論 文
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선형시스템을 위한 개선된 수렴속도를 갖는 기준모델 적응제어 -Parameter Adaptation Method

Model Reference Adaptive Control for Linear System with Improved Convergence Rate- Parameter Adaptation Method

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요 약

잡음에 의하여 교란되고 미지의 계수를 갖는 선형시스템에 대한 적응제어기가 PARAMETER ADAPTATION 방법에 의하여 설계된다. 이 제어기는 LYAPUNOV DIRECT METHOD에 기초하며, 기준모델의 추종오차를 줄이고 수렴속도를 향상시키기 위하여 간접-보조최적해를 구한다. 시변 계수의 영향과 PLANT의 교란에 대응하는 적절한 보상이 이루어지며, 모든 설계를 통하여 미지의 계수에 대한 IDENTIFICATION을 요하지 않는다.

Abstract-Adaptive controllers for linear unknown coefficient system, that is corrupted by disturbance, are designed by parameter adaptation model reference adaptive control(MRAC). This design is stemmed from the Lyapunov direct method. To reduce the model following error and to improve the convergence rate of the design, an indirect-suboptimal control law is derived. Proper compensation for the effects of time-varying coefficients and plant disturbance are suggested. In the design procedure no complete identification of unknown coefficients are required.

1. Introduction

A major part of the adaptive control schemes are concerned around model reference adaptive control (MRAC). This method is extensively used by several researchers in conjunction with various applications. There are a number of ways, as indicated in the list of references,[2), 3), 5)~20), 22), 23)], that MRAC can be set for an application. Some of these schemes have been actually developed from stability point of view. In any event the stability analyses of these designs must thoroughly be reviewed. The Lyapunov direct method and the Popov hyperstability method are perhaps the most widely used approaches to analyze the stability issues of an MRAC design. Since the MRAC method have been extensively used as an analytical tool to design various controllers from the stability point of view and based on the

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Lyapunov direct method, therefore it will be concerned with the design aspect of the controller. That design will become stable in the sense of Lyapunov. The interesting feature of the applications of the Lyapunov method in MRAC design is that it also enables us to have a measure of convergence rate of the adaptive scheme for analysis[1], although this task is not trivial. Such design will find many interesting applications[5]. One contribution of this paper is to solve, although indirectly, for an optimal measure of the convergence rate of the adaptive schemes that are designed based on the Lyapunov direct method. These controllers have developed with applications of adaptive control theory to robot manipulator systems in[9)~11)]. The results are, however, general enough to be used in a number of other dynamical systems.

Before presenting the results, two different methods of parameter adaptation and signal synthesis adaptation are stated. In parameter adaptation method, feedforward and / or feedback gain matrices are adjusted so as to reduce the generalized error between the plant and the corresponding reference model. This method, in general, assures asympototic stability[8), 15)], but this method requires perfect model matching for asymptotic stability[13)]; direct adjustability and matchability of parameters[15)]. In this paper, parameter adaptation method is studied to improve the system performance. Compensation against the effects of time- varying coefficients and system uncertainties are suggested.

The organization of this paper is as follows. In Section 2, the problem statement is presented. In Section 3, parameter adaptation MRAC based on the Lyapunov direct method and without identification of unknown coefficients is developed. Conclusions are deferred to Section 4.

2. Problem Statement

Consider a plant which has unknown time-varying cofficients as follows.

$$P : \dot{x}_p(t) = A_p(t)x_p(t) + B_p(t)u(t) + \mathbf{v}(t) \quad (1)$$

where $A_p(t) \in R^{n\times n}$, $B_p(t) \in R^{n\times r}$ are time-varying unknown coefficient matrices, such that the piar (A_p, B_p) is completely controllable for $n \geqq r$: $x_p(t) \in R^n$ is directly measurable state vector: $u(t) \in R^r$ is the adaptive control input vector to be adjusted by certain adaptive mechanism described in the sequel: and $v(t) \in R^n$ is uncertainty vector representing unknown additive environmental disturbance such that

$$\| v(t) \| \leqq \| v(t) \|_{max} \triangleq \zeta_v,$$

where $\| \cdot \|$ represents Euclidean norm, and subscript max is maximum value of the norm.

