목재공학16(1)：62~67(原著)
Wood Sci. & Tech. 16(1)：62~67. 1988

크기가 相異한 割裂 両端部에서의 應力強度因子에 관한 연구 *1

차 재 경 *2

The Study of Stress Intensity Factors Near Tips of Crack of Different Size *1

Jae Kyung Cha *2

要 約

本研究는 應力强度因子 및 割裂 크기간의 기본적인 관계를 밝히기 위하여 실시하였으며, 이 應力强度因子는 割裂의 크기와 위치를 포함한 材料의 幾何學的 특징을 갖는 有限要素法에 의해 分析하였다. 그 결과 높은 應力이 割裂이 일어난 端部에서 일어났으며, 應力强度因子의 변화는 割裂 端部로부터의 거리에 변화하였다. 割裂 端部 要素의 크기는 應力强度因子에 현저하게 영향하였으며, 有限要素法에서의 要素크기는 割裂 길이의 절반의 약 10% 정도였다.

INTRODUCTION

As a result of considerable research efforts during the past decade, linear elastic fracture mechanics (LEFM) can now be used to solve many practical engineering problems in failure analysis, material selection, and structural life prediction. LEFM can also be extended to solve fracture problems involving moderate plastic yielding by incorporating various plasticity correction factors provided fracture occurs prior to large scale yielding of the structural member.

For the purposes of analyzing fracture problems associated with brittle behavior, a knowlege of the stress field near the crack front is a prerequisite. This discussion will be limited to a remote tensile stress acting normal to the crack length, i.e. to the opening mode for which K is traditionally written as K_I. The amplitude of this stress field is described by K, the stress intensity factor (SIF). The accuracy with which the SIF describes the fracture behavior of real material depends on how well the SIF represents the conditions of stress and strain inside the fracture propagation. In this sense the SIF gives an exact representation only in the limit of zero plastic strain. However, for many practical purposes a sufficient degree of accuracy may be obtained if the crack front plastic zone is small in comparision with the vicinity around the crack in which the SIF yields a satisfactory approximation of the exact elastic stress field. The loss of accuracy associated with increasing the relative size of the plastic zone is gradual, and it is not possible at the present time to prescribe limits on the applicability of LEFM by means of theoretical considerations.

The objective of this study was to compare

*1. 接受 2月 11日, Received February 11, 1988.
*2. 노스캐롤라이나 州立大學, Dept. of Wood & Paper Science, North Carolina State University, Raleigh, NC 27695, U. S. A.

the SIFs obtained from two different procedures, namely, by experimental K-calibration and by a theoretical method. Another aim was to determine the optimum procedure for evaluating K_I by studying the SIF at the different points near the crack tip.

LITERATURE

Experimental K-calibration

There are several expressions for estimating the SIF for the central crack problem. However, the most commonly used experimental method for measuring the SIF is known as K-calibration. K-calibration requires determination of the critical opening mode strain release rate, G_I, for cracks of several lengths. The value of G_I is given by

$$G_I = (1/2) P^2 (d\lambda/da) \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where P is the applied load, and λ is compliance of the cracked specimen, a is length of crack, and $d\lambda/da$ is the slope of the experimental curve λ vs. a. K_I is then determined from G_I by following equation.

$$K_I = \sqrt{G_I E} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The SIF corresponding to the start of crack growth is called the "fracture toughness" of the material.

Measurements are made of the compliance (reciprocal of the stiffness) of a specimen having a narrow machined slot which is incrementally extended between successive measurements. The machined slot is used to simulate a crack primarily because it is not feasible to produce plane cracks of sufficient size and accuracy by overstressing the specimen. The experimental data are treated by expressing the specimen compliance as a function of crack length and then obtaining the derivative of this function with respect to crack length. To conduct a compliance calibration with good accuracy it is necessary to use a sensitive, accurate gauge and pay careful attention to detail. It is always an advantange to use as large a specimen as possible for compliance measurements because the displacements will be a proportionately large and can be measured with correspondingly good accuracy.

