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REALCOMPACT CONVERGENCE ORDERED SPACES

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In this paper, we deal with the Wallman-type ordered realcompactification of a convex convergence ordered space which is a generalization of an ordered realcompactification of topological ordered space.

For a convex convergence ordered space $(X, \leq, \rightarrow)$, let $X^* = \{(\dot{d}(x), \dot{i}(x)) | x \in X\} \cup \{(\mathscr{F}, \mathscr{G}) | (\mathscr{F}, \mathscr{G})$ is a maximal closed bifilter with the countable intersection property on X such that $\mathscr{F} \vee \mathscr{G}$ fails to converge$\}$, and give the convergence structure $\xrightarrow{*}$ on X^* and order relation $\leq^*$ on X^*. Then we obtain the Wallman-type ordered realcompactification $(X^*, \leq^*, \xrightarrow{*})$ for a convex convergence ordered space $(X, \leq, \rightarrow)$.

DEFINITION 1. If $\mathscr{F}$ (resp. $\mathscr{G}$) is a decreasing (resp. increasing) (closed) filter on X, then a pair $(\mathscr{F}, \mathscr{G})$ is called a (closed) *bifilter* on X if $\mathscr{F} \vee \mathscr{G}$ exists where $\mathscr{F} \vee \mathscr{G}$ is the filter generated by $\{F \cap G | F \in \mathscr{F}$ and $G \in \mathscr{G}\}$. A bifilter $(\mathscr{F}, \mathscr{G})$ on X is said to *converge* if the filter $\mathscr{F} \vee \mathscr{G}$ converges in X. If $\mathscr{F} \vee \mathscr{G}$ has an adherent point in X, then we say that the bifilter $(\mathscr{F}, \mathscr{G})$ has an *adherent point* in X.

For $\mathscr{F} \in F(X)$, we denote by $i(\mathscr{F})$ the filter generated by $\{i(F) | F \in \mathscr{F}\}$ where $i(F)$ is increasing set; the filters

$d(\mathscr{F})$ and $c(\mathscr{F})$ are defined analogously.

A filter $\mathscr{F}$ is called a *convex filter* if it has a filterbase of convex sets, i.e., $c(\mathscr{F})=\mathscr{F}$. Note that $\mathscr{F}$ is a convex filter iff $\mathscr{F}=i(\mathscr{F})\vee d(\mathscr{F})$. The bifilter $(\mathscr{F},\mathscr{G})$ is called *convex* if $\mathscr{F}\vee\mathscr{G}$ is a convex filter on X. A convergence ordered space $(X,\leq,\rightarrow)$ is said to be *convex* if every convex bifilter converges in X.

Given a convergence ordered space $(X,\leq,\rightarrow)$, let $\Delta=\{(x,y)|x\leq y\}$ be the graph of the partial order $\leq$ on X.

For filters $\mathscr{F},\mathscr{G}$ on X, define $\mathscr{F}\circledleq\mathscr{G}$ to mean that $(\mathscr{F}\times\mathscr{G})\vee\dot{\Delta}\neq\phi$, i.e., $(F\times G)\cap\Delta\neq\phi$ for $F\in\mathscr{F}$ and $G\in\mathscr{G}$.

DEFINITION 2. Let $(X\leq,\rightarrow)$ be a convergence ordered space. Then $(X,\leq,\rightarrow)$ is said to be T_1*-ordered* if $\mathscr{F}\longrightarrow x$, $\mathscr{F}\circledleq\dot{y}$ implies that $x\leq y$, and similarly, if $\mathscr{G}\longrightarrow x$, $\dot{y}\circledleq\mathscr{G}$ implies that $y\leq x$. If $x\leq y$ whenever $\mathscr{F}\longrightarrow x$, $\mathscr{G}\longrightarrow y$, and $\mathscr{F}\circledleq\mathscr{G}$, then $(X,\leq,\rightarrow)$ is defined to be T_2*-ordered*.

