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Testing Whether a Specific Treatment is Better Than the Others

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ABSTRACT

Experimenters often want to test whether a specific treatment is really better than the others. In such a problem we derive the likelihood ratio test and compare the result with other multiple comparisons procedures. A nonparametric procedure based on ranks is also considered. Pitman efficiency of the rank-sum procedure relative to the likelihood ratio test is computed.

1. INTRODUCTION

In comparing k treatments, experimenters are often interested in identifying the best treatment. For such a purpose, several multiple comparisons procedures and selection procedures are available.

The so-called all-pairwise multiple comparisons (MCA) procedures can be used to get the over-all picture about the pairwise comparisons of treatments (see, for example, Miller (1981)). The parameters of interest in MCA procedures are $\theta_i-\theta_j$ for all $1\leq i<j\leq k$ where $\theta_1, \cdots, \theta_k$ denote the treatment effects.

When a specific treatment can be regarded as a control, the so-called multiple comparisons procedures with a control (MCC) can be used for the simultaneous comparisons of treatments with a control. The well-known MCC procedures are those by Dunnett (1955, 1964) and by Steel (1959) among others. The parameters of interest in MCC procedures are $\theta_i-\theta_1$ for $i=2, ..., k$ where θ_1 denotes the effect of the control and $\theta_2, \cdots, \theta_k$ are the treatment effects.

Recently Hsu (1984) showed that the MCA procedures can be improved when the interest is in the comparison with the UNKNOWN best treatment. He devised the so-called multiple comparisons procedure with the best (MCB), and the parameters of interest in this procedure are $\theta_i-\max_{j\neq i}\theta_j$ for $i=$

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$1, \cdots, k$.

This paper considers a situation when the experimenter wants to test whether a specific treatment is really better than others. In practice, experimenters often have some prior knowledge regarding the treatments to be compared, and they often have a candidate for the best treatment prior to the experiment.

In such a case, the problem can be formulated as a testing problem for the following hypotheses:

$$H_0 : \theta_1 \le \max_{2\le i\le k} \theta_i \text{ vs } H_1 : \theta_1 > \max_{2\le i\le k} \theta_i \tag{1.1}$$

where θ_1 denotes the effect of a specific treatment which is designated as a candidate for the best treatment prior to the experiment, and $\theta_2, \cdots, \theta_k$ are the effects of other treatments.

Section 2 derives the likelihood ratio test (LRT) for the hypotheses (1.1) under the normal model, and compares the result with other multiple comparisons procedures. The aspect of sample size determination is also considered to control the power.

Section 3 considers a nonparametric procedure based on ranks, and compares with other nonparametric multiple comparisons procedures. Pitman efficiency of the rank-sum procedure in Section 3 relative to the procedure in Section 2 is also computed.

Section 4 applies the results in previous sections to a real data set, and conclusions are drawn.

2. LIKELIHOOD RATIO TEST

Cosider the usual one-way model

$$X_{ij} = \theta_i + \varepsilon_{ij},\ j=1, \cdots, n_i; i=1, \cdots, k \tag{2.1}$$

where ε_{ij}'s are independent and identically distributed normal random variables with mean 0 and common unknown variance σ^2 Denoting treatment 1 as a specific treatment that is believed to be the best, we would like to test

$$H_0 : \theta_1 \le \max_{2\le i\le k} \theta_i \text{ vs } H_1 : \theta_1 > \max_{2\le i\le k} \theta_i \tag{2.2}$$

To derive the likelihood ratio test, let

$$L(\underset{\sim}{\theta}, \sigma^2; \underset{\sim}{x}) = (2\pi\sigma^2)^{-N/2} \exp\left[-\sum_{i=1}^{k}\sum_{j=1}^{n_i} (x_{ij} - \theta_i)^2/2\sigma^2\right] \tag{2.3}$$

denote the likelihood function where $N = \sum_{i=1}^{k} n_i$. Since

$$\bar{X}_i = \sum_{j=1}^{n_i} X_{ij}/n_i (i=1, \cdots, k) \text{ and } \hat{\sigma}^2 = \sum_{i=1}^{k}\sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2/N$$

are the maximum likelihood estimators of $\theta_i (i=1, \cdots, k)$ and σ^2, respectively, the maximum of the likelihood over the whole parameter space is given by

$$\max_{\Omega} L(\underset{\sim}{\theta}, \sigma^2; \underset{\sim}{x}) = (2\pi\hat{\sigma}^2)^{-N/2} e^{-N/2} \tag{2.4}$$

where

$$\Omega = \{(\theta_1, \cdots, \theta_k, \sigma^2) : -\infty < \theta_i < \infty,\ i=1, \cdots, k,\ \sigma^2 > 0\}.$$

