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Design of Decentralized State Observer for Large Scale Interconnected System

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ABSTRACT

A design method of decentralized state observer for large scale interconnected systems is proposed by the use of interconnection rejection approach and interconnection modelling technique. The proposed design method is developed based on the interconnection partitioning. Therefore partitioning conditions are suggested. And the conditions for observer pole assignment and observer parameter determination procedures are described for possible interconnection patterns. The decentralized state observer gives good estimates without any information on the interconnection variables and estimation performance may not be affected by arbitrary structural perturbation of the interconnections. In addition, a numerical example is given to explain the design procedures and to show the estimation performance of the decentralized observer.

1. Introduction

A design method of decentralized observer that must be included in the decentralized state feedback control system is proposed in this paper. The decentralized control method have been proposed and developed, with hierachical control method, for the purpose of controlling large scale interconnected systems.[1] And information exchange between each subsystem cannot be performed and all the subsystems are controlled by the use of their own informations only under decentralized control environments.

Almost all of the studies in this area have been treated the controller design problems and the conditions for pole assignment and stabilization were established and various algorithms have been developed for controller design.[2) 3) 4) 7)]

However, the problem of decentralized estimation has not been received much attention in spite of the fact that the estimator is an indispensible component in the decentralized control systems. D.D. Siljak and his coworkers have proposed useful algorithms for the design of decentralized estimators by considering the interconnection terms as the structural perturbation of the system parameters.[5),6),8)] But there is some deficiencies that the convergence of the estimates is guaranteed only when the interconnections satisfy some restrictive norm condition.[8)] And the algorithms are applicable only to weakly interconnected systems.

Recently, P. Ficklscherer suggested a design method in which observer gain matrix is determined algebraically to remove all the effects of interconnections on the state estimates.

However the method also is with the difficulties in applications due to the fact that the rejection of the effect of interconnections on estimates and the pole assignment of the observer must be simultaneously per-

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formed by a constant gain matrix.

The purpose of this study is to propose a new design method of decentralized state observer by using the appropriate combination of interconnection rejection method [9][10] and interconnection modelling method [4] such that the resultant decentralized observer has simple and mild existence condition and gives good estimates even in the cases that no information of interconnection variables are available.

The contents of this paper is as follows. Firstly, system and problem descriptions are given is section(2). Secondly design procedure of the proposed decentralized observer is described in section(3). Thirdly, interconnection partitioning conditions and observer pole assignability conditions are suggested and the matrix equations for observer parameters are derived in section(4). And a numerical example is given to show the effectiveness of the design method and the estimation performance of the decentralized state observer.

2. System and problem descriptions

Consider a linear system driven by following equations.

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y = Cx(t) \tag{1}$$

where $x(t) \in R^n$ is state vector, $y(t) \in R^n$ is output vector and $u(t) \in R^m$ is input vector. And A, B, C are (n, n), (n, m) and (p, n) dimensional matrices, respectively. If Eq(1) is an interconnected system with s-subsystems, then system parameters can be represented as follows.

$$A = \begin{bmatrix} A_1 & A_{12} \cdots A_{1s} \\ A_{21} & A_{22} \cdots A_{2s} \\ \cdots & \cdots \\ A_{s1} & A_{s2} \cdots A_{ss} \end{bmatrix}, C = \begin{bmatrix} C_1 & C_{12} \cdots C_{1s} \\ C_{21} & C_2 \cdots C_{2s} \\ \cdots & \cdots \\ C_{s1} & C_{s2} \cdots C_s \end{bmatrix},$$
$$B = \mathrm{diag}(B_i) \tag{2}$$

And the representation of ith subsystem becomes as Eq(3).

