□論 文□

기계적 격자이론 도로포장 구조물

機械的 格子理論에 의한 道路鋪裝 構造物의 疲勞壽命과 累積損失分析

피로수명 누적손실분석

Fatigue Life and Cumulative Damage Analysis in the Pavement Structure by Mechano-Lattice Theory

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요 약

포장 파괴 소성

부적정한 道路鋪裝 構造物의 設定 및 維持保守의 適定管理 未洽으로 表面의 破壞와 塑性變形이 장기간 발생된다. 이로 인한 가요성 鋪裝 構造物의 파괴 原因은 一般的으로 鋪裝材料의 同質性, 선형탄성 狀態의 假定下에서 分析되었다. 그러나 아스팔트 材料의 特性은 엄밀히 分析해서 完全한 선형탄성이라고는 볼 수 없음은 잘 알려져 있다.

즉 오류 금번

따라서 根本的으로 鋪裝體의 壽命과 破壞 預測에 誤謬 發生可能性이 높다 하겠다. 今番 研究는 이와 같은 종전의 經驗적인 선형탄성 方法이 아닌 彈性－塑性 狀態下의 격자(mechano-lattice) 이론이란 새로운 技法을 導入하였다.

특히 마이너(Miner's Law) 理論의 累積損失과 確率을 適用하여 포장체의 疲勞壽命과 損失을 預測할 수 있다.

누적 피로

今番 理論은 實際로 濠洲 빅토리아주의 멜보른(Melbourne)시 一部 地役區間을 母型으로 選定되었다.

호

分析結果 가장 최적화된 道路鋪裝 각층의 두께와 材料 選定을 하기 위하여 일정기간의 교통량, 相對的 損失指數와 잔여응력 및 表面 變位, 대기온도 그리고 濕度의 影響을 종합적으로 高慮하여야 한다.

습

1. Introduction

Most of current design procedures are based on empirical relationship derived from experience and observation. These are applicable to only a limited range of material characteristics(e.g., elastic or visco-elastic). It is assumed that the materials are elastic, homogenous and isotropic. The theory of linear elasticity has been used largely to developed relatively rational flexible pavement techniques. The Shell(1) and VESYS II(2) methods are good examples.

In such methods, there are some contradictions between these assumptions and the actual behaviour which leads to pavement failure, since the asphalt properties are strictly not elastic.

Thus, error may occur in predicting the actual pavement performance. The main objective of this study is to describe how the life of pavement can be maximized by selecting pavement materials for their plastic behaviour as well as for their modulus. To do this the three dimensional mechano-lattice analysis is developed by Yandell(3) was employed.

Unlike the theory of linearized elasticity this technique is able to predict the accumulations of residual stresses and strains which influence the pavement performance throughout the road life. This is a functional method putting some realism into the basis for pavement design by not treating the pavement structure as a structure of layered elastic materials.

The layers can interact with each other in their plastic behaviour.

The technique described here is to build the elasto-plastic model to simulate flexible pavement structures.

2. Mechano-lattice theory

The mechano-lattice analysis uses a finite unit which, though more complex than the model assumed in linear elastic analysis, is very close to truly simulating an elasto-plastic as well as other types of continum.

Fig. 1 shows the rheological system of the components chosen to represent the response of the element. Each unit is a frictionless jointed framed structure of 28 rheological elements.

For the simulation of an elasto-plastic material, each rheological element has a greater unloading than loading modulus,

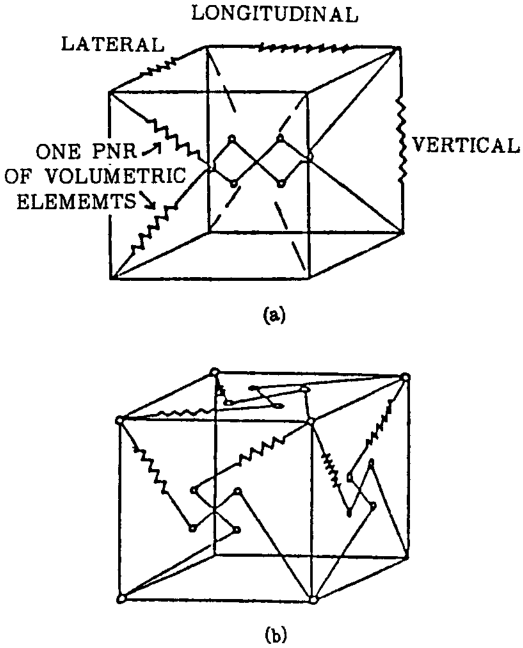


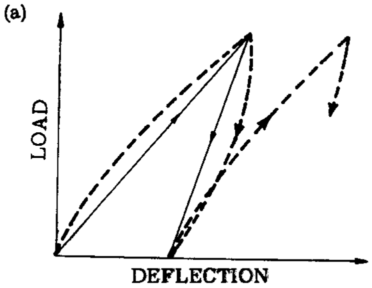
Fig 1. Three dimensional simulating unit for three dimensional, mechano-lattice analysis : (a) Volumetric and rectalinear elements(b)