DEFINITION 3. Let X and Y be convergence ordered spaces. If $f\colon X\longrightarrow Y$ is an order isomorphism and homeomorphic embedding, and if Y is compact and $f(X)$ is dense in Y, then (Y,f) will be called a convergence ordered *compactification* of X.

DEFINITION 4. Let $(X,\leq,\rightarrow)$ be a convergence ordered space. Then $(X,\leq,\rightarrow)$ is called a *realcompact* convergence ordered space if every maximal closed bifilter with countable intersection property converges in X. In definition of a convergence ordered compactification of X, if Y is realcompact convergence ordered space then (Y,f) is called a convergence ordered *realcompactification* of $(X,\leq,\rightarrow)$.

In [6] and [7], a Wallman-type ordered compactification is given in the topological setting. These ideas are used here.

THEOREM 5. Let $(X, \leq, \rightarrow)$ be a T_1-ordered convergence space. Then for each $x \in X$, $(\dot{d}(x), \dot{i}(x))$ is a maximal closed bifilter with the countable intersection property.

PROOF. Suppose that $(\mathscr{F}, \mathscr{G})$ is a closed bifilter on X such that $(\dot{d}(x), \dot{i}(x)) \subseteq (\mathscr{F}, \mathscr{G})$. Then $\dot{d}(x) \subseteq \mathscr{F}$ and $\dot{i}(x) \subseteq \mathscr{G}$. If $F \in \mathscr{F}$ then there exists a decreasing closed set $F_1 \in \mathscr{F}$ with $F_1 \subseteq F$. Since F_1 is decreasing, $F_1 = d(F_1)$. We know that $d(F_1) \cap i(x) \neq \phi$. Hence $x \in d(F_1)$.

It follows that $d(x) \subseteq d(F_1) \subseteq F$. Thus $F \in \dot{d}(x)$, so $\mathscr{F} \subseteq \dot{d}(x)$. Similarly, $\mathscr{G} \subseteq \dot{i}(x)$. Therefore $(\dot{d}(x), \dot{i}(x))$ is a maximal closed bifilter. It is obvious that it has the countable intersection property.

From now on, the space is a T_1-ordered convergence space. Let $X' = \{(\mathscr{F}, \mathscr{G}) | (\mathscr{F}, \mathscr{G})$ is a maximal closed bifilter with the countable intersection property on X such that $\mathscr{F} \vee \mathscr{G}$ fails to converge$\}$.

Define $X^* = \{(\dot{d}(x), \dot{i}(x)) | x \in X\} \cup X'$ and an order relation $\leq^*$ on X^* as follows: $(\mathscr{F}_1, \mathscr{G}_1) \leq^* (\mathscr{F}_2, \mathscr{G}_2)$ if and only if $\mathscr{F}_2 \subseteq \mathscr{F}_1$ and $\mathscr{G}_1 \subseteq \mathscr{G}_2$ for any $(\mathscr{F}_1, \mathscr{G}_1)$ and $(\mathscr{F}_2, \mathscr{G}_2)$ in X^*. Then $(X^*, \leq^*)$ is a partially ordered set.

For given decreasing subset A and increasing subset B of $(X, \leq)$, we define the sets $A^d = \{(\mathscr{F}, \mathscr{G}) \in X^* | A \in \mathscr{F}\}$ and $B^i = \{(\mathscr{F}, \mathscr{G}) \in X^* | B \in \mathscr{G}\}$.

Let $\mathscr{F}$ (resp. $\mathscr{G}$) be a decreasing (resp. increasing) filter on X. Then $\mathscr{F}^d$(resp. $\mathscr{G}^i$) denotes the filter on X^* whose base is $\{F^d | F\varepsilon\mathscr{F}\}$ (resp. $\{G^i | G\in\mathscr{G}\}$).