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{s} A_{ij} x_j(t) \tag{3}$$

$$y_i(t) = C_i x_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{s} C_{ij} x_j(t)$$

where $x_i \in R^{n_i}$, $x_i \in R^{p_i}$, $u_i \in R^{m_i}$ and $x_j (j=1, 2, \cdots i-1, i+1, \cdots S)$ are state, output, input and interconnection vector respectively. And following relations exist between Eq(1) and Eq(3).

$$n = \sum_{i=1}^{s} n_i, \quad p = \sum_{i=1}^{s} p_i \text{ and } m = \sum_{i=1}^{m} m_i$$

At this point, redefine the interconnection matrices as follows for convenience.

$$AM_i = (A_{i,1} \cdots A_{i,i-1} \; A_{i,i+1} \cdots A_{i,s}), \; AM_i \in R^{(n_i, n-n_i)} \tag{4}$$

$$CM_i = (C_{i,1} \cdots C_{i,i-1} \; C_{i,i+1} \cdots C_{i,s}), \; CM_i \in R^{(p_i, n-n_i)} \tag{5}$$

Now, the design problem of decentralized observer for Eq(1) is reduced to the design problem of independant observers (one for each subsystem) so that each observer gives good state estimates even in the presence of unknown nonmeasurable interconnection variables.

In the following sections, a design method of decentralized observer that its applicability is not restricted by the pattern and magnitude of interconnections and can therefore be applicable to almost all of the interconnected systems is proposed. Since the design method includes interconnection partitioning procedure by which interconnections are partitioned into two parts where one is the variables that can be rejected by observer gain matrix and the other is the variables that cannot be eliminated by the matrix and must be modelled, the partitioning conditions are established. In addition, the conditions for pole assignment of the observer are derived and the details of observer parameter determination are described.

3. Design procedure of the proposed decentralized observer

The design method contains following procedures:

a. Transformation of interconnection variables

b. Interconnection partitioning and augmented system construction.

c. Determination of observer parameters such that remaining interconnections are removed from the observer dynamic equation and that convergence of the estimates can be guaranteed.

It is note worthy that the existence of the observer may be provided by the conventional observability condition if the interconnection partitioning is performed to meet the partitioning conditions that will be given in section(4).

(a) Transformation of interconnection variables

If the given interconnected system satisfies the relation, $n_i < n - n_i$ there exists $((n-n_i), (n-n_i))$ dimensional nonsingular tranformation matrix $\wedge_i$ such that

$$(D_i : 0) = AM_i \wedge_i \quad (6)$$
$$(H_i : 0) = CM_i \wedge_i \quad (7)$$

and transformed variables can be difined as Eq(8).[10]

$$[x_1, x_2, \cdots x_{i-1}, x_{i+1}, \cdots x_s] = \wedge_i \begin{pmatrix} \eta_i \\ \mu_i \end{pmatrix} \quad (8)$$

where each element of μ_i is linear combination of that of η_i and $\eta_i \in R^{q_i}$, $\mu_i \in R^{(n-n_i-q_i)}$. And D_i, H_i are (n_i, q_i), (p_i, q_i) dimensional matrices. By substituting above relations, a new representation of Eq(3) is obtained as Eq(9).

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + D_i \eta_i(t) + B_i u_i(t) \\ y_i(t) &= C_i x_i(t) + H_i \eta_i(t), \quad i = 1, 2, \cdots, s \end{aligned} \quad (9)$$

(b) Interconnection partitioning modelling and construction of augmented system

Although the next step of the design procedure is interconnection partitioning,it is assumed that the partitioning is completed as Eq(10) because the partitioning conditions have not been established.(It will be described in section (4).

$$\eta_i(t) = \begin{bmatrix} \eta_{i1}(t) \\ \eta_{i2}(t) \end{bmatrix} \quad (10)$$

where $\eta_{i1} \in R^{r_i}$ is a vector that can be rejected by observer gain matrix and $\eta_{i2} \in R^{q_i - r_i}$ is a vector that cannot be rejected. By applying Eq(10) into Eq(9), following equations are obtained.