a procedure which simplifies a loading-unloading cycle of an elasto-plastic material triaxial test.

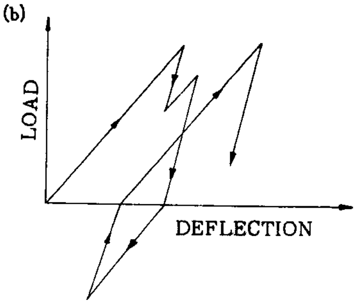
Thus, when an element is subjected to tension and compression cycles, permanent deformations of hysteresis loop can result as shown in Fig. 2.

The forces in the element which are due to displacement at each joint of the unit are analysed. A cartesian coordinate system is adopted for displacement components.

To find the stresses and permanent deformation of a repeatedly traversed two or three layer pavement, the units are arranged as shown in Fig. 3.



(a) Simplification of elasto-plastic hysteresis loop :



(b) Possible load deflection behaviour of an element

Fig 2. Simplification of elasto-plastic hysteresis loops and possible load-deflection behaviour of one of 28 elements :

Fig. 3 can be regarded as a representation of the distortion of the units for a particular position of the wheel load. However, the figure also shows the sequential loading of a typical element of a unit as the wheel traverses from right to left ; in fact, the problem is solved by imagining the pavement structure moving from left to right with the wheel load considered to be fixed in position. For example, a component of the unit 1, 2, 3 and 4 has already been subjected to a loading history. As a result there is a residual stress state represented by point A of the inset figure of Fig. 4.

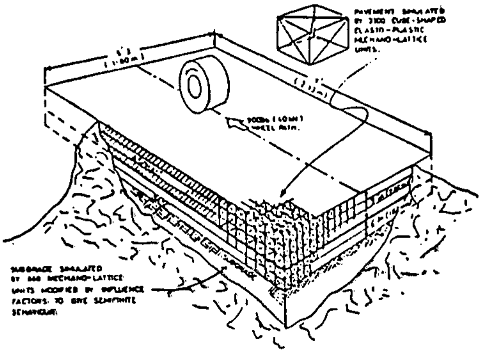


Fig 3. Assembly of units to simulate three layered pavement

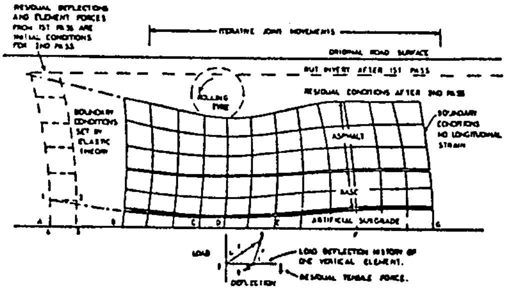


Fig 4. Diagrammatic longitudinal section of three dimensional pavement analysis showing boundary conditions

As the unit moves relative to the wheel from position A to position B the component becomes subject to a load level represented by point B.

Thus, as the load completely traverses the pavement, the loading of the component follows the path a, b, c, d, e, f and g thus leading to a residual load. The computer program performs a similar, though more complex, task after each cycle of element length-load calculation in which the forces at each joint, emanating from their attached element, are resolved into vertical, longitudinal and lateral components.

Similar behaviour occurs in the other 27 element of the unit as it moves toward, under and way from the wheel load.

3. Model Formulation

For the simulation of an elasto-plastic materials, the three hypothetical pavement structures(two and three layers) were selected from of the Shell design chart HN84 as shown in Fig. 5. The design chart was modified for a MAAT of 23 c and one base modulus 4×10^{8} Kpa(58,000 psi). In Fig. 5 three points(A, B, C) lying on one life line ($N=10^{7}$) of this Shell design chart were examined for variations fatigue life in conjunction with uncertainties(e.g., temperature and moisture content) by a probabilistic approach and for two fixed extreme relative plastic behaviour mechanisms.

