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THE IMAGE CONFIGURATION OF MICRO LENSING WITH AN EXTENDED SOURCE

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ABSTRACT

This study presents the specific rule governing the image configurations of an extended source for micro lensing of the two-body gravitational lens system developed by Chang and Refsdal (1979). Various topological situations of a source are considered in relation to the regions bounded by the so-called critical curves.

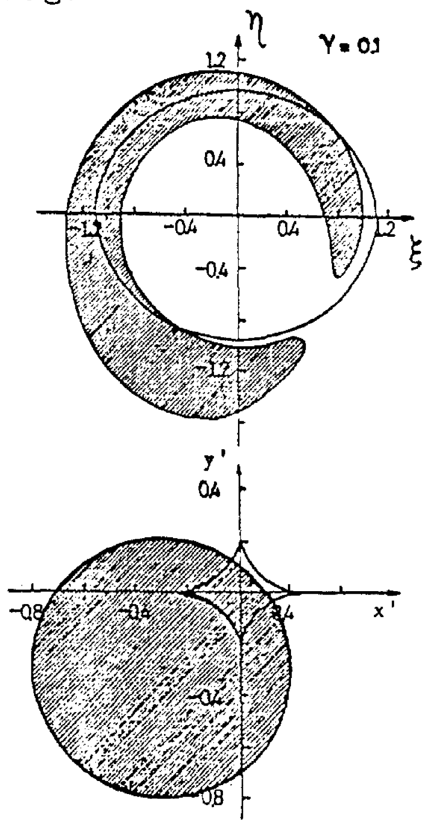
I. INTRODUCTION

After developing a model of micro lensing first by Chang and Refsdal (1979) (hereafter Paper I), astrophysical application of micro lensing has been intensively studied during last one decade, such as cosmological distance determination, mass determination of the lens, the morphological structure of the light source (e.g. QSOs). Furthermore, the theory of micro lensing has become a very helpful tool to solve many body lensing problems. In many body gravitational lensing there are mathematical difficulties in dealing with the structural complexities of n gravitating bodies simultaneously. Such difficulties, particularly in numerical procedure, have been reduced by means of the model of micro lensing. The basic concept of micro lensing constructed by Chang and Refsdal in 1979 is as follows: one may split into two major contributions of light bending due to the gravitational potential field. That is, one is due to an axially symmetrical extended mass (a stellar system with a mass larger than about $10^{11} M_{\odot}$), whereas the other, due to a discrete small mass (about Jupiter mass up to mass of a globular cluster) lying close to the intrinsic light rays in the plane where the massive gravitating body is located.

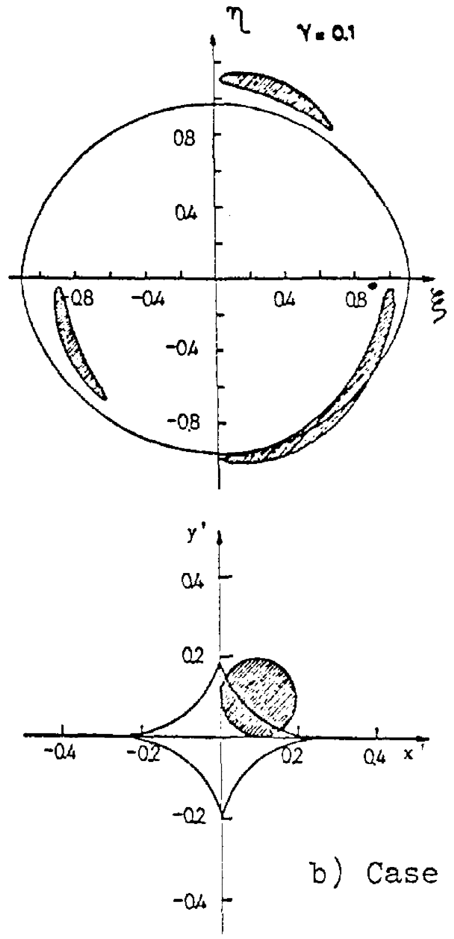
There are a couple of works on many body lensing problems, which are based on the model described above, for instance, Young (1981), Kayser, Refsdal, and Stabell (1986) (hereafter KRS 1986). The latter work presents the most detailed investigation on n-body problem and suggests various astrophysical applications.