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + (D_{i1} : D_{i2}) \begin{bmatrix} \eta_{i1}(t) \\ \eta_{i2}(t) \end{bmatrix} + B_i u_i(t) \\ y_i(t) &= C_i x_i(t) + (H_{i1} : H_{i2}) \begin{bmatrix} \eta_{i1}(t) \\ \eta_{i2}(t) \end{bmatrix}, \quad i = 1, 2, \cdots, s \end{aligned} \quad (11)$$

In Eq(11) H_{i1} is (p_i, r_i) dimensional matrix with rank $(H_{i1}) < p_i$. And D_{i1}, D_{i2} and H_{i2} are (n_i, r_i), $(n_i, q_i - r_i)$ and $(p_i, q_i - r_i)$ dimensional matrices, respectively. Since $\eta_{i2}(t)$ is a nonrejectable vecctor in the observer equations it must be modeled as following dynamic equations.

$$\begin{aligned} \dot{z}_{i2}(t) &= E_{i2} z_{i2}(t) \\ \eta_{i2}(t) &= F_{i2} z_{i2}(t) \end{aligned} \quad (12)$$

where E_{i2} and F_{i2} are $(\sigma(q_i - r_i), \sigma(q_i - r_i))$, $(q_i - r_i, \sigma(q_i - r_i))$ dimensional matrices and the elements of the matrices can be selected as Eq(13) by considering the fact that all the variables in $\eta_{i2}(t)$ are the linear combination of the states of other subsystems and the fact that these are unavailable quantities.

$$E_{i2} = \begin{bmatrix} 0 & I_{(\sigma-1)(q_i - r_i)} \\ 0 & 0 \end{bmatrix}, \quad F_{i2} = (I_{q_i - r_i} : 0) \quad (13)$$

where σ is the order of the dynamic equation for one variable which can be determined from the waveform structure of the interconnection variables and desired accuracy of the estimates.

Then augmented system is obtained as Eq(14) by substitution of Eq(12) into Eq(11).

$$\begin{aligned} \dot{\bar{x}}_i(t) &= \bar{A}_i \bar{x}_i(t) + \bar{D}_{i1} \eta_{i1}(t) + \bar{B}_i u_i(t) \\ y_i(t) &= \bar{C}_i \bar{x}_i(t) + H_{i1} \eta_{i1}(t) \end{aligned} \quad (14)$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & D_{i2} F_{i2} \\ 0 & E_{i2} \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$$
$$\bar{C}_i = (C_i : H_{i2} F_{i2}), \quad \bar{D}_{i1} = (D_{i1}^T : 0)^T \quad (15)$$

In Eq(14) (15), $\bar{x}_i = (x_i^T : z_{i2}^T)$ is l_i dimensional vector where l_i is defined as follows.

$$l_i = n_i + \sigma(q_i - r_i)$$

Then A_i, B_i, C_i, D_i and H_{i1} are (l_i, l_i), (l_i, m_i), (p_i, l_i), (l_i, r_i) and (p_i, r_i) dimensional matrices, respectively.

(c) General form of the decentralized observer

The decentralized observer for the augmented subsystem of Eq(14), (15) can be described as following dynamic equations.

$$\dot{z}_i(t)=(\bar{A}_i-L_i\bar{C}_i)z_i(t)+((\bar{A}_i-L_i\bar{C}_i)M_i+L_i+N_i)y_i(t)+T_i\bar{B}_iu_i(t)$$
$$\hat{\bar{x}}_i(t)=z_i(t)+M_iy_i(t) \quad i=1,2,\cdots\cdots s \tag{16}$$

where $z_i(t)$ is l_i dimensional state vector of the observer and $\hat{\bar{x}}_i$ is estimated vector of x_i

Now, the remaining problem is to determine the observer parameters L_i, M_i, N_i, T_i such that following conditions are satisfied.

$$\text{(i)}\ \ \mathrm{Re}(\lambda_k(\bar{A}_i-L_i\bar{C}_i)<0,\ \ k=1,2,\cdots,l_i \tag{17}$$
$$\text{(ii)}\ \ \lim_{t\to\infty}(\hat{\bar{x}}_i(t)-\bar{x}_i(t))=\lim_{t\to\infty}e_i(t)=0 \tag{18}$$

The existence of the observer parameters that satisfy Eq (17), (18) is directly related with the result of interconnection partitioning. And the interconnection partitioning conditions and pole assignability conditions will be set up in following sections.