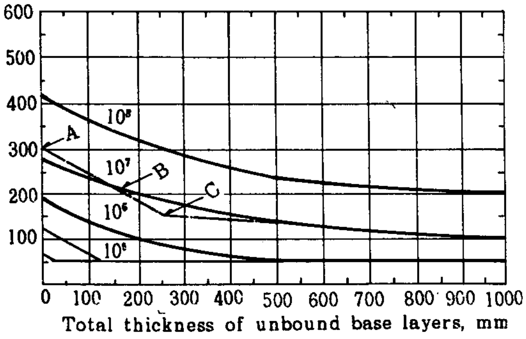


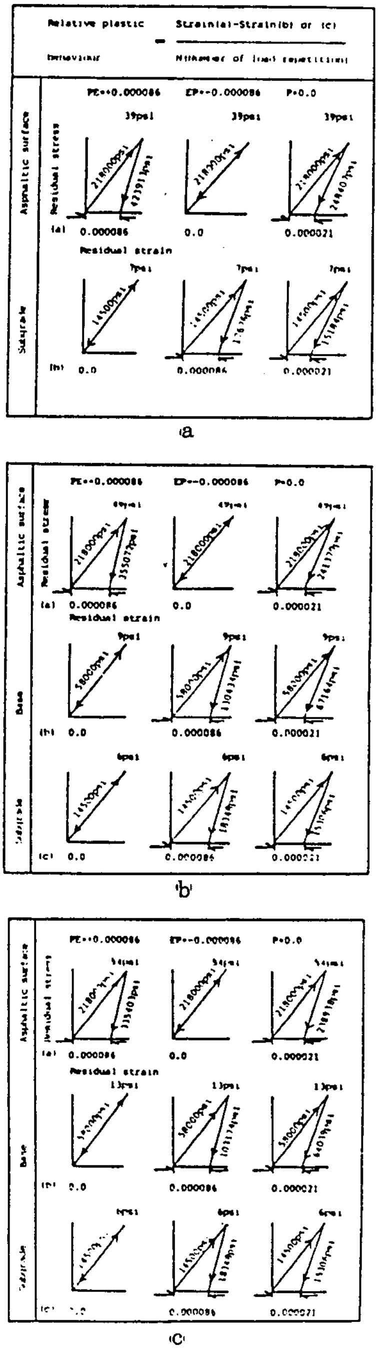
Fig 5. Modified of Shell design chart HN 84

4. Definition of relative plasticity

Although other definition may be also appropriate, relative plastic behaviour is defined here with reference to Fig. 6. It is equaled to (a)-(b) or (c) per loading repetition, "N", where (a), (b) and (c) are residual strain in triaxial test after the application and removal of a deviator stress on the A/C, the base and subgrade materials respectively. The deviator stress is equal to the maximum average experienced by that material, when in its appropriate layer of the pavement, it is traversed by a standard wheel load. It will be seen that when(a) is greater than (b) and (c), a positive(PE) plastic behaviour value results. When (a) is equal to (b) and (c), a zero (P) plastic behaviour value causes negligible residual stress, although residual strain and rutting do continue.

5. Mechano-lattice stress-strain analysis

When the relative plastic behaviour is positive(PE), the residual compressive stress in the bottom of the A/C layer reduces the transient tensile stress under the wheel. Fig. 7(a) shows that for example of section B($PE=1.5$, $H_1/H_2=9''/6''$) there is a lateral compression of 75psi at the lower asphalt surface. As calculation cycles are repeated, the transient tension is reduced from about



(a)

(b)

(c)

Fig 6. Loading-unloading stress-strain path for calculation pass : (a) Section A ; (b) Section B ; (c) Section C

24psi to 13psi.

Conversely, the residual lateral compression is increased from about 9psi to 18psi. This indicates that as the residual compressive stress is increased and accumulated, the horizontal transient tension is reduced. It is important to note here that the reducing tension under load due to increased residual compressive stress, as obtained by the mechano-lattice technique, is significantly different behaviour from that predicted by the conventional linear elastic and visco-elastic concepts. This radical difference could, because of the reduced tensile stress, lengthen the predicted fatigue life of flexible pavements.