High amplification phenomena due to the lens effect have brought us a great attention, because of their detectability by observation and astrophysical information from the light curve, for example, the structure of the source, the mass of the lens.

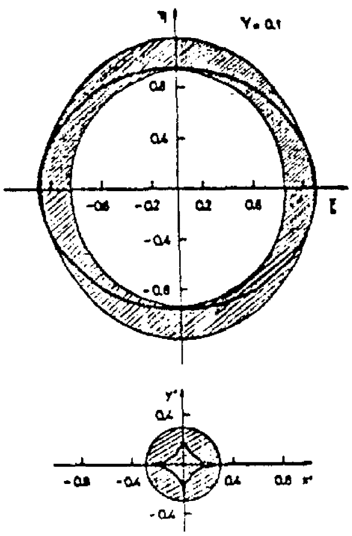
Fig. 1



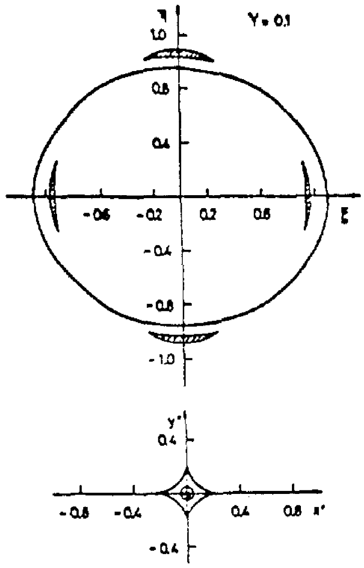
a) Case Ic



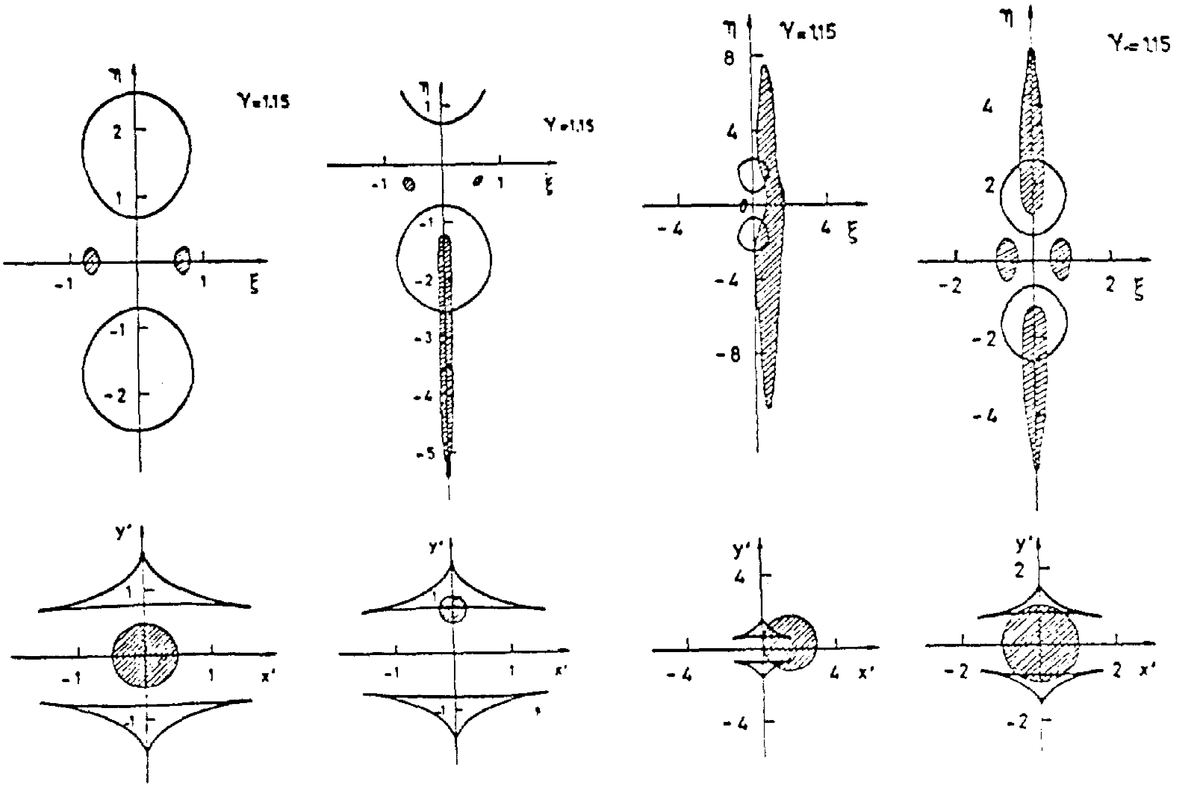
b) Case Ie



c) Case Ia



d) Case If



e) Case IIIq f) Case Ii g) Case IIn h) Case IIo

Fig. 1. Analytical solutions of the inverse mapping (i.e. $\vec{x}'$ (x′, y′) $\rightarrow \vec{b}(\xi, \eta)$) for extended sources. The source is represented by a shaded circular disk in the (x′, y′) plane (the source plane); and the images in the (ξ , η) plane (the deflector plane), represented by shaded areas. The critical curves are denoted by a dark solid line in the both planes. The coordinates of both planes are normalized by b_0 in the deflector plane and by $L/\lambda\, b_0$ in the source plane. Fig. 1a) represents to Case Ic in Table 1; Fig. 1b). Case Ie, in Table 1, etc., as indicated in Fig. 1. All the cases considered in Fig. 1, the observer is at $\vec{x}$ (x, y) = $\bar{\vec{x}}$ (0, 0).

Contrary to this, the morphological structure of the images has been paid little attention, because of micro angular separations between neighbouring sub-images as shown in Figure 1.