4. Interconnection partitioning conditions and observer pole assignability conditions

4.1 Interconection partitioning conditions

As previously described, the interconnection partitioning must be performed before the augmented subsystem is constructed. And it is a very important one in a view point that the procedure must be performed so as to meet Eq(18).

In this section, the conditions for arbitrary partitioning, say Eq(10), to meet Eq(18) are derived at first by the use of the augmented subsystem parameters. And the paritioning conditions that are described by the original system parameters that can be applied before the augmented subsystem construction are derived from the above conditions. In the subsequent developments, we assume the relation, $\mathrm{rank}(H_{i1})<p_i$ and existence of H_{i1}^+ and $\bar{C}_i^T$, where $\bar{C}_i^+=\bar{C}_i^T(\bar{C}_i\bar{C}_i^T)^{-1}$ and $H_{ij}^+=(H_{ij}^TH_{ij})^{-1}H_{ij}^T$ for $j=1,2$.

Theorem(1) : Interconnection partitioning condition (with augmented subsystem parameters)

For (p_i, l_i) dimensional matrix of Eq(19) defined by the augmented system parameters.

$$V_i=(Ip_i-H_{i1}H_{i1}^+)\bar{C}_i\bar{A}_i(I_{l_i}-\bar{C}_i^+\bar{C}_i) \tag{19}$$

if there exists (l_i, p_i) dimensional matrix K_i such that

$$K_iV_i=0 \tag{20}$$
$$(I_{li}-K_i(Ip_i-H_{i1}H_{i1}^+)\bar{C}_i(Ir_i+H_{i1}^+H_{i1})=0 \tag{21}$$

then, $\eta_{i1}(t)$ in Eqs(10)(11) and Eq(14) can be completely removed by observer gain from the observer dynamic equation.

Proof : Define the estimation error vector as follows.

$$e_i(t)=\hat{\bar{x}}_i(t)-\bar{x}_i(t) \tag{22}$$

Then, following error dynamic equation can be derived from Eq(14) and Eq(16)

$$\dot{e}_i(t)=(\bar{A}_i-L_i\bar{C}_i)e_i(t)+(T_i+M_i\bar{C}_i-I_{l_i})\bar{B}_iu_i(t)+M_iH_{i1}\dot{\eta}_{i1}(t)+(N_i\bar{C}_i+M_i\bar{C}_i\bar{A}_i)x_i(t)+(L_iH_{i1}+N_iH_{i1}+M_i\bar{C}_i\bar{D}_{i1}-\bar{D}_{i1})\eta_{i1}(t) \tag{23}$$

And following matrix equations most be satisfied in order to remove the effects of $\eta_{i1}(t)$ on the estimation error.

$$M_iH_{i1}=0 \tag{24a}$$
$$N_iC_i+M_i\bar{C}_i\bar{A}_i=0 \tag{24b}$$
$$L_iH_{i1}+N_iH_{i1}+M_i\bar{C}_i\bar{D}_{i1}-\bar{D}_{i1}=0 \tag{24c}$$
$$T_i\bar{B}_i-(I_{l_i}-M_i\bar{C}_i)\bar{B}_i=0 \tag{25}$$

Then the observer parameters can be obtained from Eqs(24), (25) by the assumption $\mathrm{rank}(H_{i1})=r_i<p_i\cdots(26)$ as follows.