To find the maximum likelihood over the null parameter space, let [2], [3], ..., [k] denote the unknown indices such that $\theta_{[2]} \leq \theta_{[3]} \leq \cdots \leq \theta_{[k]}$ are the ordered $\theta_2, \theta_3, \cdots, \theta_k$. Then the null parameter space w can be written as

$$w = \{(\theta_1, \cdots, \theta_k, \sigma^2) : \theta_1 \leq \theta_{[k]},\ \sigma^2 > 0\}.$$

It is obvious from (2.3) that we need to minimize

$$\sum_1^k n_i(\bar{x}_i - \theta_i)^2 = n_1(\bar{x}_1 - \theta_1)^2 + \sum_2^k n_{[i]}(\bar{x}_{[i]} - \theta_{[i]})^2 \tag{2.5}$$

subject to $\theta_1 \leq \theta_{[k]}$ in order to find the maximum likelihood over the null parameter space w. Then, it is not difficult to show that, for fixed [2], ..., [k], (2.5) is minimized at the following θ values:

(Case I) If $\bar{x}_{[k]} \geq \bar{x}_1$, then $\hat{\theta}_1 = \bar{x}_1$, $\hat{\theta}_{[k]} = \bar{x}_{[k]}$ and $\hat{\theta}_{[i]} = \bar{x}_{[i]}$ $(i=2, \cdots, k-1)$.

(Case II) If $\bar{x}_{[k]} < \bar{x}_1$, then $\hat{\theta}_1 = \hat{\theta}_{[k]} = (n_1\bar{x}_1 + n_{[k]}\bar{x}_{[k]})/(n_1 + n_{[k]})$ and

$\hat{\theta}_{[i]} = \bar{x}_{[i]}$ $(i=2, \cdots, k-1)$.

Therefore, for fixed [2], ..., [k], the minimum of (2.5) over w is given by

$$\frac{n_1 n_{[k]}}{n_1 + n_{[k]}} (\bar{x}_1 - \bar{x}_{[k]})^2\ I(\bar{x}_1 > \bar{x}_{[k]}).$$

Thus the maximum likelihood estimate ($m.l.e.$) of σ^2 under w is given by

$$\hat{\sigma}_w^2 = \hat{\sigma}^2 + \min_{2 \leq j \leq k} \{\frac{n_1 n_j}{n_1 + n_j} (\bar{x}_1 - \bar{x}_j)^2\ I(\bar{x}_1 > \bar{x}_j)\}/N, \tag{2.6}$$

and the maximum likelihood over w is given by

$$\max_{w} L(\underset{\sim}{\theta}, \sigma^2; \underset{\sim}{x}) = (2\pi\hat{\sigma}_w^2)^{-N/2} e^{-N/2}. \tag{2.7}$$

It follows from (2.4), (2.6) and (2.7) that the likelihood ratio test rejects H_0 when

$$\min_{2 \leq j \leq k} \frac{n_1 n_j}{n_1 + n_j} (\bar{x}_1 - \bar{x}_j)^2\ I(\bar{x}_1 > \bar{x}_j) > (d\hat{\sigma})^2$$

where a positive constant d is chosen to satisfy the given level. Thus rejection region of the likelihood ratio test is given as follows:

$$\bar{X}_1 > \max_{2\le i\le k}(\bar{X}_i + dS\sqrt{\frac{1}{n_1}+\frac{1}{n_i}}) \tag{2.8}$$

where $S^2 = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(X_{ij}-\bar{X}_i)^2/(N-k)$ is the pooled sample variance.

The rejection probability of the likelihood ratio test is given by

$$P_{\underset{\sim}{\theta},\sigma}\{\bar{X}_1 > \max_{2\le i\le k}(\bar{X}_i + dS\sqrt{\frac{1}{n_1}+\frac{1}{n_i}})\},$$

which is easily seen to be non-decreasing in θ_1 and non-increasing in $\theta_2, \cdots, \theta_k$. Therefore, the maximum type I error probability of the likelihood ratio test is attained when $\theta_1 = \theta_{[k]}$ and $\theta_{[2]} = \cdots = \theta_{[k-1]} = -\infty$. Thus the maximum type 1 error probability is given by

$$\max_{\theta_1\le\theta_{[k]}} P_{\underset{\sim}{\theta},\sigma}\{\bar{X}_1 > \max_{2\le i\le k}(\bar{X}_i + dS\sqrt{\frac{1}{n_1}+\frac{1}{n_i}})\}$$

$$= P_{\theta_1=\theta_{[k]},\sigma}(\bar{X}_1 > \bar{X}_{[k]} + dS\sqrt{\frac{1}{n_1}+\frac{1}{n_{[k]}}}).$$

