

# 圓筒型 2次元木材의 數學的 研究 \*1

차 재 경 \*2

## Mathematical Study for Cylindrically Orthogonal Log \*1

Jae Kyung Cha \*2

### 要 約

木材의 應力과 變形에 미치는 함수율 및 온도변화의 효과를 有限要素 분석법에 의해 측정하였다. 목재는 春材 및 秋材를 나타내는 層構造의 圓筒型으로 모델화 하였으며, 線形的 彈性體 그리고 圓筒型 異方性 재료로 가정하였다.

徑斷面에서의 變形은 함수율 및 온도와 밀접한 관계가 인정되었으며, 最大의 壓縮應力은 最內層인 晩材層에서 일어났다. 또한 최대의 觸斷面應力은 春材部의 最內層에서 일어났다. 徑斷 方向과 觸斷 方向의 應力間의 차이는 外層에서 가장 크게 나타났으며 이와같은 應力의 차이가 變形을 일으키는 主要因임이 밝혀졌다.

### INTRODUCTION

Wood is used under a wide range of service conditions like many structural materials. Two types of conditions predominate: the interior of buildings, and external service characterized by several ranges of seasonal weather conditions. It is the range of temperature and relative humidity fluctuation of exposure to sun and rain which distinguishes these different environments. It is well known that both temperature and relative humidity, but particularly the latter, control the level of moisture adsorbed by wood. Adsorbed moisture and temperature have a significant bearing on the physical and mechanical properties of wood.

The magnitude of drying stresses in the R, T-

plane is governed primarily by the severity of the moisture gradient as well as by the elastic parameters of wood. Schedules of temperature and relative vapor pressure are used during drying to control the gradient and minimize the state of internal stress. As the moisture content nears and passes below fiber saturation, shrinkage of surface layer takes place. Since the dimensional change is restrained by the wet, nonshrinking core, the outer surface is stressed in tension and compression stress resides in the core material. Inadequate control leads to the development of large stresses, which may exceeds the fracture strength of wood across the grain and lead to both internal and external checking.

The objective of this study was to study the transverse stress distribution under thermal load.

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\*2 . 노스캐롤라이나 州立大學, Dept. of Wood & Paper Science, North Carolina State University, Raleigh, NC 27695, U. S. A.

## MECHANICAL PROPERTIES OF WOOD UNDER MC AND TEMPERATURE

MC and temperature have important effects on mechanical properties of wood. In estimating the mechanical properties of any material, two sorts of quantities have to be measured, the elastic moduli and strength values.

The influence of moisture on mechanical properties of wood, in particular strength and elastic behavior, has been catalogued for many woods. Green lumber used in the framing of residence might dry to MC as low as 5% in the structure. The deformation which results in quite different from that of lumber dried before use. Under conditions of constant load wood exhibits unusual creep deflections with cyclic change of moisture. The complexities introduced by simultaneous time variation of load, deformation and MC require special attention.

Data recently suggest that certain mechanical properties especially strength, may attain a maximum value at approximately 5% MC and decline at lower levels of MC. Other data reveal no maximum, the strength properties continuing to increase until all moisture has been removed. Some investigators believe that a decline in strength at MCs below 5% may be due to microfailures in the cell which occur during drying. The phenomenon is complex and remains unresolved at present. Wangaard [17] and Markwardt [1] found that the change in MOR and MOE was a decrease of 4 and 2% for mechanical properties due to an average increase in 1% MC below fiber saturation.

Mark et al. [7] and Tang et al. [13] were determined on plate shear specimens during dryout from green condition. Tang et al. showed that modulus of rigidity (G) of scarlet red oak varies with MC. G of scarlet red oak determined at various moisture contents.  $G_{LT}$ ,  $G_{LR}$  and  $G_{RT}$  showed a same trend of Wangaard's for MOE with decreasing MC below fiber saturation

point.

The expansion which occurs during the heating of material results from the increased oscillation of atoms, the increase in the distance between atoms results in linear and volumetric expansion. The increase in length (L) is proportional to the change in temperature ( $\phi$ ).

$$\Delta L/L = \alpha \Delta \phi$$

With few exceptions the algebraic sign of  $\alpha$  is negative because of the decrease in mechanical properties with increasing temperature.