The reference model for the above plant is described by

$$M : \dot{x}_m(t) = A_m(t)x_m(t) + B_m(t)w(t), \quad (2)$$

where $A_m \in R^{n\times n}$, $B_m \in R^{n\times r}$ are constant matrices such that the pair (A_m, B_m) is completely controllable, and A_m is hurwitzian matrix: $x_m(t) \in R^n$ is the state vector: and $w(t) \in R^r$ is the reference input vector such that

$$\| w(t) \| \leqq \| w(t) \|_{max} \triangleq \zeta_w$$

The objective of this study is to design adaptive controller to force the state of the plant (1) to follow that of the reference model (2). Furthermore, this design will result in fast-converging error between the above two states. These problems are addressed in parameter adaptation method which is stemming from the Lyapunov direct method.

3. Parameter Adaptation Method

3.1 Introduction

In this section, a plant that has following properties is considered. Study of this properties has organized with the author's research in design of controllers for mechanical manipulators. These dynamical systems enjoy special properties that are the motivating factors in development of the present paper. Suppose $B_p = [B^T_{p1}, B^T_{p2}]^T$, where $B_{p1} \in R^{(n-r)\times r}$ and $B_{p2} \in R^{r\times r}$. If B_{p1} were a null matrix and

$B_{p2}=B^T_{p2}>0$, then this would have been correspond to the dynamic equation of a mechanical manipulator[9),10)]. In thelater case the uncertainty vector v is regarded as $v=[v^T_1, v^T_2]^T$ with $v_1=0_{n-r,1} \in R^{n-r}$ and $v_2 \in R^r$.

Employing adaptive feedback gains $K \in R^{r\times n}$ and $H \in R^{r\times r}$ to control plant as that of Fig. 1, perfect model matching can be achieved when $K=\bar{K}$ and $H=\bar{H}$ provided that $v(t)=0$ and $u_s=0$. Here $\bar{K}=B_p^+(A_m-A_p)$ and $\bar{H}=B_p^+ B_m$. The superscript $^+$ denotes the left Penrose pseudo-inverse.

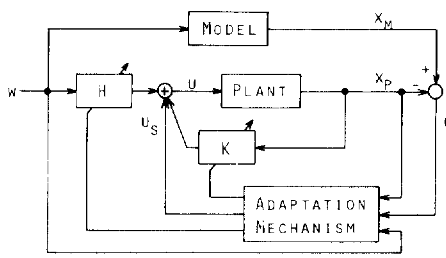

Fig.1 Parameter adaptation MRAC.

The plant (1) with control law $u=Kx_p+H_w+u_s$ becomes

$$\dot{x}_p=[A_p+B_pK]x_p+B_pHw+B_pu_s+v \tag{3}$$

Defining the error vector $e \triangleq x_m-x_p$, then we have

$$\dot{e}=A_me+B_p\Phi_r-B_pu_s-v, \tag{4}$$

where $\Phi \triangleq [\Phi_1, \Phi_2]=[\bar{K}-K, \bar{H}-H]$ and $r \triangleq [r_1^T, r_2^T]^T = [x_p^T, w^T]^T$.

In the following adaptive laws are derived for constant (or slowly-varying) coeficients A_p and B_p, with $v=0$, The corresponding adaptive laws for time-varying coefficient system with $v\neq 0$ is studied subsequently.

3.2. System of Slowly-Varying Coefficients Without Uncertainty Vector

3.2.1 Stable Adaptive Law

Consider B_p to be constant during the adaptation process and $v=0$. Then the adaptive control law for state error equation (4) are obtained from the following lemmas. For the first step of developing an adaptive law, we regard that B_p is known to us.

Lemma 1 : Conventional Adaptive Control[9),10)]

The system of differential equation (4) with $u_s=0$, and

$$\dot{\Phi}=-S^{-1}B_p^T\, Per^T \tag{5}$$

is asymptotically stable, where $0<P=P^T \in R^{n\times n}$ is the solution of

$$A_m^T\, P+PA_m+Q=\theta,\ \ Q=Q^T>\theta \tag{6}$$

and $0<S=S^T \in R^{r\times r}$.