Conceptually, it appears that residual compressive stress is accumulated at the bottom of the asphaltic surface and reduces the transient tensile stress under passing traffic loading. On the other hand, the tensile residual stress which accumulate on the top of the asphaltic surface may also shorten the separate top cracking fatigue life partly offsetting the lengthen of cracking life in the lower surface of the asphaltic concrete. But, this study does not include the buile-up of tensile stresses in the top surface of the A/C as a cause of top surface rather than under surfacing cracking.

Hugo and Kennedy(4) studied this problem in the field and theoretically using Yandell's mechano-lattice analysis to determine the top surface tension resulting from the various layer A/C acting with different plastic behaviours. When the relative plastic behaviour is negative(EP) the transient and residual tension at the bottom of the A/C are increased(Fig. 7(b)). The important

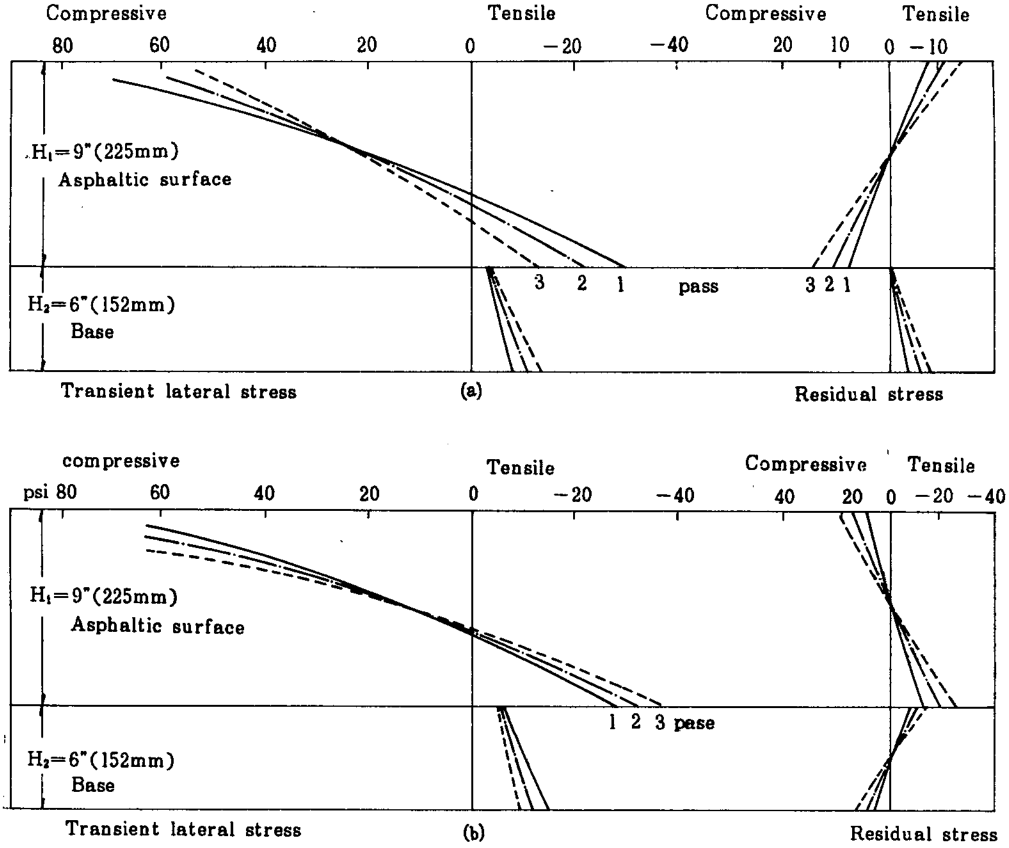


Fig 7 . Transient and residual lateral stress for first to third calculation passes on Section B : (a) Positive (PE=1.5) ; (b) Negative (EP=1.5) ;

point is that the increased magnitude of the transitory cycles of tensile stress would help decrease the fatigue life of the pavement. This behaviour cited above has been demonstrated and confirmed by Yandell, in simulating one millon traverses of a wheel load along an asphaltic pavement. It appears that the horizontal tension beneath the load is proportional to wheel traffic loading.

In the absence of initial negative plasticbehaviour this may lead to premature cracking in the asphaltic layer due to weakening of the bound materials.

6 . Fatigue life prediction

According to relative plasticity definition, the positive plasticity value and asphalt modulus is presented as shown in Fig. 8 when the A/C acts more plastically than the base and subgrade. As seen, the relative plasticity is increaseb as the asphalt modulus is decreased and the unloading modulus is increased and provided the base and subgrade remain constant. The Shell organization has presented the relationship between the bituminous mix stiffness and temperature variation in asphalt pavement.