DEFINITION 6. We define a convergence structure $\xrightarrow{*}$ on X^* as follows: For a filter $\mathscr{H}$ on X^*,

(1) $\mathscr{H}\xrightarrow{*}(\dot{d}(x), \dot{i}(x))$ in X^* if and only if there exists a filter $\mathscr{F}\longrightarrow x$ in X such that $(d\mathscr{F})^d \vee (i\mathscr{F})^i \subseteq \mathscr{H}$;

(2) $\mathscr{H}\xrightarrow{*}(\mathscr{F}, \mathscr{G})$ in X^*, $(\mathscr{F}, \mathscr{G})\in X'$ if and only if $\mathscr{F}^d \vee \mathscr{G}^i \subseteq \mathscr{H}$.

Then $(X^*, \leq^*, \xrightarrow{*})$ is called a *convergence ordered space.*

THEOREM 7. Let $(X, \leq, \rightarrow)$ be a convergence ordered space. Then $A=d(A)$ and $A\subseteq X$ is closed in X if A^d is closed in X^*.

PROOF. Suppose that A^d is closed in X^*. Let $x\varepsilon\bar{A}$. Then there exists a filter $\mathscr{F}\longrightarrow x$ such that $A\in\mathscr{F}$. Thus $(d\mathscr{F})^d \wedge (i\mathscr{F})^i \xrightarrow{*} (\dot{d}(x), \dot{i}(x))$ and $A^d\in(d\mathscr{F})^d$. Hence we have a filter $(d\mathscr{F})^d \vee (i\mathscr{F})^i \xrightarrow{*} (\dot{d}(x), \dot{i}(x))$ such that $A^d\in(d\mathscr{F})^d \vee (i\mathscr{F})^i$, that is, $(\dot{d}(x), \dot{i}(x))\in\bar{A}^d = A^d$ and so $d(x)\subseteq A$, i.e., $x\in A$, since A is decreasing. Thus A is closed in X.

THEOREM 8. Let $(X, \leq, \rightarrow)$ be a T_1-ordered convergence space. Then $(X^*, \leq^*, \xrightarrow{*})$ is a realcompact convergence ordered space.

PROOF. Let $(\mathscr{H}, \mathscr{K})$ be a maximal closed bifilter with the countable intersection property, and $\mathscr{F}$ (resp. $\mathscr{G}$) be the filter generated by $\{A\subseteq X | A^d\in\mathscr{H},\ A=d(A)\}$ (resp. $\{B\subseteq X | B^i\in\mathscr{K},\ B=i(B)\}$). Then $\mathscr{F}$ (resp. $\mathscr{G}$) is a decreasing (resp. increasing) closed filter with the countable intersection property. If $F\in\mathscr{F}$, $F=d(F)$ and $G\in\mathscr{G}$, $G=i(G)$ then $F^d\in\mathscr{H}$ and $G^i\in\mathscr{K}$. Since $(\mathscr{H}, \mathscr{K})$ is a bifilter, $F^d\cap G^i \neq \phi$

and so we have some maximal closed bifilter $(\mathcal{M},\mathcal{N})\in F^d\cap G^i$. Hence $F\cap G\neq\phi$. Therefore $(\mathcal{F},\mathcal{G})$ is a closed bifilter. Suppose that $(\mathcal{F}_1,\mathcal{G}_1)$ is a closed bifilter with the countable intersection property on X such that $(\mathcal{F},\mathcal{G})\subset(\mathcal{F}_1,\mathcal{G}_1)$. If $A=d(A)\in\mathcal{F}_1$ such that $A\notin\mathcal{F}$, then $X-A$ is increasing and since $i(x)$ is a increasing closed set for some $x\in X-A$, $X-A\in\mathcal{G}\subseteq\mathcal{G}_1$. It is contradiction to the fact $\mathcal{F}_1\vee\mathcal{G}_1\neq\phi$. Thus $(\mathcal{F},\mathcal{G})$ is a maximal closed bifilter. If $\mathcal{F}\vee\mathcal{G}\longrightarrow x$ in X, then $(\mathcal{H},\mathcal{K})\xrightarrow{*}(d(x),\ i(x))$ in X^*, since $(\mathcal{F}^d,\mathcal{G}^i)\subseteq(\mathcal{H},\mathcal{K})$. If $\mathcal{F}\vee\mathcal{G}$ fails to converge in X, then $(\mathcal{F},\mathcal{G})\in X'$ and so $(\mathcal{H},\mathcal{K})\xrightarrow{*}(\mathcal{F},\mathcal{G})$ in X^*. Therefore $(X^*,\leq^*,\xrightarrow{*})$ is a realcompact convergence ordered space.