Mathematical analysis of this case was conducted by Forman and Kobayasi [3], and Ishida. Brown and Srawley [1] obtained the following expression by using the least squares best fit procedure for the result of Ishida. The following expression fitted Ishida's results to within 0.5% over the range of a/W from 0 to 0.7.

$$K_I = \sigma\sqrt{0.5a}\{1.77 + 0.227(a/W) - 0.51(a/W)^2 + 2.7(a/W)^3\} \quad \ldots\ldots\ldots (3)$$

where W is the width of specimen.

2.2 Elastic Stress Field Approach

Owing to the practical difficulties of the energy approach, a major advance was made by Irwin who showed from linear elastic theory that the stresses in the vicinity of the crack tip take the form

$$\sigma = (K_I/\sqrt{2\pi r})\ f(\theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where r, θ are the cylindrical polar coordinates of a point with respect to the crack tip as shown in figure 1, $f(\theta)$ is a function of θ and K is the SIF at the point.

Equation 4 shows that as r tends to zero the stresses become infinite, i.e. there is a stress singularity at the crack tip. Westergaard [2] solved the stress function using complex variables to obtain the following relationships for the tensile stresses σ_{xx} and σ_{yy} induced perpendicular and parallel to the crack, respectively and the shear stress σ_{xy} at the point (r, θ).

$$\sigma_{xx} = [K_I/\sqrt{2\pi r}]\ \cos\theta/2\ [1 - \sin\theta/2 \sin 3\theta/2]$$
$$\sigma_{yy} = [K_I/\sqrt{2\pi r}]\ \cos\theta/2\ [1 + \sin\theta/2 \sin 3\theta/2]$$
$$\sigma_{xy} = [K_I/\sqrt{2\pi r}]\ \cos\theta/2\ \sin\theta/2\ \cos 3\theta/2 \quad \ldots (5)$$

Equation 5 gives the stresses as products of geometrical term $f(\theta)/\sqrt{2\pi r}$ and K_I.

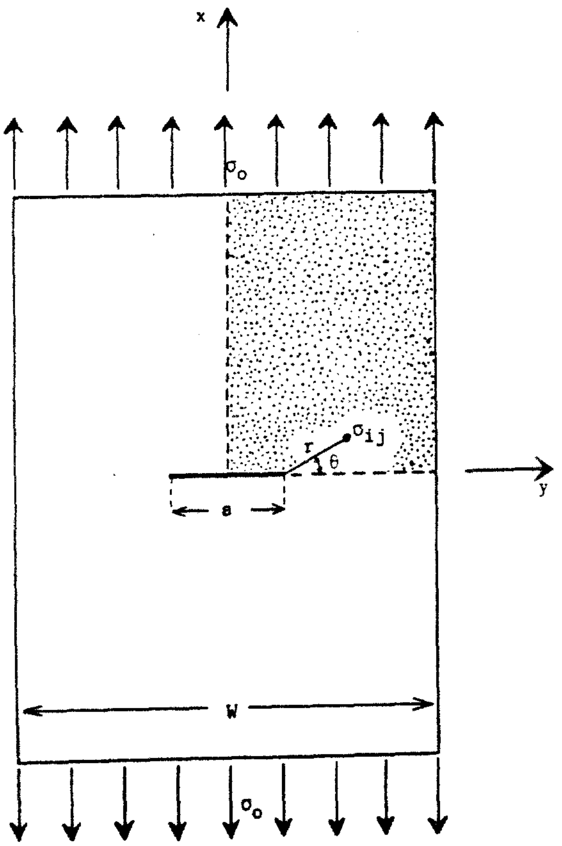

Fig. 1. Stress at a Point ahead of a Crack Tip.

PROCEDURE

Stress analysis by applying fracture mechanics to engineering problems consists of determining the SIF induced by a cracks. Unfortunately, analytical methods fail to yield the SIF except for a limited number of simple problems. For most industrial applications of fracture mechanics where a 5-15% error is tolerated, a numerical solution to actual problems in two dimensions appears to be more palatable than lengthy exact solutions to idealized problems which only approximate the actual conditions. The numerical method which has been widely accepted in industry is the method of finite element analysis.