Thus, it is seen that the plasticity of the asphalt surface layer can be increased due to higher temperature.

Fig. 9, which were produced by the mechano-lattice stress-strain analysis, shows that the residual stress increases as the positive increases. In order for the effect of temperature variation to be applied to the change of asphalt modulus and hence change in plasticity and prediction of fatigue life, temperature changing regimes, which were measured at a site near Melbourne(9, Jan. 1966) have been selected for this application(5). The collected data on temperature gradient for single were obtained from layers of asphalt concrete 100 mm thick. One daily temperature variation represents one day of a hot summer period and another day of temperature variation is assumed to be similar to the first day of summer.

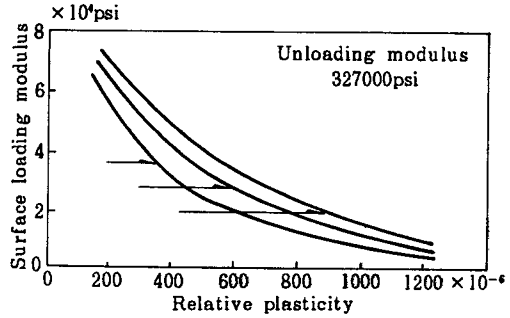


Fig 8. Relationship between surface loading modulus and relative plasticity

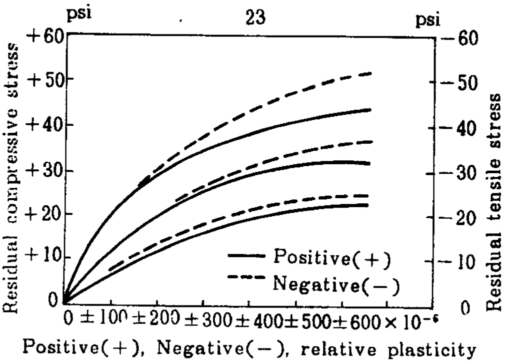


Fig 9. Residual compressive and tensile stresses ith change of relative plasticity on Section B (a) Positive (PE=1.5) ; (b) Negative(EP=1.5)

Therefore, it does not give any seasonal temperature difference over other seasons of the year. Dormon and Metcalf (6)have indicated that the effect of the modulus of the asphalt bound layer is associated with temperature at a depth equal to one-third of the surface layer thickness. Thus, the temperature variation data for a 100mm depth of the A/C layer is used for section A where the A/C is 300mm thick.

Similarly, the temperature variation data for section B and C are obtained from the temperature 75mm depth(e.g, 75mm= $H_1/3=225/3$) and 50mm depth(e.g, 57mm= $H_1/3=171/3$) of the A/C layers, respectively.

Yandell(3) has shown that the fatigue life determinded by A/C cracking would be due to the residual lateral compressive stress(positive). The fatigue failure envelope is taken directly from the Shell chart M-4(10) after the first converting strain into stress($T=23°C$, Smix $=1.5\times10^9$N/㎡ =218, 000psi). Based on the above, the fatigue stress is calculated from the Shell elastic stress(31psi, $N=10^7$) minus of the mechano-lattice calculated residual compressive stress. When the A/C acts less plastically than the base and subgrade, the negative plasticity value is increased with a decrease in subgrade dynamic loading modulus provided unloading modulus remain constant as assumed(Fig. 10). Fig. 10 also shows that the residual tensile stress is

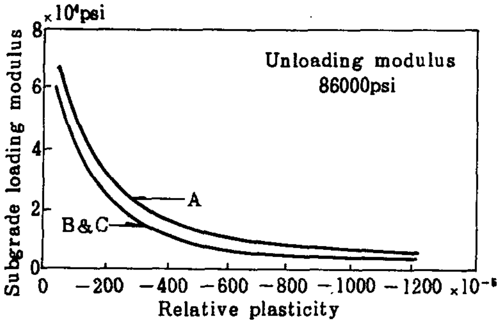


Fig 10. Relationship between subgrade loading modulus and relative plasticity

markedly increased when the negative plasticity of the lower layers are increased.

Davis(7) has established the relationship between the CBR of unsoaked soil specimens, moisture contents and density for six British soils ranging from a heavy clay to a gravel. For comparison purpose, the sensitivity of the log CBR value to unit change in void ratio varies little between the different dry densities. The relationship for the sandy soil between log CBR and moisture content is obtained by averaging the dry densities of the 4 lines and plotting. Also, the Shell organization, from extensive dynamic field testing, presented the empirical relationship, the subgrade modulus is increased as the moisture content decreased.