Proof : Defining a positive definite function V_1 as the Lyapunov function

$$V_1=e^TPe+Tr\{\Phi^TS\Phi\}, \tag{7}$$

where Tr denotes the trace of matrix. Then the derivative of (7) along (4) results in

$$\dot{V}_1=-e^TQe+2Tr\{\Phi^T(S\dot{\Phi}+B_p^T\, Per^T)\}. \tag{8}$$

Substituting (5) into (8) yields $\dot{V}_1=-e^TQe\leq 0$, and the equality holds if and only if all elements of e are zero. Q.E.D.

The next step is to derive an adaptive law for the plant whose coefficients B_p are unknown, but those certain characteristics are known to us.

Remark 1 : From the above lamma, if we select $S=\gamma_1 YB_{p2}$ for $B_{p1}=0$ and symmetric definite matrix B_{p2}, then the adaptive law (5) is represented by

$$\dot{\Phi}=-\gamma_1^{-1}YLPer^T, \tag{9}$$

where $L=[0_{r,n-r}, I_r]$ ($0_{r,n-r} \in R^{r\times(n-r)}$ is a null matrix and $I_r \in R^{r\times r}$ is an identity matrix), and $0<\gamma_1$ is a weighing factor. Here Y is defined as follows.

$$Y=\begin{cases} 1, & \text{if } B_{p2}>0, \\ -1, & \text{if } B_{p2}<0. \end{cases} \tag{10}$$

The result in Remark 1 can be found in[9), 10)]. If components of r(t) are composed of distinct frequences, then dynamic systems described by (4) and (5) or (9) are uniformly asymptotically stable in the space of $\{e, \Phi\}$,[15)]. An integral control law $u_S=u_Z$ is applied to the input stage to improve the performance of adaptive system.

Lemma 2 : The system of differential equations (4), (5) or (9) and

$$\dot{u}_Z=-m(t)u_Z+2U^{-1}B_p^T Pe \tag{11}$$

is stable for $0<U=U^T \in R^{r\times r}$, if

$$m(t)>-\lambda_{min}(Q)\|e\|^2/\{\lambda_{min}(U)\|U_Z\|^2\}, \quad \text{for } t\in[t_0, \infty) \tag{12}$$

Furthermore, the system is asymptotically stable if $m(t)>0$.

Proof : Defining a positive definite function V_2 as the Lyapunov function

$$V_2=e^TPe+Tr\{\Phi^T S\Phi\}+1/2u_Z^T Uu_Z, \tag{13}$$

then derivative of (13) along (4) results in

$$\dot{V}_2=-e^TQe+2Tr\{\Phi^T(S\dot{\Phi}+B_p^T Per^T)\}+u_Z^T(U\dot{u}_Z-2B_p^T Pe). \tag{14}$$

Substituting (5) or (9) and (12) into (14) yields

$$\dot{V}_2=-e^TQe-m(t)u_Z^T Uu_Z \leqq -\lambda_{min}(Q)\|e\|^2-m(t)\lambda_{min}(U)\|u_Z\|^2\leqq 0$$

The equality ($\dot{V}=0$) holds if and only if all elements of e and u_Z are zero for $m(t)>0$. Therefore the system is asymtotically stable. Q.E.D.

Remark 2 : From the above lamma, if we select $U=2\gamma_2 YB_{P2}$ for $B_{P1}=0$ and symmetric definite matrix B_{P2}, then the adaptive law (11) is replaced by

$$\dot{u}_Z=-m(t)u_Z+\gamma_2^{-1}YLPe. \tag{15}$$

Here $0<\gamma_2$ is a weighting factor.