Theorem 9. Let $(X,\leq,\rightarrow)$ be a convex T_1-ordered convergence space. Then the natural map ϕ: $(X,\leq,\rightarrow)\longrightarrow(X^*,\leq^*,\xrightarrow{*})$ is a dense embedding.

Proof. Suppose that $\mathcal{F}\longrightarrow x$ in X. Then $\phi(\mathcal{F})\supseteq\phi\,(d\mathcal{F}\vee i\mathcal{F})\supseteq\phi\,(d\mathcal{F})\vee\phi\,(i\mathcal{F})\supseteq(d\mathcal{F})^d>(i\mathcal{F})^i\xrightarrow{*}\phi\,(x)$ in X^*. Thus ϕ is a continuous function. Suppose that $\phi(\mathcal{F})\xrightarrow{*}\phi(x)$ in X^*. Then $\phi\,(\mathcal{F})\supseteq(d\mathcal{G})^d\vee(i\mathcal{G})^i$ for some $\mathcal{G}\longrightarrow x$ in X. Thus $\mathcal{F}\supseteq\phi^{-1}((d\mathcal{G})^d\vee(i\mathcal{G})^i)=\phi^{-1}((d\mathcal{G})^d)\vee\phi^{-1}((i\mathcal{G})^i)=d\mathcal{G}\vee i\mathcal{G}\longrightarrow x$, since X is convex. Hence ϕ is an embedding. Let $(\mathcal{F},\mathcal{G})\in X'$, then $\phi\,(\mathcal{F}\vee\mathcal{G})\supseteq(d(\mathcal{F}\vee\mathcal{G}))^i\vee(i(\mathcal{F}\vee\mathcal{G}))^i=\mathcal{F}^d>\mathcal{G}^i$, since $(\mathcal{F},\mathcal{G})=(d(\mathcal{F}\vee\mathcal{G}),\ i(\mathcal{F}\vee\mathcal{G}))$. Thus $\phi(\mathcal{F}\vee\mathcal{G})\xrightarrow{*}(\mathcal{F},\mathcal{G})$ in X^*. We conclude that ϕ is a dense embedding.

By Theorem 8 and Theorem 9, we have the following.

Theorem 10. Let $(X,\leq,\rightarrow)$ be a convex T_1-ordered convergence space. Then $(X^*,\leq^*,\xrightarrow{*})$ is a realcompactification of $(X,\leq)$.

Suppose that $(\mathcal{F},\mathcal{G})$ and $(\mathcal{H},\mathcal{K})$ are two bifilters. We define the relation $\leq^*$ on X^* as follow: $(\mathcal{F},\mathcal{G}) \circledleq (\mathcal{H},\mathcal{K})$ if $\mathcal{G}\vee\mathcal{H}\neq\phi$.

DEFINITION 11. Let $(X,\leq,\rightarrow)$ be a T_1-ordered(resp. T_2-ordered) convergence space. Then $(X,\leq,\rightarrow)$ is said to be *strongly* T_1*-ordered* (reps. *strongly* T_2*-ordered*) if

(1) if $(d\mathcal{F},\ i\mathcal{F})\longrightarrow x$ and $(d\mathcal{F},\ i\mathcal{F}) \circledleq (\mathcal{H},\mathcal{K})$, then $(\dot{d}(x),\dot{i}(x))\leq^* (\mathcal{H},\mathcal{K})$;

(2) if $(d\mathcal{F},\ i\mathcal{F}) \longrightarrow x$ and $(\mathcal{H},\ \mathcal{K}) \circledleq (d\mathcal{F},i\mathcal{F})$, then $(\mathcal{H},\mathcal{K}) \leq^* (\dot{d}(x),\dot{i}(x))$.