Some lensing systems show us very distinct structure of the lensed images, e.g. QSO 0957 + 561 A, B, QSO 2237 + 0305, QSO 115 + 080. Even the recent study (Hewitt *et al.* 1988) has suggested that the radio-morphology of ring-like structure of MG 1131 + 0456 may be an Einstein Ring (the luminous ring produced with an alignment between the light source, the gravitational lens, and the observer) (see, e.g. Einstein 1936; Refsdal 1964).

This study demonstrates some examples of image configuration due to micro lensing under the given idealized physical conditions. The discussion is limited to the case of a single discrete mass acting as a micro lens at a time.

II. THE NORMALIZED LENS EQUATION

All the geometries and the assumptions adopted in this section are exactly the same with Paper I and Chang and Refsdal (1984) (hereafter Paper II). The contour of the image configuration has been analytically obtained by using the well-known gravitational lens equations with an extended source. For simplification the extended source is assumed to be a circular disk. With the observer at $\vec{x}(x, y) = \vec{x}(0, 0)$ the lens equations are given by

$$\vec{x}' = \begin{pmatrix} 1+\gamma_\xi & 0 \\ 0 & 1-\gamma_\eta \end{pmatrix}\vec{b} - s(\vec{b}) \tag{1}$$

where $\vec{x}'$ is $\vec{x}'$ (x′, y′) in the source plane, and $\vec{b}$ is $\vec{b}(\xi, \eta)$ in the deflector plane, and $s(\vec{b})$, the bending effect due to a micro lens under consideration. $\vec{x}'$ (x′, y′) denotes the position of the source point in the source plane, and $\vec{b}$ (ξ, η), the impact parameter in the deflector plane.