The finite element method (FEM) is a procedure for obtaining approximate solutions to continuum problems. It involves conceptually dividing the body under consideration into elements and assuming an approximate form for the solution within each element. A primary advantange of the FEM is that a general computer program which is applicable to a wide range of specimen geometries and loading conditions has been developed.

Two important considerations in the development of finite element analysis for fracture mechanics are the proper modeling of the crack tip singularity and the interpretion of the results in terms of a SIF or a crack driving force. If the state of stress in the vicinity of the crack tip can be determined within a reasonable degree of accuracy, the SIF can be computed by equation 5. The finite element analysis must then produce sufficiently accurate states of stress within the local region where these equations are valid.

To determine the optimum procedure for evaluating SIF, a finite width tension plate with a central crack was considered. A quadrant of this plate was initially divided into 88 elements by the subroutine. The quadrilateral type was used for all the elements. The nodal point numbering and corresponding coordinates shown in figure 2 was automatically done within a subroutine of the program. The lines of symmetry where normal displacements and tangential forces vanish are representing by rollers. So nodes number (5, 6, 7, ...,12) were fixed only in the x direction. Nodes number (1, 13, 25, 37, 49, 61, 73, 85, and 97) were fixed in the y direction. The half crack size represents the distance from node 1 to 5. Since the stress distributions of σ_{xx}, σ_{yy}, and σ_{xy} along the y axis are required, the elements along this axis are well defined. Cracks of length equal to 10%, 20%, 30%, 40%, 50%, and 60% of the width of the specimen were chosen. Finite element models for specimens with these six different crack lengths had to be developed. They were automatically obtained by a computer program which developed nodes and elements of the mesh for these different crack lengths. The specimen size analyzed was 2 inch wide, 1/8 in thick. Calculations were based on

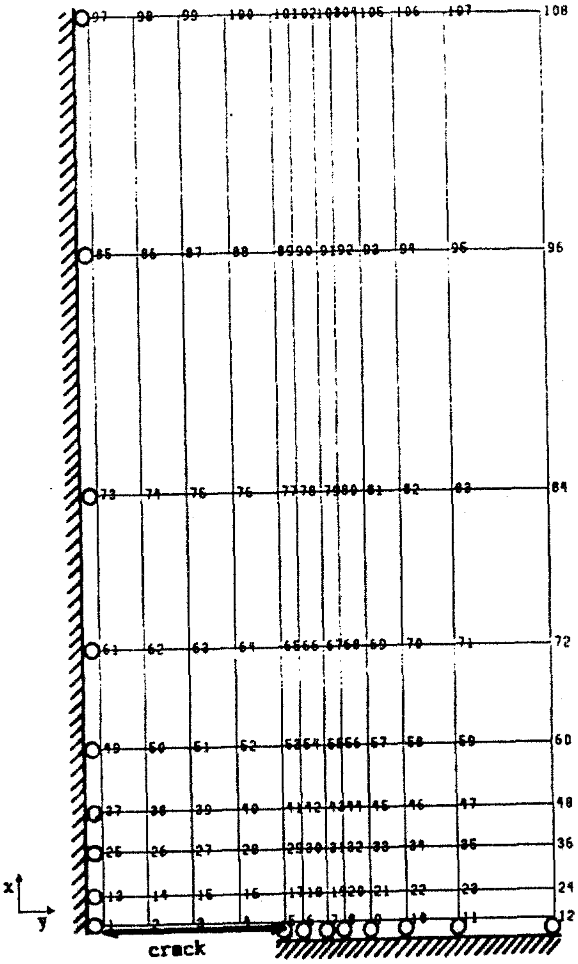

Fig. 2. Generalized mesh Generization for central Crack Problem.