$$M_i=K_i(Ip_i-H_{i1}H_{i1}^+) \tag{27}$$
$$N_i=-K_i(Ip_i-H_{i1}H_{i1}^+)\bar{C}_i\bar{A}_i\bar{C}_i^+ \tag{28}$$
$$L_i=D_{i1}H_{i1}^++K_i(Ip_i-H_{i1}H_{i1}^+)(\bar{C}_i\bar{A}_i\bar{C}_i^+H_{i1}-\bar{C}_i\bar{D}_{i1})H_{i1}^++S_i(Ip_i-H_{i1}H_{i1}^+) \tag{29}$$
$$T_i=I_{l_i}-K_i(Ip_i-H_{i1}H_{i1}^+)\bar{C}_i \tag{30}$$

and S is (l_i, p_i) dimensional matrix with arbitrary elements. Now, direct substitution of Eqs(27), (28) into

Eq(24b) gives condition (19), (20) and the condition (21) is obtained by direct substitution of Eqs(27), (28), (29) into Eq(24c). The Proof is completed.

Although Theorem(1) can effectively be used for the determination of decentralized observer parameters, it may not be applicable for the interconnection partitioning because it is described by agumented system parameters.

Therefore, the partitioning condition must be described by the parameters of original system description (11). And the condition is given by Collorary (1). In there, mathematical model for $\eta_{i2}(t)$ given by Eq(12) and Eq (13), is used without loss of generality.

Corollary (1). Interconnection partitioning condition (with original system parameters)

For (p_i, l_i) dimensional matrix given by Eq(31) defined by the original system parameters,

$$V_i = (Ip_i - H_{1i}H_{i1}^+)(C_i(A_i(In_i - C_i^+C_i) - D_{i2}H_{i2}^+C) : C_i(D_{i2}(I^{(q_i - r_1)} - H_{i2}^+H_{i2}) - A_iC_i^+H_{i2})F_{i2}] \quad (31)$$

$$K_iV_i = 0 \quad (32)$$

$$(I_{l_i} - K_i(Ip_i - H_{i1}H_{i1}^+)(C_i : 0)\begin{pmatrix} D_{i1} \\ 0 \end{pmatrix}(I_{r_i} - H_{i1}^+H_{i1}) = 0 \quad (33)$$

then, there exist observer parameters that satisfy Eq(24) and Eq(25). The equivalence of Eq(31) and Eq(19) can be shown by direct substitution of Eq(15) into Eq(19) and is omitted.

4.2 Conditions for observer pole assignment

When the Interconnection partitioned by applying collorary (1), an augmented subsystem (15) is constructed and observer parameters satisfying Eqs(24), (25) can be determined. However, the convergence of estimates can be provided only when there exists observer gain matrix L_i that satisfies Eq(17). Therefore, existence condition of such L_i must be established.

The condition is dependant on the parameters H_{i1} and

To derive the conditions for each cases, following representation of $(\bar{A}_i - L_i\bar{C}_i)$ is used.

$$(\bar{A} - L_i\bar{C}_i) = (\bar{A}_i - \bar{D}_{i1}H_{i1} + \bar{C}_i) - K_i(Ip_i - H_{i1}H_{i1}^+)(\bar{C}_i\bar{A}_i\bar{C}_i^+H_{i1} - \bar{C}_i\bar{D}_{i1})H_{i1}^+\bar{C}_i - S_i(Ip_i - H_{i1}H_{i1}^+)\bar{C}_i \quad (34)$$

And the conditions for each case are obtained as follows.

(a) $H_{i1}=0$

Since the R.H.S of Eq(34) becomes as $(R_i - S_iC_i)$ and since S_i s a matrix with arbitrary elements, it is always possible to set $L_i = S_i$. Therefore, arbitrary pole assignment can be achieved iff$(\bar{C}_i : \bar{A}_i)$ is completely observable pair.