We have taken for convenience the observer's position at the origin of the observer's plane, i.e. $\vec{x}$ (x, y) = $\vec{x}$ (0, 0). In eq. (1) the first term in the r.h.s. represents the effects only due to the extended massive gravitating body, and the second term of the r.h.s. is the effects only due to the micro lens.

III. CLASSIFICATION OF THE GRAVITATIONAL LENS MODEL (γ-PARAMETER)

In the previous works (Paper I and II) γ-parameter is introudced to classify the model of the deflector under consideration, which has the information on the physical properties of the deflector, such as mass density distribution and shears of light bundles as passing close to a perturbing star (micro lens) lying in the deflector plane. KSR (1986) and Packzinsky (1986) have re-defined γ-parameter in order to reduce the number of free parameters in the lens equations. Their redefinition of γ-parameter, of course, has the same physical meaning with that appeared in Papers I and II. However, there is only one difference. Papers I and II have adjusted γ-parameter for a given value of σ_c, the surface mass density of continuousely distributed matter in the deflector plane where perturbing stars (micro lenses) are embedded.

Let us now briefly introduce the definition of γ-parameter. By introducing the shear parameter γ, which is then given by

$$\gamma = \frac{\gamma_\xi - \gamma_\eta}{2} \tag{2}$$

and

$$\sigma = \frac{\sigma_s}{1 - \sigma_c} \tag{3}$$

One may employ γ and σ in the lens equations instead of γ_ξ, γ_η, σ_s appeared in the lens equations of Papers I and II. γ_ξ and γ_η are the shear components in ξ -and η -axis of the deflector plane, respectively, and σ_s, the smeared out surface density of stars around the light bundle (for details, see, Chang 1981; Papers I and II; KRS 1986).

γ_ξ and γ_η are then related to γ, σ_s, and σ_c in following way:

$$\gamma_\xi = \gamma - (\sigma_s + \sigma_c) \quad (4a)$$

$$\gamma_\eta = -\gamma - (\sigma_s + \sigma_c) \quad (4b)$$

For the demonstration of the image configuration, we have taken $\gamma = \gamma_\xi = -\gamma_\eta = 0.1$ and 1.15, which are arbitrarily chosen for demonstration.

IV. THE NUMBER OF IMAGES

The number of images depends on the topological situation between the source and the regions bounded by the critical curves, i.e. the critical regions. With a point source in relation to the critical curves it has been discussed in great detail in Papers I, II, and in Chang (1981). In the present study we would like to present the algebraic relation which governs the number of images of an extended source. In micro lensing with only one micro lens there are one or two critical regions in the three geometrical planes, i.e. the deflector-, the source-, and the observer-plane, respectively.

The specific rule governing the number of images, which we have confirmed numerically, are as follows:

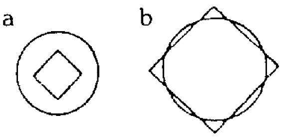
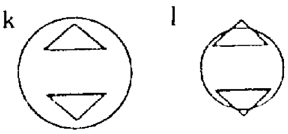
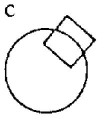
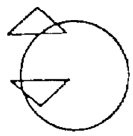
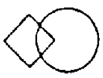
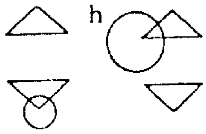
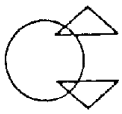
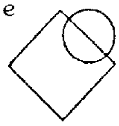
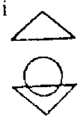
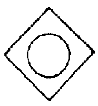
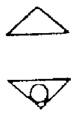
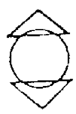
$$N_i = 2(1 + N_r) - N_c \quad (5)$$

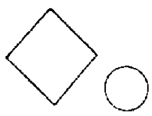
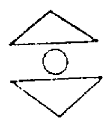
where N_i is the number of images, and N_r, the number of critical regions in the source plane, and N_c, the number of the critical curves covered by the source. For instance, let us consider the case of no critical region being topologically related to the source. Then, we have $N_r = 0$ and $N_c = 0$, which yields $N_i = 2$.