Since $(\bar{X}_1 - \bar{X}_{[k]})/S\sqrt{\frac{1}{n_1}+\frac{1}{n_{[k]}}}$ has a t-distribution with $(N-k)$ degrees of freedom when $\theta_1 = \theta_{[k]}$, the likelihood ratio test of level α is given by the rejection region (2.8) with $d = t_\alpha(N-k)$, the upper α-quantile of the t-distribution with $(N-k)$ degrees of freedom.

Summarizing the discussions so far, we can state that the level α likelihood ratio test of H_0 vs H_1 in (2.2) rejects H_0 when

$$\bar{X}_1 > \max_{2\le i\le k}(\bar{X}_i + t_\alpha(N-k)S\sqrt{\frac{1}{n_1}+\frac{1}{n_i}}). \tag{2.9}$$

Next, we consider the aspect of determining the sample size n to control the power of the likelihood ratio test in the case of balanced one-way model with common sample size n. Since the power function of the likelihood ratio test in (2.9) is non-decreasing in θ_1 and non-increasing in $\theta_2, \cdots, \theta_k$, the following result is easily obtained: For

$$\Omega(\delta) = \{(\theta_1, \cdots, \theta_k, \sigma); \theta_1 - \max_{2\le i\le k}\theta_i \le \delta\sigma\},$$

$$\inf_{(\underset{\sim}{\theta},\sigma)\in\Omega(\delta)} P_{\underset{\sim}{\theta},\sigma}(\bar{X}_1 > \max_{2\le i\le k}\bar{X}_i + t_\alpha(\nu)\, S\sqrt{2/n})$$

$$= \int_0^\infty \int_{-\infty}^\infty \Phi^{k-1}(x + \sqrt{n}\delta - \sqrt{2}w\, t_\alpha(\nu))\, d\Phi(x)\, dQ_\nu(w) \tag{2.10}$$

where $Q_\nu(w)$ is the cdf of S/σ, and $\nu = k(n-1)$.

For selected values of k, n, δ and α, the minimum power in (2.10) has been computed. Figure 1 shows the power charts for $k = 6$, $\alpha = 0.05, 0.01$ and $\delta = 0.50, 0.75, 1.00, 1.25$. In these computations, Gauss-Hermite and Gauss-Laguerre quadratures were used to evaluate the inner integral and the outside integral, respectively.