THEOREM 12. Let $(X,\leq,\rightarrow)$ be a convex convergence space. If X is strongly T_1-ordered, then X^* is T_1-ordered.

PROOF. Suppose that X is T_1-ordered. Let $\mathcal{H}\xrightarrow{*}(\dot{d}(x), \dot{i}(x))$ and $\mathcal{H} \circledleq (\dot{d}(y),\ \dot{i}(y))$. Then there exists a filter $\mathcal{F} \longrightarrow x$ such that $(d\mathcal{F})^d \vee (i\mathcal{F})^i \subseteq \mathcal{H}$, and so $(d\mathcal{F})^d\vee (i\mathcal{F})^i \circledleq (\dot{d}(y),\ \dot{i}(y))$. It follows that $\{((dF)^d \cap (iF)^i)\times (\dot{d}(y),\ \dot{i}(y))\}\cap \Delta^* \neq \phi$ for each $F\in\mathcal{F}$, where Δ^* is the graph of order in X^*. This implies that there exists a maximal closed bifilter $(\mathcal{M},\mathcal{N})\leq^* (\dot{d}(y),\ \dot{i}(y))$ such that $dF\in\mathcal{M}$ and $iF\in\mathcal{N}$ for each $F\in F$. By definition of $\leq^*$, $\dot{d}(y)\subseteq \mathcal{M}$ and $\mathcal{N} \subseteq \dot{i}(y)$. Thus $d(y)\in\mathcal{M}$ and $iF\in\mathcal{N}$ for each $F\in\mathcal{F}$. Since $(\mathcal{M},\mathcal{N})$ is a bifilter, $iF \cap d(y) \neq \phi$ for each $F\in\mathcal{F}$. Hence we have a $z\in F$ with $z\leq y$ for each $F\in\mathcal{F}$. This means that $\mathcal{F}\circledleq\dot{y}$. Since X is T_1-ordered and $\mathcal{F}\longrightarrow x$, $\mathcal{F} \circledleq \dot{y}$, and $x\leq y$. Thus $(\dot{d}(x),\dot{i}(x)) \leq^* (\dot{d}(y),\dot{i}(y))$. Similarly, if $\mathcal{H}\xrightarrow{*}(\dot{d}(x),\dot{i}(x))$ in X^* and $(\dot{d}(y),\dot{i}(y))\circledleq$

$\mathcal{H}$, then $(\dot{d}(y), \dot{i}(y)) \leq^* (\dot{d}(x), \dot{i}(x))$. The natural map f is increasing.

Suppose that $\mathcal{H} \xrightarrow{*} (\mathcal{F}, \mathcal{G})$, $(\mathcal{F}, \mathcal{G}) \in X'$ and $\mathcal{H} \circledS (\dot{d}(x), \dot{i}(x))$. Then $\mathcal{F}^d \vee \mathcal{G}^i \circledS (\dot{d}(x), \dot{i}(x))$. This means that there exists a maximal closed bifilter $(\mathcal{M}, \mathcal{N}) \in F^d \cup G^i$ with $(\mathcal{M}, \mathcal{N}) \leq^* (\dot{d}(x), \dot{i}(x))$ for each $F \in \mathcal{F}$ and $G \in \mathcal{G}$. If $G \in \mathcal{G}$ then $G \in \mathcal{N} \subseteq \dot{i}(x)$. Thus $G \subseteq \dot{i}(x)$. Since $F \in \mathcal{M}$ and $d(x) \in \mathcal{M}$, $F \cap d(x) \in \mathcal{M}$. For $F \in \mathcal{F}$ and $G \in \mathcal{G}$, $d(x) \cap F \cap G \neq \phi$, since $(\mathcal{M}, \mathcal{N})$ is a maximal closed bifilter. Thus $d(x) \in \mathcal{F}$ and so $\dot{d}(x) \subseteq \mathcal{F}$. Therefore $(\mathcal{F}, \mathcal{G}) \leq^* (\dot{d}(x), \dot{i}(x))$. Similarly, if $\mathcal{H} \xrightarrow{*} (\mathcal{F}, \mathcal{G})$ in X^* and $\mathcal{H} \circledS (\mathcal{F}, \mathcal{G})$, $(\mathcal{F}, \mathcal{G}) \in X'$ and $(\dot{d}(x), \dot{i}(x)) \circledS \mathcal{H}$, then $(\dot{d}(x), \dot{i}(x)) \leq^* (\mathcal{F}, \mathcal{G})$.