We should remind us of the characteristic feature of the critical regions of Chang-Refsdal model of micro lensing. The projected surface mass-density in the deflector plane would be the major factor to determine the shape of the critical regions. The critical regions will be bounded by the critical curves with three or four cusps.

We have presented three major topological situations in Table I, such that i) only one critical region, ii) two critical regions, and iii) no critical region are related to the source Figure 1 shows various image configurations of the circular source seen by the observer at $\dot{x}$ (x, y) $= \dot{x}$ (0, 0).

Table 1. The Number of Images N_i for Different Topological Situations between the Source and the Critical Regions and the Region outside the Critical Regions. The Boundary of the Source is indicated by a Circle. The Critical Curves are denoted by Solid Line with Cusps.

N_i	case I		case II	N_i
	$\gamma < 1$	$\gamma > 1$	$\gamma > 1$	
Ring image 0	a b 		k l 	Ring image 0
1	c 		m 	1
2	d 	g h 	n 	2
3	e 	i 		3
4	f 	j 	o 	4

N_i	case III	
	$0 < \gamma < 1$	$\gamma > 1$
2	p 	q 

V. THE NORMALIZED UNITS

The diameter of the source in the normalized unit, D', is converted into

$$D = D' \frac{L}{\lambda} b_0 \; ; \; b_0 = \sqrt{\frac{4GM}{c^2} \frac{(L - \lambda)}{L}} \; (1 + z) \tag{6}$$

where D is the actual demension of the source diameter, and b_0 is the radius of luminòus ring (the so-called Einstein ring). L and λ are the affine parameters to the source and to the deflector from the observer, respectively. In the deflector plane the coordinates are normalized by b_0. M and z are the mass and the redshift of the micro lens, respectively.

VI. DISCUSSION

Multiple image systems due to micro lensing have been predicted by the theory developed in earlier works (Papers I, II ; Chang 1981). After detecting QSO 0975 + 561 A, B in 1979 by Walsh *et al.* (1979), there have been several important detections and discussions on micro lensing (see, e.g. Fey *et al.* 1985; Huchra *et al.* 1985). If we take the lens system consisting of a deflector at the distance of the half of the source distance from the observer. The double QSO 0957 + 561 A, B belongs to such a system.

We take the cosmological model of $q_0 = \sigma_0 \simeq 0$ with $H_0 = 60$ km s^{-1} Mpc^{-1}, and the mass of micro lens 1 $M_\odot$ at the red-shift 0.4. With the affine distances about 2000 Mpc to the source, and about 1000 Mpc to the deflector, the units of length are 0.04 ly in the deflector plane, and 0.07 ly in the source plane. They are corresponding to 2.09×10^{-6} second of arc in the deflector plane, and 3.67×10^{-6} second of arc in the source plane. Taking a globular cluster of $M \sim 10^5 M_\odot$ acting as a micro lens, for the system given above, the unit length in the source plane is 22.1 ly, and 12.6 ly in the deflector plane. With a globular cluster acting as a perturbing mass lying close to the light path, we would expect to have a better angular separation of the sub-images for resolution.

In conclusion, we give a brief discussion on image properties due to n-body micro lensing. If one takes into account n stars lying close to the light path at a time, the axial symmetrical properties of the critical curves should be violated. The characteristic contour of the critical curves with cusps would still be presented, as the case of one perturbing star. The critical regions, however, are not any longer two or one region in the deflector-, the source-, and the observer-plane, respectively. There would be more than two critical regions. They would be then randomly scattered (see, e.g. KRS 1986). Concerning the number of images in n-body problem it may not be possible to give a simple algebraic form presented in eq. (5). However, there would be a certain algebraic relation between the number of deflectors and the number of images.

This problem will be considered in the next paper (in preparation).

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