Thus, it is seen that the residual tension in the bottom of the A/C is dependent upon the unbound layer modulus in terms of relative plasticity, in conjunction with seasonal moisture contents.

All climatic factors such as the moisture content have some effect on pavement materials because of their water susceptibility.

In particular, under Australian environmental conditions it is necessary to know the moisture content of granular materials affects the pavement. To do this, 36 sites in Victoria State(8) were selected from a study done in 1966/67. The fatigue life also would be shortened due to the residual tensile stress by Yandell(negative).

The fatigue stress is calculated from the Shell converted elastic stress(31psi) plus the residual tensile stress. Thus, the fatigue life would be expected to have values than the Shell fatigue criterion($N=10^7$) due, in this case, to the moisture content changes.

7. Probabilistic solution

Due to the variation and uncertainties associated with load repetition, a probabilistic method has been extensively used for road design based on prediction of fatigue failure. Particularly, fatigue life can be only estimated at a certain confidence level.

The confidence interval established is called a two side confidence interval, because it includes the upper and the lower limit that bound the value of pouplation mean(μ). In this case, we will be interested in the lower limit of the mean because, for the specification purposes, the road engineer is required to support a specified lower confidence limit of the mean pavement yield strength. For such purposes, the(1-α) lower confidence, denoted$<\mu)_{(1-\alpha)}$, for the mean (μ) as follows (9);

$$<\mu)_{1-\alpha}=(\bar{x}-K_{\alpha}\frac{\sigma}{\sqrt{n}}) \cdots\cdots\cdots\cdots\cdots(1)$$

where 1-α=the specified confidence level ; and

$$K_a=\phi^{-1}(1\text{-}\alpha) \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

On the other hand, the two sided confidence interval(1-α) for the population variance using Chi-square distribution can be obtained as follow ;

$$<\sigma^2)_{1-a}=\left\langle \frac{(n-1)S^2}{X_{1-(1/2a),\ n-1}} ; \frac{(n-1)S^2}{X_{(1/2a),\ n-1}} \right\rangle \cdots (3)$$

Where, S=the sample variance.

Then, the exact(α) lower confidence limit for σ^2 of a normal population is

$$<\sigma^2)_{1-a}=\left\langle \frac{(n-1)S^2}{C_{(1-a,\ n-1)}} ; \infty \right\rangle \cdots\cdots\cdots\cdots (4)$$

Where, $C_{(1-a,\ n-1)}$=the variance of the $X^2_{(n-)1}$ variate at the cumulative probability of 1−α. Hence, the mean and the variance of the predicted fatigue life of the three pavement sections A, B and C are calculated from the previous references using equation(1) and(3).

This is presented in Fig. 11.

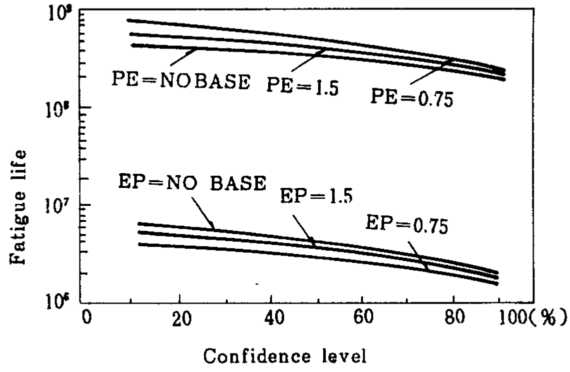


Fig 11. Fatigue life change by confidence level on Section A, B and C.

8. Fatigue life anlysis

As seen in Fig. 11, the fatigue life is higher when the A/C layer behaves more plastically (by the magnitude assumed) than the base and subgrade. An example for PE=1.5(H_1/H_2=9"/6") on section B shows that the fatigue life(99% confidence limit) is increased to about 33 times the Shell value (10^7). Moreover, as the confidence level is decreased to 10%, the fatigue life is increased about 57 times.

This is due to the residual compressive stress build up at the lower asphalt layer.

In the opposite case(EP=1.5, H_1/H_2=9"/6", 99% confidence limit) the fatigue life has dropped to about 68% of the Shell value when the A/C layer is non-plastic. At the 10% confidence level, it is decreased to about 55% of the Shell value. This is due to the fact that residual tensile stress has the effect of increasing the magnitude of the transient tensile stress under a rolling wheel when the A/C layer acts less plastically than the base and subgrade. This reduced fatigue life may lead to premature cracking.

