (Tc + 4b_1)/r$$
$$x_1,\ x_2,\ x_3 > 0$$
N, N_1, $N_2 > 0$ and integer

where
N_1 = number of loads high, N_2 = number of bays,
N = number of cranes are the decision variables.

4. Solution algorithm for the model

Substituing the equations for x_1, x_2, and x_3 into the objective function, and transforming constraints to reveal the decision variables, the optimal design model of AS/RS becomes a function of N_1, N_2 and N which can be stated as

$$\text{Min. } TC = a^*N_1 + b^*N_2 + c^*N_1N_2 + d^*/N_1 + e^*/N_2 + f^*/(N_1N_2) + g^*N_1^2 + h^*$$

where

$$a^* = a_2V + C_5(b_4+2b_7)(b_8+b_9)$$
$$b^* = (C_3+C_4+C_8)b_4(b_5+b_6) + 2C_5(b_5+b_6)b_{10}$$
$$c^* = 2C_5(b_5+b_6)(b_8+b_9)$$
$$d^* = V(C_3+C_4+C_8)(3b_2+d_3)(b_5+b_6)/2$$
$$e^* = VC_5(3b_2+b_3)(b_8+b_9)/2$$

$$f^{*}=V(C_1+(C_3+C_4+C_8)(3b_2+b_3)b_7+C_5(3b_2+b_3)b_{10}+2C_6(3b_2+b_3)+fC_7+\sum_{t=1}^{T} PC_1(1+i)^t)/2$$
$$g^{*}=a_3V$$
$$h^{*}=(a_1+(C_3+C_4+C_8)b_4b_7+C_5(b_4+2b_7)b_{10}+2Cb_4+\sum_{t=1}^{T} S/(1+i)^t$$

subject to

$$N_1\leq\frac{e_3-b_{10}}{b_8+b_9} \cdots\cdots\cdots(1)$$

$$N_2\leq\frac{e_2-b_7}{b_5+b_6} \cdots\cdots\cdots(2)$$

$$N\leq\frac{e_1-b_4}{3b_2+b_3} \cdots\cdots\cdots(3)$$

$$N\geq\frac{V}{2N_1N_2} \cdots\cdots\cdots(4)$$

$$N\geq \mathrm{LD}(Tc+4b_1)/r \cdots\cdots\cdots(5)$$

N_1, N_2, $N>0$ and integer$\cdots\cdots\cdots(6)$

Note that value of Tc depends upon N_1 and N_2, and true decision variables are N_1 and N_2.
Now, a solution algorithm is proposed to solve this model which consists of three phase.

***Phase 1 : Determine the feasible upper bounds of N_1, N_2, and N. Let UBN_1 denote the feasible upper bound for N_1, UBN_2 for N_3, and UBN for N with epuation (1)−(3), Each bound is determined as follows ;**

$$UBN_1=FIX\left(\frac{e_3-b_{10}}{b_8+b_9}\right)$$

$$UBN_2=FIX\left(\frac{e_2-b_7}{b_5+b_6}\right)$$

$$UBN=FIX\left(\frac{e_1-b_4}{3b_2+b_3}\right)$$

where FIX is the function of an IBM microcomputer to truncate a number to an integer (e.g. FIX(7. 6)=7)

***Phase 2=Find the feasible pairs of N_1 and N_2**

Let LBN_1 denote the feasible lower bound for N_1, LBN_2 for N_2 based on the constraints (4)−(6). The feasible pairs of N_1 and N_2 are found by the following steps ;

step (1) $LBN_1 = FIX(V/(2 \times UBN \times UBN)) + 1$

step (2) assign $N_1 = LBN_1$

step (3) $LBN_2 = FIX(V/(2 \times N_1 \times UBN)) + 1$

step (4) assign $N_2 = LBN_2$

step (5) $N_d = V/(2 \times N_1 \times N_2)$

$N_0 = LD \times (Tc + 4b_1)/r$

$N = FIX(max(N_d, N_0)) + 1$

step (6) if $N \leq UBN$, keep the valve of N_1, N_2

otherwise, increase N_2 one by one

and GO TO step (5) until $N_2 = UBN_2$

step (7) increase N_1 one by one

and GO TO step (3) until $N_1 = UBN_1$

**Phase 3 : Calculate the value of objective function(TC) and find the optimal solution of N_1 and N_2*

Calculate TC as to each feasible pair of N_1 and N_2 which was found in Phase 2 and then choose the pair giving the minimum value of TC.