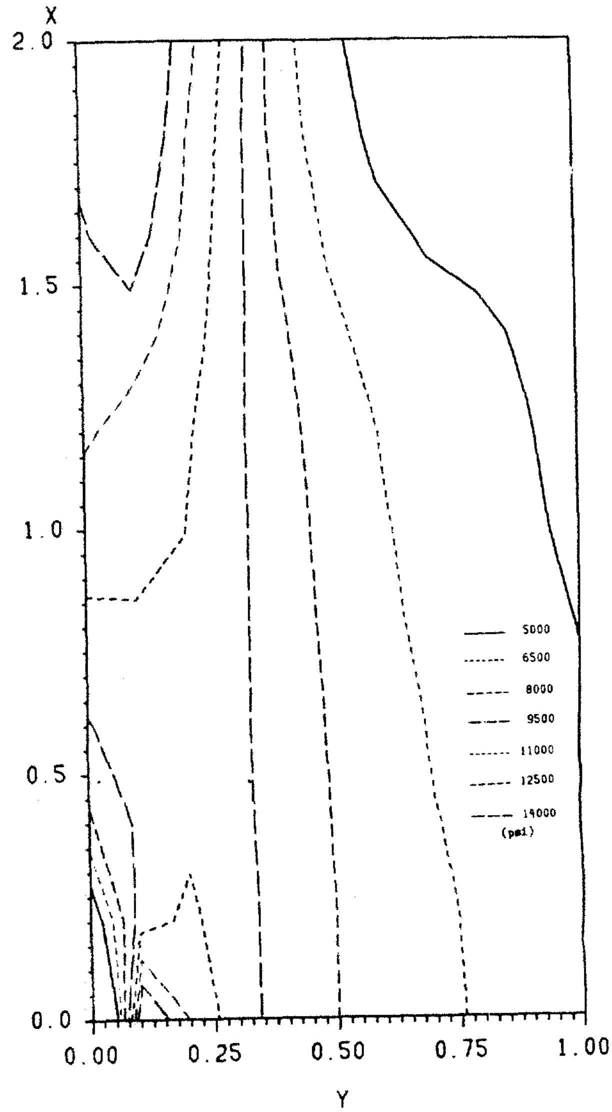

Fig. 3. The Stress (σ_{xx}) Distribution for Crack Length (a/W=0.1)

a uniform tensile stress of 8000 psi applied in the x-direction at the ends of the specimen.

RESULTS AND DISCUSSION

Crack Tip Stress Distribution

The stress distributions near the crack tip are the most important factors in fracture mechanics, because they determine whether a crack advances. The normal crack tip stresses σ_{xx} are shown in figures 3 and 4. High stresses are shown the near the crack tip, then tbe stresses gradually decrease far from the crack tip.

Stress Intensity Factor

The SIF was calculated by the equation 3 and 5. For calcuation of SIF by using equation 5, two different kinds of r values were used. Firstly, a constant r value was used for all the cases. Then the two diferent kinds of SIFs were compared as shown in table 1. In using the stress field approach of equation 5, it was found that the SIF was significantly affected by the ratio of distance from end crack to half crack length. The