At a constant MC and below about 400°F mechanical properties are essentially linearly related to temperature. The change in properties that occurs when wood is quickly heated or cooled and then tested at that condition is termed "immediate effect". At temperature below 200°F the immediate effect is essentially reversible; that is, the property will return to the value at the original temperature if the temperature change is rapid [14]. Sano [11] found that strength properties linearly decreased with increasing temperature for ash and spruce.

Although similarities can be noted between the effects of temperature and absorbed moisture on the properties of woods, the molecular mechanisms involved are quite different. Nonetheless, the effects of temperature and moisture changes are frequently dramatic, difficult to predict, and complex to interpret.

The effects of temperature and moisture on wood properties are not always easily determined. Although the equilibrium MC of wood depends on the relative humidity, the level of adsorbed moisture also varies with temperature. Both temperature and moisture interact to modify wood properties. Wood properties are dependent on the particular level of each; their interaction at elevated level is unusually complex.

## PROCEDURE

Numerical solutions are essentially identical

for both the finite element method (FEM) and the matrix analysis of structures. Most structural problems involving a small number of members can be solved. The structural Analysis Program IV (SAP IV) available at NCSU contains a finite element program which possesses all the basic capabilities needed for the problem at hand. Linear elastic action was assumed and the SAP IV program was run for all the models.

#### Material properties

The two major assumptions about the mechanical properties are considered:

1. material homogeneous (cases 1 and 3),
2. the nonuniform distribution of MC and temperature effect, so nonuniform material property (cases 2 and 4).

Therefore, four different cases are considered in this study as shown in table 1.

The elastic and thermal expansion coefficients are different for each cases for earlywood and latewood as shown in table 2, 3, and 4. Homogeneous material the elastic coefficients (case 1 and 3) are only different between earlywood and latewood. The nonuniform material properties (case 2 and 4) assume that the MC control the elastic coefficients. The MC is distributed linearly from pith to bark and elastic coefficients is linearly dependent on the MC. The thermal expansion coefficients for case 1 and 2 was used same values for earlywood and latewood as shown in table 3. To compare the thermal effect, different thermal expansion coefficients were also used for the earlywood and latewood for case 3 and 4 as shown in table 4.

Table 1. Description of Loading and Material Property Types

Case	Loading Method	Material property
1	thermal***	uniform*
2	thermal	nonuniform**
3	thermal	uniform
4	thermal	nonuniform

\* uniform: all the earlywood or latewood

layers have the same elastic coefficients as shown in table 2.

\*\* nonuniform: all the layers has different elastic coefficients. The outer layer of elastic coefficients have the same value as shown in table 2. Then linearly decrease to the inner layer. So the innerest layers have the half value of outer layers.

\*\*\* Case 1 and 2 for thermal load have the same thermal expansion coefficients for earlywood and latewood as shown in table 3. Case 3 and 4 for thermal load have different thermal expansion coefficients for earlywood and latewood as shown in table 4.

Table 2. Elastic Coefficients of Earlywood and Latewood for uniform Material Properties.

Variables	Earlywood	Latewood
$E_R$ ( $10^5$ psi) [1]	0.5	1.5
$E_T$ ( $10^5$ psi) [1]	0.3	1.0
$E_L$ ( $10^6$ psi) [1]	0.5	2.0
$V_{RT}$ [1]	0.55	0.50
$V_{RL}$ [1]	0.04	0.03
$V_{TL}$ [1]	0.03	0.02
$G_{RT}$ ( $10^4$ psi) [1]	0.15	0.6

Table 3. Thermal Expansion Coefficients of Earlywood and Latewood for Case 1 and Case 2.

Variables	Earlywood	Latewood
$\alpha_R$ ( $10^{-4}$ in./ $^{\circ}$ C) [6]	-0.11	-0.11
$\alpha_T$ ( $10^{-4}$ in./ $^{\circ}$ C) [6]	-0.177	-0.177
$\alpha_L$ ( $10^{-5}$ in./ $^{\circ}$ C) [6]	-0.8	-0.8

#### Numbering of Nodes and Elements

The following assumptions are made regarding the wood geometry referred to figure 1;

1. the tangential direction ( $\theta$ ), corresponds to the direction of tangent to the growth rings,

Table 4. Thermal Expansion Coefficients of Earlywood and Latewood for Case 3 and Case 4.