(b) $H_{i1} \neq 0$, $V = 0$

Since $V_i = 0$, K_i is matrix with arbitrary elements. It is possible to set $S_i = K_i$. Therefore, all terms that are related to K_i and S_i must be used for pole assignment. And arbitrary pole assignment is possible if the pair in Eq(35) is completely observable.

$$[(Ip_i - H_{i1}H_{i1}^+)(Ip_i + \bar{C}_i\bar{A}_i\bar{C}_i^+H_{i1}H_{i1}^+ - \bar{C}_i\bar{D}_{i1}H_{i1})\bar{C}_i : (\bar{A}_i - \bar{D}_{i1}H_{i1}^+\bar{C}_i))] \quad (35)$$

(c) $H_{i1} \neq 0$, $V_i \neq 0$

In this case, pole assignment via K_i is impossible because K_i is matrix with fixed elements. But, if (i) rank $(V_i) < P_i$ and (ii)

$$[(Ip_i - H_{i1}H_{i1}^+)\bar{C}_i : (\bar{A}_i - \bar{D}_{i1}H_{i1}^+\bar{C}_i) - K_i(Ip_i - H_{i1}H_{i1}^+)(\bar{C}_i\bar{A}_i\bar{C}_i^+H_{i1} - \bar{}_i\bar{D}_{i1})H_{i1}^+\bar{C}_i] \quad (36)$$

(36) is completely observable pair, then pole assignment via S_i is possible.

4.3 Determination of observer parameters.

The observer parameters can be determined by using Eqs(27), (30) in general. It can, however, be simplified as follows according to the parameter H_{i7} and V_i.

(a) $H_{i7}=0$ $(L = S_i)$

In this case Eq(21) becomes as $K_iC_iD_{i1} = D_{i1}$ and the general solution is obtained as Eq(37)

$$K_i = \bar{D}_{i1}(\bar{C}_i\bar{D}_{i1}) + \bar{K}_i(Ip_i - (\bar{C}_i\bar{D}_{i1})(\bar{C}_i\bar{D}_{i1})^+) \quad (37)$$

where k_i is (l_i, p_i) dimensional arbitrary matrix.

The other parameters M_i, N_i, T_i are determined as Eq (38) in terms of K_i.

$$M_i = K_i \tag{38a}$$
$$N_i = -K_i\bar{C}_i\bar{A}_i\bar{C}_i^+ \tag{38b}$$
$$T_i = I_{1i} - K_i\bar{C}_i \tag{38c}$$

And L_i can be obtained by pole assignment algorithm after substitution of K_i into Eq(34) is performed.

(b) $H_{i1} \neq 0$, $V_i \neq 0 (K_i = S_i)$

In this case, K_i must be computed at first by pole assignment algorithm. Then, remaining parameters M_i N_i, T_i. L_i are determined by following equations.

$$M_i = K_i(Ip_i - H_{i1}H_{i1}^+) \tag{39a}$$
$$N_i = -M_i\bar{C}_i\bar{A}_i\bar{C}_i^+ \tag{39b}$$
$$T_i = I_{1i} - M_i\bar{C}_i \tag{39c}$$
$$L_i = \bar{D}_{i1}H_{i1}^+ + M_i(Ip_i + \bar{C}_i\bar{A}_i\bar{C}_L^+H_{i1}H_{i1}^+ - \bar{C}_i\bar{D}_{i1}H_{i1}^+) \tag{39d}$$

(c) $H_{i1} \neq 0$, $V_i = 0$ (Rank $[V_i] < p_i$)

Firstly, compute K_i from the equation $K_iV_i = 0$. Then, compute M_i, N_i, T_i by using the Eqs(27), (28), (30) and K_i. Thirdly, compute S_i by some pole assignment algorithm. Finally, find L_i from Eq(29) by substiting all the obtained parameters.