Suppose that $\mathcal{H} \xrightarrow{*} (\mathcal{F}, \mathcal{G})$, $(\mathcal{F}, \mathcal{G}) \in X'$ and $\mathcal{H} \circledS (\dot{\mathcal{S}}, \mathcal{T})$. Then $\mathcal{F}^d \vee \mathcal{G}^i \circledS (\dot{\mathcal{S}}, \mathcal{T})$. It follows that there exists a maximal closed bifilter $(\mathcal{M}, \mathcal{N}) \in F^d \cap G^i$ for each $F \in \mathcal{F}$ and $G \in \mathcal{G}$ such that $(\mathcal{M}, \mathcal{N}) \leq^* (\mathcal{S}, \mathcal{T})$. If $S \in \mathcal{S}$, then $S \in \mathcal{M}$ and $S \cap F \in \mathcal{M}$. Since $(\mathcal{M}, \mathcal{N})$ is a maximal closed bifilter, $S \cap F \cap G \neq \phi$ for each $F \in \mathcal{F}$ and $G \in \mathcal{G}$. Thus $S \in \mathcal{F}$ and so $\mathcal{S} \subseteq \mathcal{F}$. If $G \in \mathcal{G}$ then $G \in \mathcal{N} \subseteq \mathcal{T}$. Thus $\mathcal{G} \subseteq \mathcal{T}$. Therefore $(\mathcal{F}, \mathcal{G}) \leq^* (\mathcal{S}, \mathcal{T})$. Similarly, if $\mathcal{H} \xrightarrow{*} (\mathcal{F}, \mathcal{G})$, $(\mathcal{F}, \mathcal{G}) \in X'$ and $(\dot{\mathcal{S}}, \mathcal{T}) \circledS \mathcal{H}$, then $(\mathcal{S}, \mathcal{T}) \leq^* (\mathcal{F}, \mathcal{G})$.

Suppose that $\mathcal{H} \xrightarrow{*} (\dot{d}(x), \dot{i}(x))$ and $\mathcal{H} \circledS (\dot{\mathcal{S}}, \mathcal{T})$, $(\mathcal{S}, \mathcal{T}) \in X'$. Then $\mathcal{F} \longrightarrow x$ and $(d\mathcal{F})^d \vee (i\mathcal{F})^i \circledS (\mathcal{S}, \mathcal{T})$. There exists a maximal closed bifilter $(\mathcal{M}, \mathcal{N}) \in (dF)^d \cap (iF)^i$ for each $F \in \mathcal{F}$ such that $(\mathcal{M}, \mathcal{N}) \leq^* (\mathcal{S}, \mathcal{T})$. Since $iF \in \mathcal{N} \subseteq \mathcal{T}$ and $\mathcal{S} \subseteq \mathcal{M}$, $iF \cap S \neq \phi$ for each $F \in \mathcal{F}$ and $S \in \mathcal{S}$. Thus $(d\mathcal{F}, i\mathcal{F}) \circledS (\mathcal{S}, \mathcal{T})$. Since X is convex, $(d\mathcal{F}, i\mathcal{F}) \longrightarrow x$.