Variables	Earlywood	Latewood
$\alpha_R (10^{-4} \text{ in./}^\circ\text{C})$	-0.13	-0.09
$\alpha_T (10^{-4} \text{ in./}^\circ\text{C})$	-0.20	-0.14
$\alpha_L (10^{-4} \text{ in./}^\circ\text{C})$	-0.10	-0.06

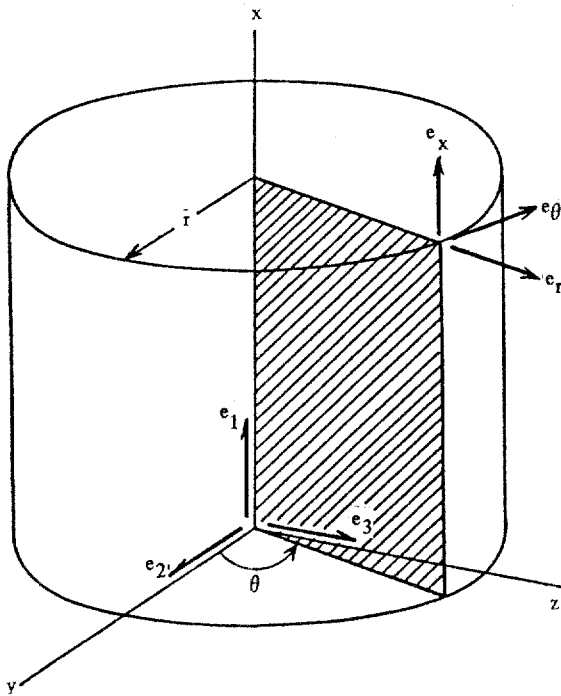


Figure 1. Cylindrical Coordinate

2. the radial direction ( $r$ ), corresponds to the direction of the rays,
3. and the longitudinal direction ( $x$ ), corresponding to the longitudinal axis.

The longitudinal effect is small, so the longitudinal effects are not considered. Therefore, this study will be a two dimensional problem. The equations of transformation from cylindrical to cartesian coordinate are;

$$x = 0 \quad y = r \cos \theta \quad z = r \sin \theta$$

The structural systems to be analyzed may be composed of combination of a number of

different structural elements. A first step in the application of this finite element method (FEM) is to divide the structure into elements. The element may be of different sizes and shapes in one version of the program. The nodal point numbering and corresponding coordinates was automatically done within as a subroutine of the program as shown in figure 2. The domain was used 1/4 of the log, because displacements of the specimen are axisymmetric along the radial and tangential plane. Since the stress distribution along the loading direction is required, the elements of this axis were well defined. After completing the finite element mesh, data were checked by the plotter.

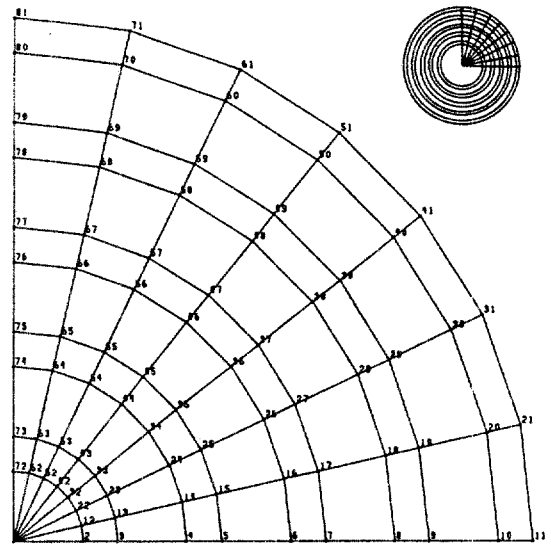


Figure 2. Generalized Mesh for 1/4 of cross section of cylindrical log.