5. A Numerical Example

In order to explain the design procedures and to show the performance of the decentralized state observer, a simple interconnected system with following parameters is considered.

$$A = \begin{bmatrix} -3 & 0 & 1 & 1 & 1 & 0 \\ 0 & -4 & 0 & 0 & 0 & 1 \\ 0 & 1 & -5 & 0 & 0 & 0 \\ 1 & 1 & 0 & -3 & 0 & 1 \\ 0 & 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{e. 1}$$

The system can be partitioned into 2 subsystems with the parameters.

$$A_i = \begin{bmatrix} -3 & 0 & 1 \\ 0 & -4 & 0 \\ 0 & 1 & -5 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad AM_i = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_i^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad CM_i^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{e. 2}$$

By applying the transformation of Eq(e.3) to above interconnection terms, D_i and H_i are abtained as Eq(e.4).

$$\Lambda_i = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \tag{e. 3}$$

$$D_i = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^T, \quad H_i = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{e. 4}$$

And following parameters are obtained by the use of partitioning conditions.

$$(D_{i1}^T : H_{i1}^T)^T = (0 \quad 1 \quad 0 \quad 1 \quad 0) \tag{e. 5}$$
$$(D_{i2}^T : H_{i2}^T)^T = (1 \quad 0 \quad 0 \quad 0 \quad 0)$$

Now, $\eta_{i2}(t)$ may be modelled by following dynamic equation.,

$$\dot{z}_{i2}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_{i2}(t), \quad \eta_{i2}(t) = (1 \quad 0)\, z_{i2}(t) \tag{e. 6}$$

and the augmented system is constructed as Eq(e.7).

$$\bar{A}_i=\begin{bmatrix}-3 & 0 & 1 & 1 & 0\\ 0 & -4 & 0 & 0 & 0\\ 0 & 1 & -5 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix},\ \bar{D}_i=\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix},\ \bar{B}_i=\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}$$

$$\bar{C}_i=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\end{bmatrix}\quad F^T_{i2}=\begin{bmatrix}1\\ 0\end{bmatrix}\qquad (e.\ 7)$$

In the augmented system, $H_{i1}=0$, $V_i=0$ and pole assignment condition of Eq(35) is stisfied. The observer parameters are determined as follows.

$$K_i^T=\begin{bmatrix}-5.5 & 1 & 0.72 & -17.72 & -10.4\\ -5.5 & 1 & 0.72 & -17.72 & -10.4\end{bmatrix}\qquad (e.\ 8)$$

$$M_i^T=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ -5.5 & 1 & 0.72 & -17.72 & -10.4\end{bmatrix}\qquad (e.\ 9)$$

$$N_i^T=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ -22 & 4 & 2.88 & -70.88 & -41.6\end{bmatrix}\qquad (e.\ 18)$$

$$L_i^T=\begin{bmatrix}5.5 & 0 & -0.72 & 17.72 & 10.4\\ -5.5 & 0 & 0.72 & -17.72 & -10.4\end{bmatrix}\qquad (e.\ 11)$$

$$T_i=\begin{bmatrix}1 & 5.5 & 0 & 0 & 0\\ 0 & 0.72 & 0 & 0 & 0\\ 0 & -0.72 & 1 & 0 & 0\\ 0 & 17.72 & 0 & 1 & 0\\ 0 & 10.4 & 0 & 0 & 1\end{bmatrix}\qquad (e.\ 12)$$

where observer poles are preselected as $(-1,\ -2,\ -4,\ -5, -6.5)$ and resultant observer is constructed as Eq (e.13).

$$\dot{z}_i(t)=\begin{bmatrix}-8.5 & 5.5 & 1 & 1 & 0\\ 0 & -5. & 0 & 0 & 0\\ 0.72 & 0.28 & -5 & 0 & 0\\ -17.72 & 17.72 & 0 & 0 & 1\\ -10.4 & 10.4 & 0 & 0 & 0\end{bmatrix} z_i(t)$$

$$+\begin{bmatrix}5.5 & 7.75\\ 0 & 0\\ -0.72 & -3.68\\ 17.72 & 16.18\\ 10.4 & 15.6\end{bmatrix} y_i(t)+\begin{bmatrix}5.5\\ 0\\ -0.72\\ 17.72\\ 10.4\end{bmatrix} u_i(t)$$