3.2.2 Indirect-Suboptimal Control Law

As in the case of signal synthesis method (refer to the companion paper[24)], a lower bound of $\eta_1=-\dot{V}_1/V_1$ is used[4)] to find a suboptimal control law u_Z viaan "optimal" m(t).

$$\eta_1\geqq\frac{\lambda_{min}(Q)\|e\|^2+m(t)\lambda_{min}(U)\|u_Z\|^2}{\lambda_{max}(P)\|e\|^2+Tr\{\Phi^T S\Phi\}+\frac{1}{2}\lambda_{max}(U)\|u_Z\|^2} \triangleq g_1(m, u_Z, e). \tag{16}$$

wrere $\lambda_{max}(\cdot)$ represents the maximum eigenvalue of $(\cdot)$. To achieve the condition of $g_1^0(m^0, u_Z^0, e)\geqq g_1(m, u_Z, e)$ for a given state e, the criterion

$$J=\frac{1}{2}\int_0^\infty\{\alpha\|u_Z\|^2+\beta m^2\}dt \tag{17}$$

is to be maximized subject to

$$\dot{u}_Z=-mu_Z+2U^{-1}B_p^T Pe\triangleq-mu_Z+f, \tag{18}$$

to find the "optimal" m(t). By treating the f in (18) as one entity, an approximated solution (discarded after tenth order of $\|u_Z\|$) is obtained by using.

Hamilton-Jacobi-Bellman equation[24)]. The procedure to solve the criterion (17) is the same in[24)], however the result can be used in parameter adaptation method as well as in signal synthesis method.

A set of indirect solutions of $g_1(m, u_Z, e)$, namely G_1°, obtained is as follows.

$$G_1^0=\{m^0 \mid \text{the solution of Max. J, for } \zeta_m>m^0\} \tag{19}$$

where $\zeta_m \; \lambda_{min}(Q)\lambda_{max}(U)/2\{\lambda_{max}(P)\lambda_{min}(U)\}$

The result is summerized in the following proposition.

proposition 1 : The system of differential equation (4) with (5) or (9) and $u_S=u_Z$ such that

$$\dot{u}_Z=-m^0 u_Z+2U^{-1}B_p^T Pe, \quad \text{with } u_Z(t_0)=0, \tag{20}$$

is asymptotically stable, where m° is described by (19). The elements of (19) are as follows.

The "optimal" m that solves Max. J is

$$m^0 = \begin{cases} -m, & \text{for } m < 0 \\ 0, & \text{for } m \geqq 0 \end{cases} \quad (21a)$$

where

$$m \simeq m_1 + m_2, \text{ for} \quad (21b)$$

$$m_1 = -\frac{\alpha}{2\beta} \| u_Z \|^4 (f\, u_Z)^{-1}, \text{ and}$$

$$m_2 = \frac{\alpha^2}{8\beta^2} \| u_Z \|^{10} (f\, u_Z)^{-3}$$

and a sufficient condition of (21a) and (21b) is

$$|m_1| + |m_2| \leqq \sqrt{\alpha/\beta}\ \| u_z \| \quad (21c)$$

Derivation of (21) is carried out by using the well known procedures in[21].

Following numerical example shows the applications of the above results.

Example 1 : Consider plant (1) with v=0, and model (2) as follows.

$$P : \dot{x}_p = -x_p + 0.5u \quad (22a)$$
$$M : \dot{x}_m = -20x_m + 2w, \quad (22b)$$

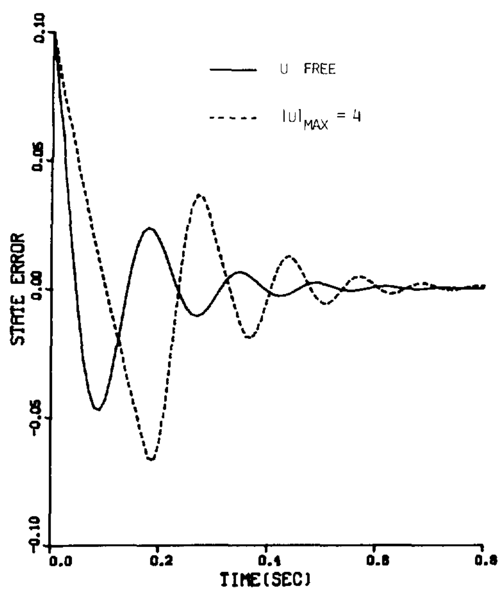

Fig. 2 Sate error with m=0.

with $K(t_0) = -30$and $H(t_0) = 3$. The numerical results shown in Fig. 2 and Fig. 3 demonstrate the improvement of systemby application of the results.