### Loads and Boundary Conditions

The log is divided as a quarter of it because of axisymmetry. For this reason the boundary conditions for this problem are fixed in the bottom part of nodes (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11) in the  $z$  direction as shown in figure 2. The leftmost nodes (1, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81) assume fixity in the  $y$  direction. The node 1 was fixed in both directions of  $y$  and  $z$ . The boundary condition was also automatically assigned by the subroutine of program.

There was some assumption for temperature distribution for thermal loading. The temperature of outermost nodes (11, 21, 31, ....., and 81) was used 155°C and innermost node 1 for half of the outermost node temperature. And linearly distributed temperature for radial direction was used.

With the coordinates of all nodal points known and the number of degrees of freedom having been established, the stiffness matrix is applicable elastic element. The only quantities in this matrix are the dimension of the element and elastic coefficients. Therefore, it is applicable to any isotropic, orthotropic and generally anisotropic materials.

### RESULTS AND DISCUSSION

#### Displacement distribution

The maximum displacement occurred in the outermost surface of log as shown in figure 3. The maximum displacement is changed by the material properties and thermal expansion coefficients as shown in table 5. The maximum radial displacement was not strongly influenced by the thermal expansion coefficients as shown in figure 3. However, the radial displacement in the middle layer of log was influenced by the thermal expansion coefficients for different earlywood and latewood.

Table 5. The Maximum Displacements.

Material Property Case	Displacement ( $10^{-3}$ in.)
Case 1	-2.726
Case 2	-2.768
Case 3	-2.726
Case 4	-2.743

#### Stress distribution

For figure 4 it is noted that the maximum

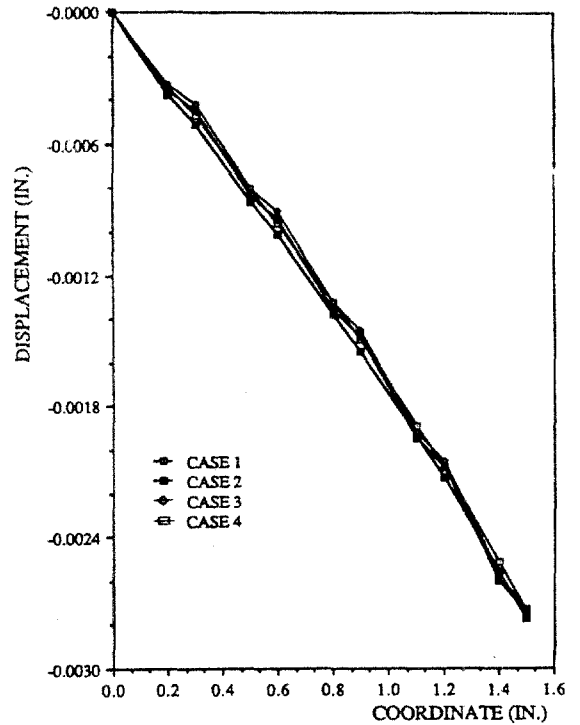


Figure 3. Radial Displacement.

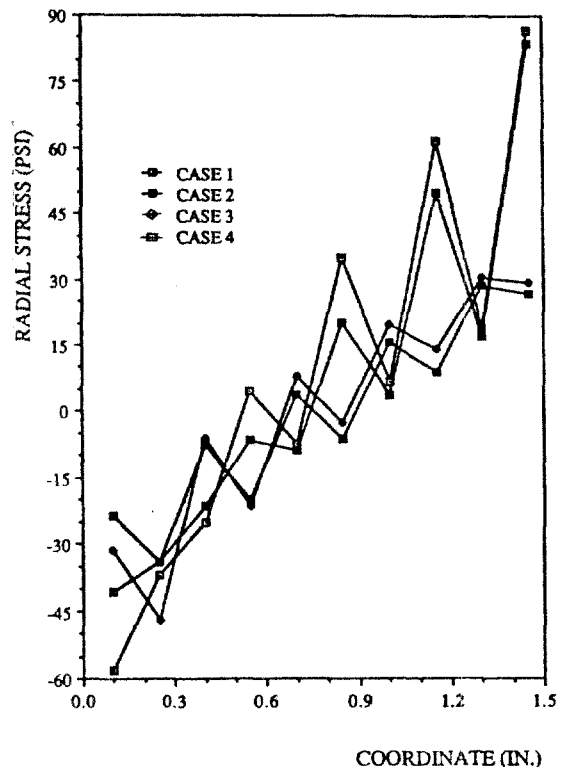


Figure 4. Radial Stress Distribution.