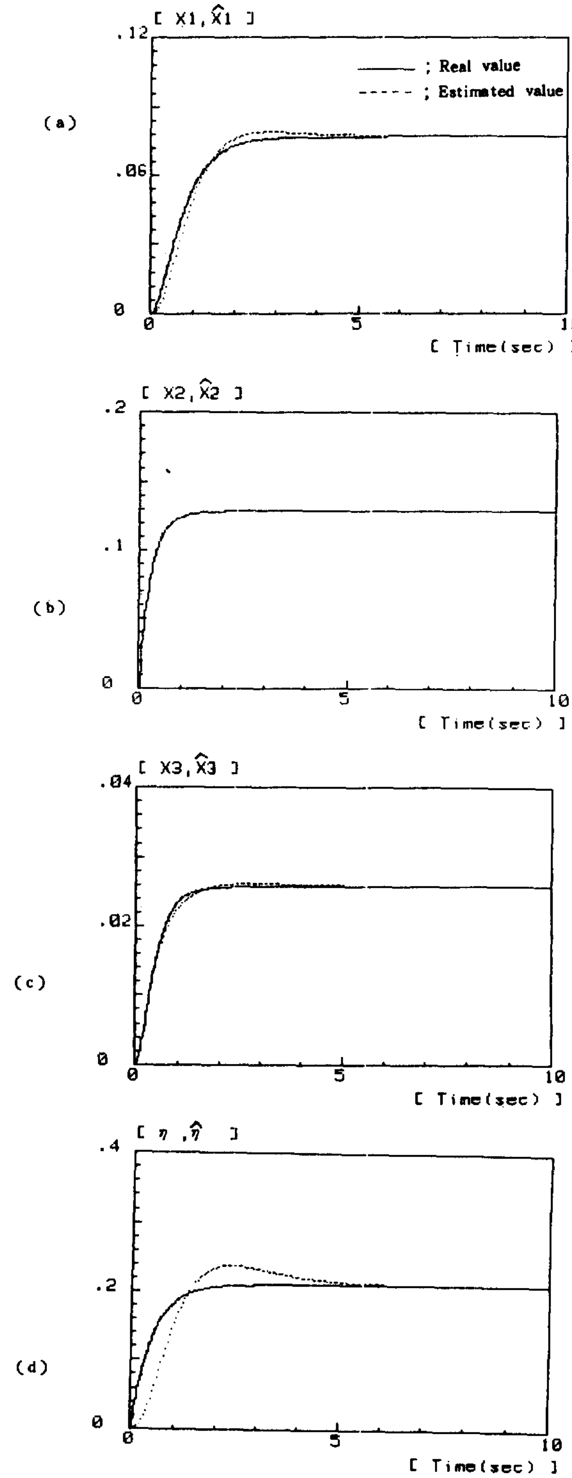

Fig. 1 Estimation results of the Decentralised Observer.

$$\hat{x}_i(t) = z_i(t) + (-5.5 \quad 1.0 \quad 0.72 \quad -17.72 \quad -10.4)' \, y_{i2}(t) \qquad \text{(e. 13)}$$

The Fig.1 shows the actual state values and estimated values when $u_1=0.5, u_2=0.5$ and $x(0)=0$. From the figures, it is observed that the decentralized state observer gives good estimates.

6. Conclusion

A design method of decentralized observer for large scale interconnected systems is proposed by the use of interconnection rejection and interconnection modelling technique. The interconnection partitioning conditions are suggested and proved in detail since the basic idea behind the design method is the interconnection partitioning. And the conditions for pole assignment and the computational procedures of observer parameters are derived for various type of interconnection patterns.

The decentralized observer does not need any information of the interconnection variables and the estimation performance of the observer may not be affected by arbitrary structural perturbation of the interconnections because the interconnections have been considered as unknown quantities in the design stage.

In addition, the decentralized observer has good estimation performance as shown in the simulation results.

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