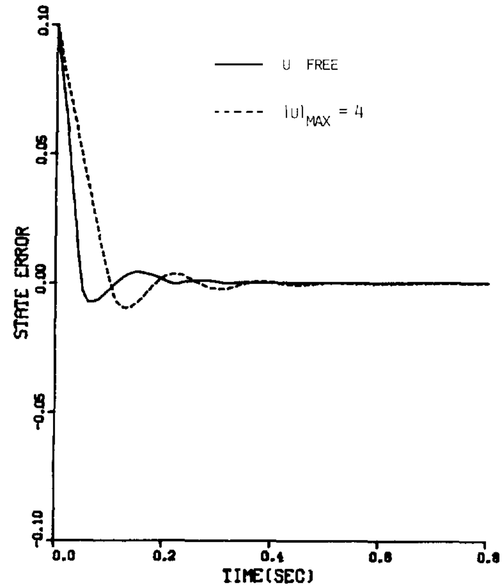

Fig. 3 State error with indirect-suboptimal m^0(cf., (19)).

3.3 System with Time-Varying Coefficients and Uncertainty Vector

In this section it is assumed that the coefficients of plant (1) are changed during the adaptation process and uncertainty vector $v \neq 0$ exists. Consider the Lyapunov function (13) with m as given by (19). In this case the increasing function of V such that the V satisfies the Lyapunov function should carefully be reviewed. In this regard the weighting matrix S and U are assumed to be satisfied in the above condition (Lyapunov function).

Derivative of this type Lyapunov function along (4), together with (5) and (20), is

$$\dot{V} = -e^T Q e - m^0 u_Z^T\ U u_Z + Tr\{\Phi^T \dot{S}\, \Phi\} + \frac{1}{2} u_Z^T\ \dot{U} u_Z + 2v^T P e. \quad (23)$$

Here S and U are in most cases constant matrices, but S and U can be changed when we select $S=\gamma_1 YB_{p2}$ and $U=\gamma_2 YB_{p2}$. The adaptive law obtained from Lemma 1 for this case becomes

$$\dot{\Psi}_b=\gamma_1^{-1}YLPer^T+\dot{\Psi}_a \tag{24}$$

here $\Psi_a=[\bar{K}, \bar{H}]$ and $\Psi_b=[K, H]$. With the assumption that the system coefficient matrices remain constant during the adaptation process, $\dot{\Psi}_a$ was regarded zero in the previous analysis (Section 3. 2). But in the present case of time-varying coefficients this is not true. Indeed, we first use

$$\dot{\Psi}_b=\gamma_1^{-1}YLPer^T \tag{25}$$

then the effect of $\dot{\Psi}_a$ in adaptive control law (24) can be compensated as follows.

But first, an extra input u_p is added to the earlier input term such that

$$u_s=u_z+u_p \tag{26}$$

The resulting derivative of Lyapunov function becomes

$$\dot{V}=-e^TQe-m^0u_z^T\,Uu_z+Tr\{\Phi^T\dot{S}\,\Phi+2\Phi^TS\,\dot{\Psi}_a\}+\frac{1}{2}u_z^T\dot{U}u_z+2v^TPe-2u_p^TB_p^TPe. \tag{27}$$

To achieve a stable control law the following criterion is chosen.

$$\underset{u_p}{\text{Min}}\ \underset{y}{\text{Max}}\ \dot{V}<0, \tag{28}$$

where y represents the uncertainties which has been caused from time-varying terms and the uncertainty vector v. The effects in the derivative of V is the third, forth and fifth term of right hand side in (27). Now two different approaches are suggested to achieve this criterion.