Therefore $(\dot{d}(x),\ \dot{i}(x)) \leq^* (\mathcal{S},\mathcal{T})$. Similarly, if $\mathcal{H} \xrightarrow{*} (\dot{d}(x),\ \dot{i}(x))$ and $(\mathcal{S},\mathcal{T}) \circledS \mathcal{H}$, $(\mathcal{S},\mathcal{T}) \in X'$, then $(\mathcal{S},\mathcal{T}) \leq^* (\dot{d}(x), \dot{i}(x))$.

We conclude that $(X^*, \leq^*, \xrightarrow{*})$ is a T_1-ordered convergence space. It is obvious that if X^* is T_1-ordered then X is T_1-ordered.

THEOREM 13. Let $(X, \leq, \rightarrow)$ be a convex convergence ordered space. If $(X, \leq, \rightarrow)$ is strongly T_2-ordered, then $(X^*, \leq^*, \xrightarrow{*})$ is T_2-ordered.

PROOF. Suppose that $\mathcal{H} \xrightarrow{*} (\dot{d}(x),\ \dot{i}(x))$, $\mathcal{K} \xrightarrow{*} (\dot{d}(y),\ \dot{i}(y))$ and $\mathcal{H} \circledS \mathcal{K}$. Then there exist $\mathcal{F} \longrightarrow x$, $\mathcal{G} \longrightarrow y$ such that $(d\mathcal{F})^d \vee (i\mathcal{F})^i \circledS (d\mathcal{G})^d \vee (i\mathcal{G})^i$. Thus we have two maximal closed bifilters $(\mathcal{M},\mathcal{N}) \in (dF)^d \cap (iF)^i$ and $(\mathcal{S},\mathcal{T}) \in (dG)^d \cap (iG)^i$ such that $(\mathcal{M},\mathcal{N}) \leq^* (\mathcal{S},\mathcal{T})$ for each $F \in \mathcal{F}$ and $G \in \mathcal{G}$. It follows that $dF \in \mathcal{M}$, $iF \in \mathcal{N}$, $dG \in \mathcal{S}$ and $iG \in \mathcal{T}$, and $\mathcal{S} \subseteq \mathcal{M}$, $\mathcal{N} \subseteq \mathcal{T}$. Thus $dG \in \mathcal{S} \subseteq \mathcal{M}$ and $iF \in \mathcal{N}$. This means that $iF \cap dG \neq \phi$ for each $F \in \mathcal{F}$ and $G \in \mathcal{G}$. Therefore $\mathcal{F} \circledS \mathcal{G}$. Since X is T_2-ordered, $x \leq y$ and so $(\dot{d}(x),\ \dot{i}(x)) \leq^* (\dot{d}(y),\ \dot{i}(y))$.

Suppose that $\mathcal{H} \xrightarrow{*} (\dot{d}(x),\ \dot{i}(x))$, $\mathcal{K} \xrightarrow{*} (\mathcal{S},\mathcal{T})$, $(\mathcal{S},\mathcal{T}) \in X'$ and $\mathcal{H} \circledS \mathcal{K}$. Then there exists $\mathcal{F} \longrightarrow x$ with $(d\mathcal{F})^d \vee (i\mathcal{F})^i \subseteq \mathcal{H}$, and $\mathcal{S}^c \vee \mathcal{T}^i \subseteq \mathcal{K}$. For each $F \in \mathcal{F}$, $S \in \mathcal{S}$ and $T \in \mathcal{T}$, $\{\{(dF)^d \cap (iF)^i\} \times \{S^d \cap T^i\}\} \cap \Delta^* \neq \phi$ and so there exist $(\mathcal{M},\mathcal{N}) \in (dF)^d \cap (iF)^i$ and $(\mathcal{U},\mathcal{V}) \in S^d \cap T^i$ such that $(\mathcal{M},\mathcal{N}) \leq^* (\mathcal{U},\mathcal{V})$. Since $iF \in \mathcal{N} \subseteq \mathcal{V}$ and $S \in \mathcal{U} \subseteq \mathcal{M}$, $i\mathcal{F} \vee S \neq \phi$ by definition of a bifilter $(\mathcal{U},\mathcal{V})$. Hence $(d\mathcal{F}, i\mathcal{F}) \circledS (\mathcal{S},\mathcal{T})$ and $(d\mathcal{F}, i\mathcal{F}) \longrightarrow x$. It follows that $(\dot{d}(x),\ \dot{i}(x))$

$\leq^* (\mathcal{S}, \mathcal{T})$.