Table 1 Size unit load and restrictions

parameter	value(m)	definition
b_2	1.2	width of a unit load
b_5	1.2	length of a unit load
b_8	1.2	height of a unit load
e_1	35	building restriction on width
e_2	125	building restriction on length
e_3	23	building restriction on height

Table 2 Clearance and building allowances

parameter	value(m)	definition
b_3	0.45	clearance for one aisle unit
b_2	0.6	allowance between rack and wall
b_6	0.2	clearance for length of a unit load
b_7	15.8	allowance for crane operation
b_9	0.25	clearance for height of a unit load
b_{10}	1.7	allowance between rack and roof

Table 3 Cost elements

parameter	value(BF)	definition
C_2	—	cost of a rack opening
C_3	4500	cost of a foundation per square metre
C_4	650	cost of roof per square metre
C_5	600	cost of wall per square metre
C_6	100000	cost of conveyor loop per metre
C_7	200000	cost of one I/O buffer position
C_8	10000	cost of land per square metre
S	2500000	yearly salary cost for order pickers

[BF : Belgian Francs 1BF=0.0244 $]

Table 4 The available crane specifications

type	cost(BF)	v_x(m/sec)	v_y(m/sec)	b_1(sec)
1	7900000	3.5	1.8	5
2	8800000	4	1.5	5

Table 5 Miscellaneous requirements

parameter	value	definition
V	14508 UL	required storage volume
LD	195 UL	load activity per hour
T	20 year	lifetime of automated warehouse
r	0.85	crane utilization rate
i	0.2	discount rate during lifetime
f	5%of LD	number of I/O buffer per a crane
P	2%	percentage for maintenance cost

[UL=unit load]

5. An illustrative example

In this example, the data required to design AS/RS are based on case studies of Ashyeri et al. [1], and Zollinger [9] and listed at tables 1 to 5.

The proposed solution algorithm is implemented through IBM PC and The results are summarized on the Tables 6 to 8. The computer run time is about 10 seconds.

Table 6 Feasible pairs of[N_1, N_2] and total cost

crane type 1				crane type 2			
N_1	N_2	N	TC	N_1	N_2	N	TC
13	76	8	2.657754E08	12	76	8	2.691350E08
13	77	8	2.677879E08	12	77	8	2.710885E08
14	71	8	2.646299E08	13	70	8	2.664715E08
14	72	8	2.667672E08	13	71	8	2.685536E08
14	73	8	2.689045E08	13	72	8	2.706357E08
14	74	8	2.710428E08	13	73	8	2.727178E08
14	75	8	2.731791E08	13	74	8	2.747999E08
14	76	8	2.753164E08	13	75	8	2.768819E08
14	77	8	2.774537E08	13	76	8	2.789640E08
				13	77	8	2.810461E08
				14	65	8	2.645773E08
				14	66	8	2.667895E08
				14	67	8	2.690017E08
				14	68	8	2.712140E08
				14	69	8	2.734262E08
				14	70	8	2.756384E08
				14	71	8	2.778506E08
				14	72	8	2.800629E08
				14	73	8	2.822751E08
				14	74	8	2.844873E08
				14*	75*	7*	2.529358E08*
				14	76	7	2.548777E08
				14	77	7	2.568196E08

[* ; optimal solution]

Table 7 Optimal design of AS/RS

parameter	optimal value	definition
N_1	14	number of loads high
N_2	75	number of bays
N	7	number of cranes
–	14700	number of total rack openings
x_w	28.35	rack width
x_k	105	rack length
x_h	20.3	rack height
x_1	28.95	building width
x_2	120.8	building length
x_3	22	building height

[Unit : height, length, width (meter)]

Table 8 Optimal cost elements

notation	optimal value	definition
CRNCOST	61.6	cost of cranes
RCOST	96.81923	cost of rack structure
FDCOST	18.01038	cost of foundation
WALLCOST	3.571261	cost of walls
CONVCOST	5.790001	cost of conveyor loop
BFCOST	14	cost of I/O buffers
LDCOST	34.97161	cost of land
DOPCOST	18.17327	discounted operating cost
TC	252.935752	optimal total cost

[Unit: million (1000000) BF]

6. CONCLUSION

This study presents an optimal design model of AS/RS which can be applied to the real world problems. We incorporate the dimensional constraints of AS/RS and detailed cost elements in this model.

A solution algorithm is proposed and implemented through IBM PC. This interactive program allows extensive sensitivity analysis of the important parameters such as available crane types, size of a unit load. The design of AS/RS can be affected by the operating policies such as the storage assignment rules and the effects of these policies can be considered as further study.

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