3.3.1 Approach-1

The inequality (28) is equivallent to

$$\underset{u_p}{\text{Min}}\ \underset{y}{\text{Max}}\ V\leq 0 \tag{29}$$

Here

$$\dot{V}_p=\dot{V}_{p1}+\dot{V}_{p2}\quad \text{with} \tag{30a}$$

$$\dot{V}_{p1}=Tr\{\Phi^T\dot{S}\,\Phi-2\Phi^TS\,\dot{\Psi}_a\}+1/2u_z^T\dot{U}u_z, \text{ and} \tag{30b}$$

$$\dot{V}_{p2}=2v^Tpe-2u_p^TB_p^TPe\ (\text{or }\dot{V}_{p2}=2v^TPe-2u_p^TB_{p2}LPe). \tag{30c}$$

To follow the same design procedure about B_p in Lemma 1-Remark 1, and Lemma 2-Remark 2, the following case studies will be the same order of procedure for known B_p first, and then unknown B_p later.

Case 1-Ⅰ : Known matrix B_p

Let $u_{p1}=\dfrac{B_p^T\ Pe}{\|B_p^T\ Pe\|}\rho_1$, for $\rho_1>0$, then from (30)

$$\dot{V}_p\leq-\frac{2e^TPB_pB_p^TPe}{\|B_p^T\ Pe\|}\rho_1+\|\dot{V}_{p1}\|+2\|v^TPe\|$$
$$\leq-2\|B_p^T\ Pe\|\rho_1+2\zeta_{p1}+2\|Pe\|\zeta_v, \tag{31}$$

where $\zeta_{p1}=\frac{1}{2}\|\dot{V}_{p1}\|_{max}$. Since ζ_{p1} consists of mismatching parameters caused from the time varying terms of parameter and weighting matrices S and U, we assumed that the resulting norm of error is represented by $\zeta_{p1}=\mu_1\|Pe\|$, for $\mu_1>0$, then

$$\dot{V}_p\leq-2\{\|B_p^TPe\|\rho_1-\|Pe\|(\mu_1+\zeta_v)\}. \tag{32}$$

If ρ_1 and u_{p1} are selected by

$$\rho_1=\frac{(\mu_1+\zeta_v)\|Pe\|}{\|B_p^T\ Pe\|},\ \text{or} \tag{33a}$$

$$u_{p1}=\|B_p^T\|(\mu_1+\zeta_v), \tag{33b}$$

then $V_p\leq 0$, and the u_{p1} in (33b) satisfies the condition (29). Notice that $\mu_1=0$ when coefficients A_p, B_p, and weighting matrices S and U are constant. Further comments on μ_1 are given later.

Case 1-Ⅱ. $B_{p2}=B_{p2}^T$ is unknown definite matrix

In this case the adaptive control law (33) can not be used bacause $B_p=[0, B_{p2}^T]^T$ is unknown. But we assume that Y is known as in the most case of mechanical system, and B_{P2} is symmetric definite matrix. Let

$$u_{P2} = \frac{YLPe}{\| LPe \|}\rho_2, \text{ for } \rho_2>0, \ \zeta_{P1}=\mu_2 \| LPe \|, \text{ for } \mu_2>0. \text{ Then from (30),}$$

$$\dot V_p \leq -\frac{2(LPe)^T\bar B_{p2}(LPe)}{\| LPe \|}\rho_2+2\zeta_{p1}+2\| LPe \|\zeta_V$$

$$= -\frac{2(Lpe)^T(\bar B_{p2}-I_r)(LPe)}{\| LPe \|}\rho_2-2[\rho_2-\mu_2-\zeta_V]\| LPe \|$$

$$\leq -2\{[\lambda_{min}(\bar B_{p2}-I_r)+1]\rho_2-\mu_2-\zeta_V\}\| LPe \| \quad (34)$$

where $B_{P2}=Y\bar B_{P2}$. If ρ_2 is selected by

$$\rho_2=\frac{\mu_2+\zeta_V}{\lambda_{min}(\bar B_{p2}-I_r)+1}, \quad (35)$$

then $\dot V_P\leq 0$ because $\lambda_{min}(\bar B_{P2}-I_r)>-1$.