Suppose that $\mathcal{H} \xrightarrow{*} (\mathcal{F}, \mathcal{G})$, $\mathcal{K} \xrightarrow{*} (\mathcal{S}, \mathcal{T})$ and $(\mathcal{F}, \mathcal{G})$, $(\mathcal{S}, \mathcal{T}) \in X'$ and $\mathcal{H} \circledS \mathcal{K}$. Then $\mathcal{F}^d \vee \mathcal{G}^i \subseteq \mathcal{H}$ and $\mathcal{S}^d \vee \mathcal{T}^i \subseteq \mathcal{K}$. We have $(\mathcal{M}, \mathcal{N}) \in F^d \cap G^i$ and $(\mathcal{U}, \mathcal{V}) \in S^d \cap T^i$ with $(\mathcal{M}, \mathcal{N}) \leq^* (\mathcal{U}, \mathcal{V})$ for each $F \in \mathcal{F}$, $G \in \mathcal{G}$, $S \in \mathcal{S}$ and $T \in \mathcal{T}$. If $S \in \mathcal{S}$ then $S \in \mathcal{U} \subseteq \mathcal{M}$. Thus $S \cap F \cap G \neq \phi$ for each $F \in \mathcal{F}$ and $G \in \mathcal{G}$, and so $S \in \mathcal{F}$. If $G \in \mathcal{G}$ then $G \in \mathcal{N} \subseteq \mathcal{V}$. Thus $G \cap S \cap T \neq \phi$ for each $S \in \mathcal{S}$ and $T \in \mathcal{T}$. Hence $G \in \mathcal{T}$. Therefore $(\mathcal{F}, \mathcal{G}) \leq^* (\mathcal{S}, \mathcal{T})$.

Suppose that $\mathcal{H} \xrightarrow{*} (\mathcal{S}, \mathcal{T})$, $(\mathcal{S}, \mathcal{T}) \in X'$, $\mathcal{K} \xrightarrow{*} (\dot{d}(x), \dot{i}(x))$ and $\mathcal{H} \circledS \mathcal{K}$. Then there exists $\mathcal{F} \longrightarrow x$ with $(d\mathcal{F})^d \vee (i\mathcal{F})^i \subseteq \mathcal{K}$ and $\mathcal{S}^d \vee \mathcal{T}^i \subseteq \mathcal{H}$. For each $F \in \mathcal{F}$, $S \in \mathcal{S}$, and $T \in \mathcal{T}$, there exist $(\mathcal{M}, \mathcal{N}) \in S^d \cap T^i$ and $(\mathcal{U}, \mathcal{V}) \in (dF)^d \cap (iF)^i$ with $(\mathcal{M}, \mathcal{N}) \leq^* (\mathcal{U}, \mathcal{V})$. It follows that $S \in \mathcal{M}$, $T \in \mathcal{N}$, $dF \in \mathcal{U}$, $iF \in \mathcal{V}$, $\mathcal{U} \subseteq \mathcal{M}$, and $\mathcal{N} \subseteq \mathcal{V}$. Thus $dF \vee \mathcal{T} \neq \phi$, and so $(\mathcal{S}, \mathcal{T}) \circledS (d\mathcal{F}, i\mathcal{F})$. Hence $(\mathcal{S}, \mathcal{T}) \leq^* (d(x), i(x))$. We conclude that X^* is T_2-ordered.

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