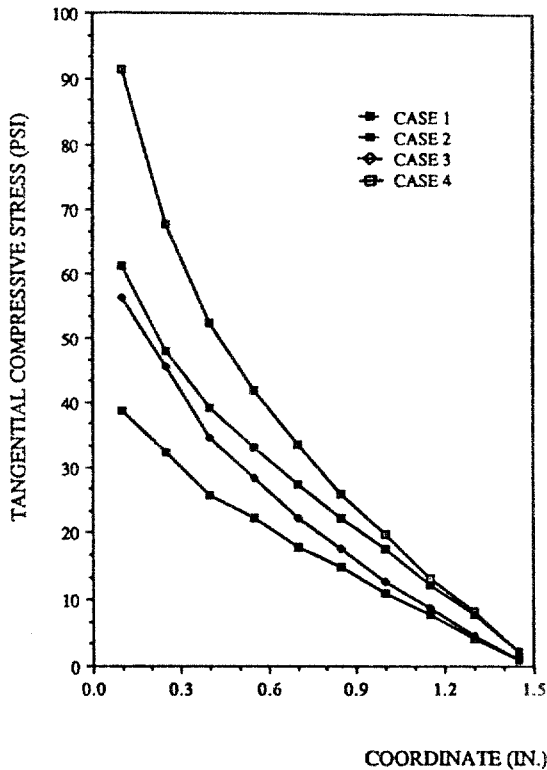


Figure 5. Tangential Stress Distribution.

radial compressive stress occurred in the innermost latewood region of the log for cases 3 and 4. The maximum tensile stress was also occurred at the outer latewood region of log. However, the maximum compressive stress was in the core region for cases 1 and 2. The maximum tensile stress also occurred in the outer layer for cases 1 and 2. The stresses of figure 4 show a mid-point of each element. This is a consequence of the fact that the axis of cylindrical orthotropy is a mathematical singular line.

The radial stress is strongly affected by the material properties depended on MC. The radial stress has a saw-tooth-like form for earlywood and latewood. The latewood had a more compressive stress than the earlywood. The radial stress of case 1 was a more compressive stress than that of case 2 at the inner layer. The outer layer also had more tensile stress for case 1 than that for case 2. The different value of thermal expansion for earlywood and latewood (cases 3 and 4) was less influenced the radial stress than that of cases 1

and 2. This shows that the radial stress is also strongly influenced by the material thermal expansion coefficients.

The maximum tangential stress occurred at the core layer. Tangential stress was also strongly affected by the material properties at inner layer. The tangential stress has smoother form for earlywood and latewood. The tangential stress of cases 3 and 4 is less than that of cases 1 and 2. This can be also explained that the tangential stress is influenced by thermal expansion coefficients.

There occurred the biggest difference in radial and tangential stress at the outer layer of log as shown in figures 3 and 4. This stress difference probably causes defect to separation of L-R plane.

## CONCLUSIONS AND SUGGESTIONS

This study has a following conclusions:

1. The radial displacement is strongly related to the material property depended on such as MC and temperature.
2. The radial stress is strongly dependent on the material properties.
3. The maximum compressive radial stress occurs at the innermost latewood layer of log. However, the maximum radial compressive stress occurs at the core layer of log of uniform thermal coefficients.
4. The stress functions have an saw-tooth-like forms for earlywood and latewood.
5. The maximum tangential stress occurs at the innermost earlywood layer of log.
6. The biggest difference between the radial and tangential stress occurs at the outer layer.

The suggestions of this study are:

1. There is a more study are required for more subdivision of log to see the interaction of earlywood and latewood.
2. More studies are required that material property changes of each layer for temperature and MC dependent.

3. There is also recommended to check the alternate radial stress function.

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