3.3.2 Approach-2

In the Approach-1, it is assumed that $\| \dot V_{P1} \| = \mu_1 \| Pe \| = \mu_2 \| LPe \|$. Detemining the value of μ_1 in certain application may not be easy. Therefore a different approach is suggested next.

Case 2-I : Known matrix B_P

Let $u_{p3}=\frac{B_p^T Pe}{\| B_p^T Pe \|}\rho_3$, for $\rho_3>0$, then from (30)

$$\dot V_{p2}\leq -2\{\| B_p^T Pe \|\rho_3-\| Pe \|\zeta_V\} \quad (36)$$

If we select $\rho_3=\| Pe \|\zeta_V / \| B_P^T Pe \|$, then $\dot V_{P2}\leq 0$. For this u_{P3}, $\dot V$ in (27) satisfies the folowing inequality.

$$\dot V\leq -e^TQe-m^0u_Z^T Uu_Z+2\zeta_\rho. \quad (37)$$

To have $\dot V<0$, it is required that

$$m^0>(\lambda_{min}(Q)\| e \|^2+2\zeta_{\rho 1})/(\lambda_{min}(U)\| u_Z \|^2) \quad (38)$$

Then the u_{P3} and condition (38) insure the overall stability of adaptive system.

Case 2-II : $B_{P2}=B_P^T{}_2$ is unknown definite matrix

Let $u_{P4}=\frac{YLPe}{\| LPe \|}\rho_4$, for $\rho_4>0$, then by the same procedure as in the the Case 1-II,

$$\dot V_{p2}\leq -2[\lambda_{min}(\bar B_{p2}-I_r)+1]\rho_4-\zeta_V]\| LPe \|. \quad (39)$$

For $\rho_4=\zeta_V/[\lambda_{min}(\bar B_{P2}-I_r)+1]$, $\dot V_{P2}\leq 0$. This u_{P4}, and condition (38) insure the overall stability of adaptive system.

In the above approaches, discontinuity of control input u_{Pi}, i=1, .., 4 may happen if $\| B_P^T Pe \|\to 0$ or $\| LPe \|\to 0$. To avoid this dificulty numerical input u_{Pi}° for sufficiently small $\delta>0$ is chosen as follows.

$$u_{pi}^0=\begin{cases}\frac{B_p^T Pe}{\| B_p^T Pe \|}\rho_i, & \text{for } \| B_p^T Pe \|\geq\delta \\ 0, & \text{for } \| B_p^T Pe \|<\delta,\ i=1,3, \text{ and}\end{cases} \quad (40a)$$

$$u_{pi}^0=\begin{cases}\frac{YLPe}{\| LPe \|}\rho_i, & \text{for } \| LPe \|\geq\delta \\ 0, & \text{for } \| LPe \|<\delta,\ i=2,\ 4\end{cases} \quad (40b)$$

Example 2 : For the following plant (41a) and the model (41b), a computer simulation study is performed using a fourth-order Runge-Kutta method.

$$P:\dot x_p=\begin{bmatrix}0 & 1\\ 2 & -1\end{bmatrix}x_p+\begin{bmatrix}0\\ 2\end{bmatrix}u+\begin{bmatrix}0\\ -0.5\cos(10t)-1.4\end{bmatrix} \quad (41a)$$

$$M:\dot x_m=\begin{bmatrix}0 & 1\\ -4 & -4\end{bmatrix}x_m+\begin{bmatrix}0\\ 3\end{bmatrix}w. \quad (41b)$$

The numerical valus of feedback gains are selected as K= [-2.2, -1] and H=1.2, with $\Gamma_1=I_2, \Gamma_2=0.5I_2$. The weighting constant P is

$$P=\begin{bmatrix}2,000 & 300\\ 300 & 30\end{bmatrix}. \quad (42)$$

The simulation results shown in Fig. 4 and Fig. 5 demonstrate the improvement of system behavior with indirect-suboptimal m°. Fig. 6 shows better results using the compensation scheme than those adaptive controllers without any compensation. In Fig. 6, it is observed that the error caused from the time varying terms are reduced.