It can be seen that the typical design life of 10^7 repetitions would be greatly influenced by the uncertainty factors affecting relative plasticity. Thus, the mechano-lattice analysis shows that a Shell design chart could be modified to take account of the uncertainties, influencing relative plastic behaviour inherent in the design parameters.

9. Cumulative Fatigue Damage

In order to assess the fatigue damage the design traffic loading is commonly referred to as the Equivalent Number of Standard Axles(ESA)

The Victoria Road Board⑽ provided the annual traffic loading on the Midland Highway(1969/1970). Miner's damage expression is

$$D = \sum_{i=1}^{m} \frac{ni}{Ni}$$

D = total accumulated fatigue damage over pavement design life

ni = number of applied loads of ith magnitude.

Ni = number of allowable service life of ith magnitude in failure.

D is not a constant for a pavement but varies depending on the randomness of ni and Ni.

The total variation in D can be determined as follows(11).

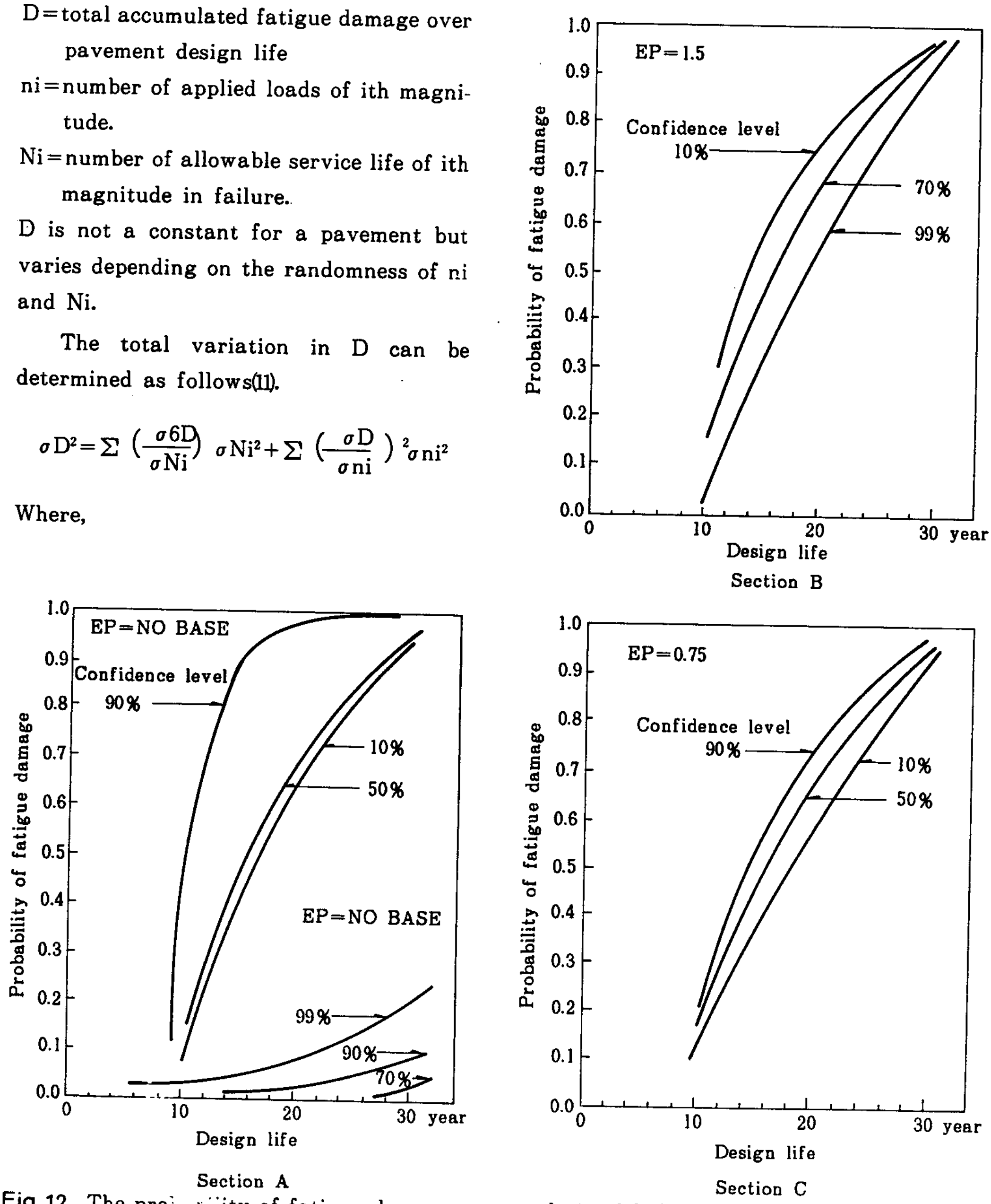
$$\sigma D^2 = \sum \left(\frac{\sigma 6D}{\sigma Ni}\right) \sigma Ni^2 + \sum \left(\frac{\sigma D}{\sigma ni}\right)^2 \sigma ni^2$$