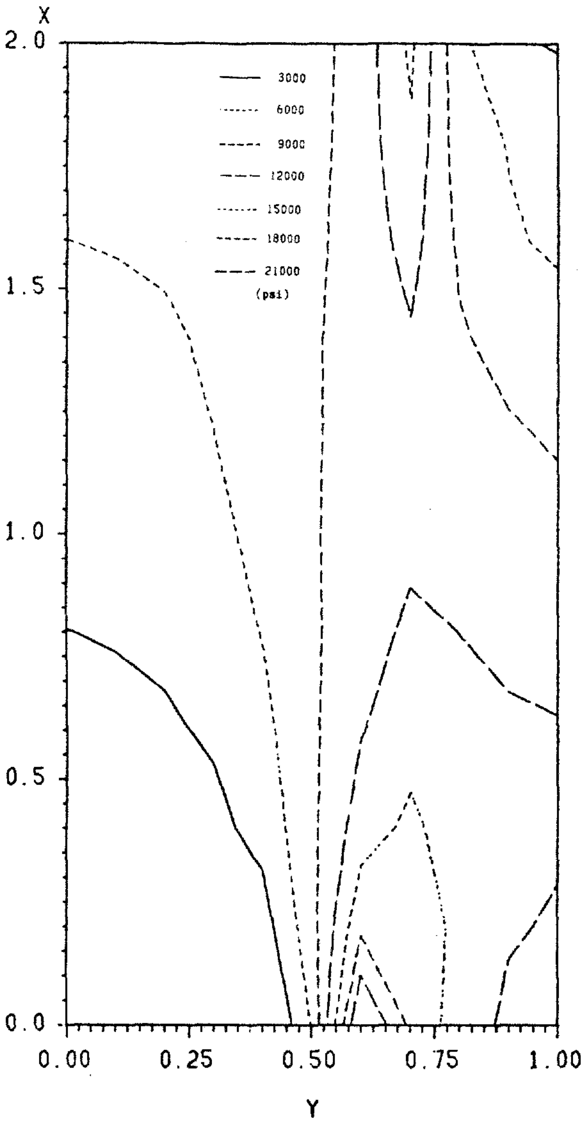

Fig. 4. The Stress (σ_{xx}) Distribution for Crack Length (a/W=0.6)

reason is that the stress function for equation 5 is strictly valid only for an infinite plate. The geometry of finite size specimens has an effect on the crack tip stress field. Therefore, the SIF was also calculated by using the same ratio (2r/a=0.1) of the distance r to half crack length to compare the two methods as shown in table 2. This shows that the difference was 2-11%, primarily due to the inability of method of finite element analysis to handle the problems with steep gradients, such as those which exist in the vicinity of a crack tip. Part of this inaccuracy was due to the stiffness matrix used which derived on the basis of uniform stress in the basic element.

From the above results, it is important to select the most suitable mesh size near the crack tip element. We can also find that the big size of crack length should have the smaller ratio of distance from end of crack to half crack length than that of small crack length as shown in table 1 and 2.

Table 1. Stress Intensity Factors for constant Distance r=W/30 from Crack Tip.

Crack Length Ratio (a/W)	$\frac{r}{W}$	SIF-value(psi$\sqrt{in.}$) Equation 5	Equation 3	SIF-ratio
0.1	1/30	8345	4529	1.84
0.2	1/30	9917	6499	1.53
0.3	1/30	11295	8172	1.38
0.4	1/30	12396	9876	1.26
0.5	1/30	14000	11842	1.18
0.6	1/30	15989	14288	1.12

Table 2. Stress Intensity Factors for Distance r=a/20 from Crack Tip.

Crack Length Ratio (a/W)	$\frac{r}{W}$	SIF-value(psi$\sqrt{in.}$) Equation 5	Equation 3	SIF-ratio
0.1	0.005	4612	4529	1.02
0.2	0.010	6885	6499	1.05
0.3	0.015	9013	8172	1.10
0.4	0.020	10904	9876	1.10
0.5	0.025	13166	11842	1.11
0.6	0.030	15737	14288	1.10

CONCLUSIONS

This study supports the following conclusions.

1. High stress concentration occurs around the crack tip.
2. The SIF changes with distance from crack tip.
3. The mesh size near the crack tip should select around 10% of the half crack size.

REFERENCES

1. Brown, W.F. & Srawley, J.E. 1967. Plane Strain Crack Toughness Testing of high Strength metallic Material. ASTM special technical Publication No. 410, Philadelphia, Pa.
2. Ewalds, H.L. & Wandill, R.J.H. 1983. Fracture Mechanics. Edwald Arnold Co, Baltimore, MD.
3. Forman, R.G. & Kobayashi, A.S. 1964. On the axial Rigidity of the perforated Strip and the Strain Energy Release Rate in the centrally notched Strip subjected to uniaxial Tension. Journal of Basic Engineering, 693-699.
4. Mandell, J.F., McGarry, F.J. Wang, S.S. & Im, J.H. 1973. Stress Intensity Factors for anisotropic Fracture Test Specimens of special Geometries. Massachussets Institute of Technology Report No. MITSG 74-10. Cambrige, Massuchussets.
5. Sih, G.C. Handbook of Stress Intensity Factor. 1973. Lehigh University Press, Bethleham, Pennsylvania.
6. Sih, G.C., Ryon, N.E. & Jones, R. 1983. Fracture Mechanics Technology applied to Material Evaluation and structural Design. Maritinus Nijhoff Publishers, Boston.