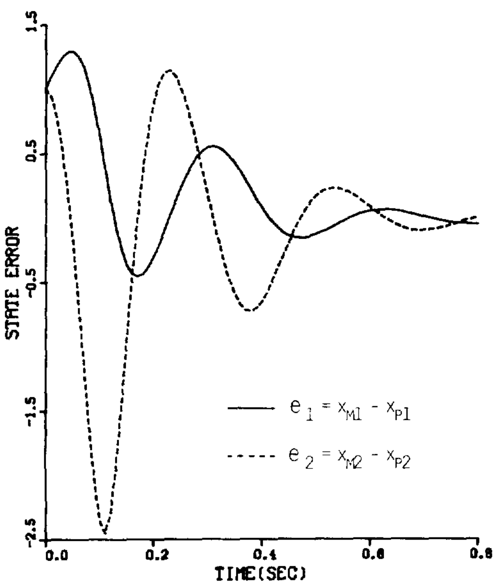

Fig. 4 State error with m=0.

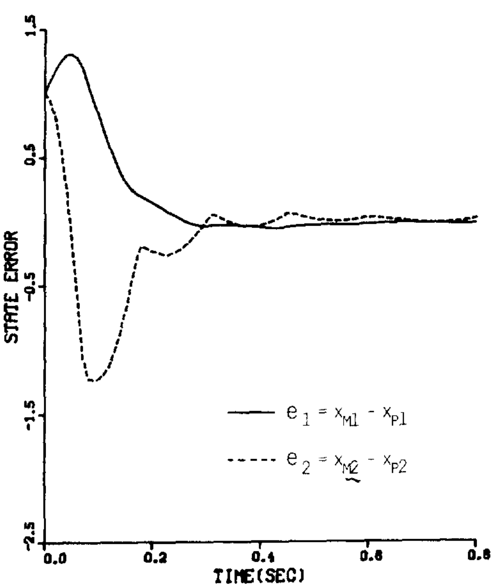

Fig. 5 State error with indirect-suboptimal m°(cf., (19)).

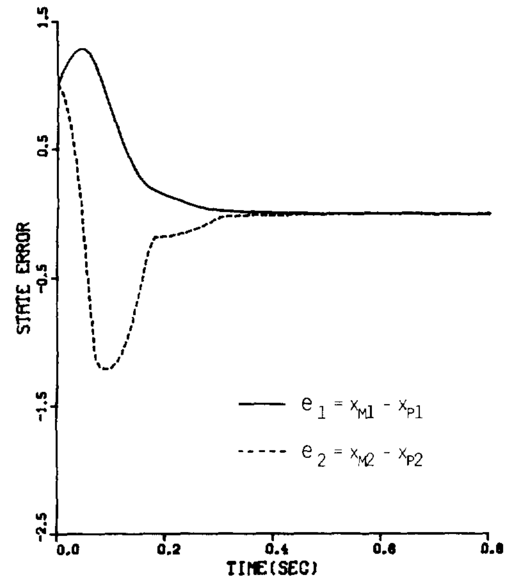

Fig. 6 State error with indirect-suboptimal m° and with compensation of uncertainty vector.

4. Conclusions

A plant with unknown coefficients and additive uncertainty vector is considered in this paper. For this system adaptive controllers are designed so that the plant state follows the state of corrsponding model. These controllers are designed based on the Lyapunov direct method and the resulting control schemes are developed by parameter adaptaion method. Simulation result shows asymptotic stsbility of state error. The integral input with indirect suboptimal solution reduces the norm of these state error substantially. This method(direct adaptation) does not require the complete identification of unknown coefficients, thus the designed controller is fast and can be used in the real-time.

In the design procedure, delay of adjustable system has not been considered, but present information is used to control the unknown plant. The controllers for the corresponding discrete systems may be designed similarly. In the above simulation numerical constraint on the input vector have improved, although the issue of design with input constraint is not discussd theoretically in this paper. These issues and the applications of this controller in design for mechanical systems are subject of future research.

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