Where,

σD^2 = variance of accumulated damage.

σNi^2 = variance in allowing fatigue application to cracking.

σni^2 = variance in estimating traffic application



Section B

Section A

Section C

Fig 12. The probability of fatigue damage versus design life for change in specified confidence level on Section A, B and C.

The probability of fatigue cracking,

$P_f(D)$ is $P_f(D) = P(D > 1) = P(\log_{10} D > 0)$

From figures the first time at which the cumulative damage exceeds 1.0 can be found. This provides the estimate of the number of years which a pavement would last before developing the specified amount of fatigue cracking.

10. Conclusion

A Shell desion chart has been re-evaluated by the Yandell's mechano-lattice analysis, in conjunction with the uncertainties of temperature and moisture contents. The temperature and moisture content records were taken directly from Melbourne in 1966 and 36 sites in Victoria State in 1966/67.

1. With respect to fatigue life under positive relative plastic behaviour (PE) conditions, when the asphaltic concrete surface layer behaved in a more plastic way than the base and subgrade, the life predicted by the mechano-lattice technique was greater than that indicated by Shell fatigue criteria(10^7) due to build-up of residual compressive stress in the bottom of the asphaltic layer.
2. With respect to fatigue life under negative relative plastic behaviour conditions, when the asphaltic concrete surface layer behaved less plastically than the base and subgrade, the life predicted by the mechano—lattice technique was lower than that of the Shell chart, due to build—up of residual tensile stresses in the bottom of the asphaltic layer.
3. The strength of pavements appears to fluctuate as a result of unknown environmental changes. It was found that, when the elasto-plastic road materials are subjected to environmental uncertainties, the mechano-lattice model was theoretically more reliable than the Shell design curves, applied directly to the flexible pavement structures. This is because the mechano-lattice takes the plasticity and relative plasticity of the layers into account a rigorous manner. The uncertainties inherent in the pavement materials, such as temperature and moisture content, can be expressed in terms of variations in relative plastic behaviour.
4. Damage analysis is treated probabilistically with development of design requirement specifying confidence level.

   The fatigue life can only be predicted at a certain confidence level assuming that failure is the result of fatigue damage.

   This damage is a consequence of stress, strain and operational environmental condition.

   This damage model section which has been presented should have its capability of yielding statistical estimates for pavement response.

This is purely theoretical study using initial assumed plastic parameters and variability taken from field observation and designed to demonstrated the possbility on increasing design and evaluation precision in terms of confidence level. Alternative is to incorporate plasticity into the design process.

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NOTATION

The following symbols are used in this paper :

A/C = Asphaltic concrete surface ;

EP = Negative relative plastic behaviour value(−) ;

EP=NO BASE= Section A(H_1/H_2=12"/0), the asphaltic surface layer acts less plastically than the subgrade ;

EP=1.5 = Section B(H_1/H_2=9"/6"), the asphaltic surface layer acts less plastically than the base and subgrade ;

EP=0.75=Section C(H_1/H_2=6.75"/9"), the asphaltic surface layer acts less plastically than the base and subgrade ;

H_1 =Surface layer thickness

(inches) ;

H_2 =Base layer thickness(inches) ;

P =Zero relative plastic behaviour value(0) ;

PE=Positive relative plastic behaviour value(+) ;

PE= NO BASE =Section A($H_1/H_2=12"/0$), the asphaltic surface layer acts more plastically than the subgrade ;

PE=1.5=Section B($H_1/H_2=9"/6"$), the asphaltic surface layer acts more plasti- subgrade ;

PE=0.75=Section C($H_1/H_2=6.75"/9"$), the asphaltic surface layer acts more plastically than the base and subgrade ;