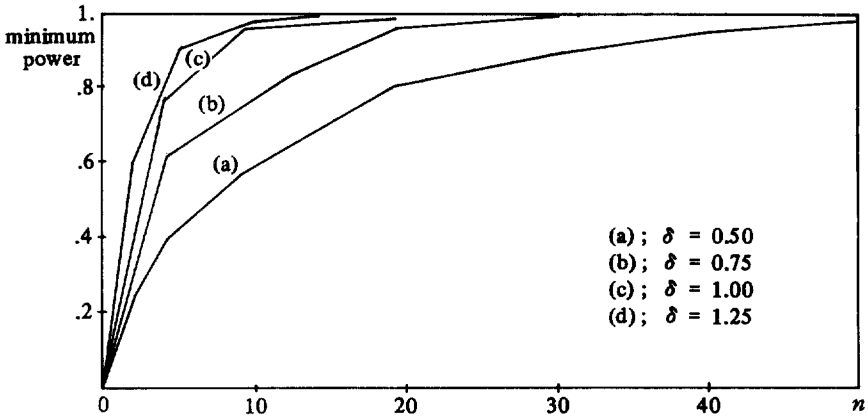

Figure 1-1. Power chart for $\alpha = 0.05$ and $k = 6$

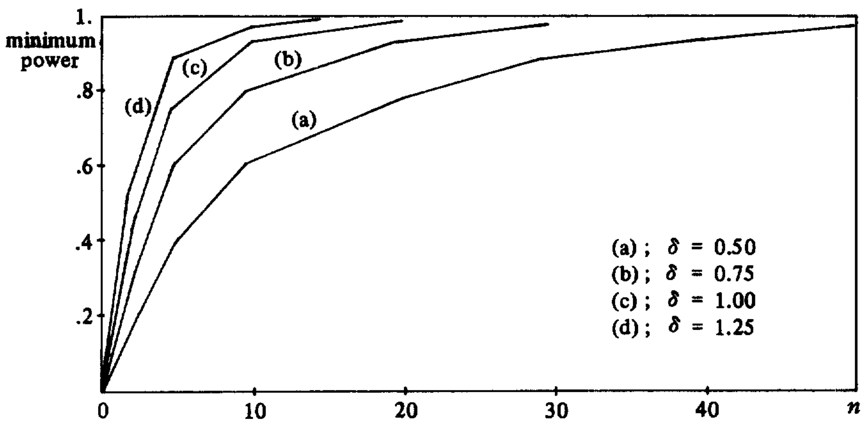

Figure 1-2. Power chart for $\alpha = 0.01$ and $k = 6$

Using the ordinary method to construct confidence set from a test, we can derive a useful lower confidence bound on $\theta_1 - \max_{2 \le i \le k} \theta_i$ from the likelihood ratio test. Since the likelihood ratio test in (2.9) is of level α, we have

$$P_{\theta_1-\max\theta_i=0}\{\bar{X}_1>\max_{2\le i\le k}(\bar{X}_i+t_\alpha S\sqrt{\frac{1}{n_1}+\frac{1}{n_i}})\}\le\alpha$$

where $t_\alpha=t_\alpha(N-k)$. Thus we have

$$1-\alpha\le P_{\theta_1-\max\theta_i=0}\{\bar{X}_1\le\max_{2\le i\le k}(\bar{X}_i+t_\alpha S\sqrt{\frac{1}{n_1}+\frac{1}{n_i}})\}$$

$$=P_{\theta_1-\delta-\max\theta_i=0}\{\bar{X}_1-\delta\le\max(\bar{X}_i+t_\alpha S\sqrt{\frac{1}{n_1}+\frac{1}{n_i}})\}$$

$$=P_{\theta_1-\max\theta_i=\delta}\{\delta\ge\bar{X}_1-\max(\bar{X}_i+t_\alpha S\sqrt{\frac{1}{n_1}+\frac{1}{n_i}})\}$$

$$=P_{\underline{\theta}}\{\theta_1-\max_{2\le i\le k}\theta_i\ge\bar{X}_1-\max_{2\le i\le k}(\bar{X}_i+t_\alpha S\sqrt{\frac{1}{n_1}+\frac{1}{n_i}})\}.$$

Therefore the 100 (1- α)% confidence lower bound on $\theta_1-\max_{2\le i\le k}\theta_i$ is given as follows:

$$\theta_1-\max_{2\le i\le k}\theta_i\ge\bar{X}_1-\max_{2\le i\le k}(\bar{X}_i+t_\alpha S\sqrt{\frac{1}{n_1}+\frac{1}{n_i}}) \qquad (2.11)$$

It should be noted that a confidence lower bound on $\theta_1-\max_{2\le i\le k}\theta_i$ can be deduced from Dunnett's (1955, 1964) MCC confidence lower bound on $\theta_1-\theta_i(i=2,\cdots,k)$, which in the case of balanced model is given by

$$\theta_1-\theta_i\ge\bar{X}_1-\bar{X}_i-d_D\sqrt{\frac{2}{n}}S \quad \text{for all } i=2,\cdots,k$$

where d_D is chosen to satisfy the confidence level $(1-\alpha)$. The deduced confidence lower bound on $\theta_1-\max_{2\le i\le k}\theta_i$ is given by

$$\theta_1-\max_{2\le i\le k}\theta_i\ge\bar{X}_1-\max_{2\le i\le k}\bar{X}_i-d_D\sqrt{\frac{2}{n}}S, \qquad (2.12)$$

which differs from (2.11) only in design constant d_D. In fact, $d_D=d_D(k,\nu,\alpha)$ increases as k becomes larger and $d_D(2,\nu,\alpha)=t_\alpha(\nu)$ with $\nu=k(n-1)$. Table 1 below gives the values of d_D (refer Gupta, Panchapakesan and Sohn (1985)) compared with the values of $t_\alpha=d_D(2,\nu,\alpha)$.

Table 1. Values of design constants $d_D = d_D(k, \nu, \alpha)$

$\alpha = 0.05$

ν \ k	2	3	4	5	6	7	8	9	10
15	1.75	2.07	2.24	2.36	2.44	2.52	2.57	2.62	2.67
∞	1.64	1.92	2.06	2.16	2.23	2.29	2.34	2.38	2.42

$\alpha = 0.01$

ν \ k	2	3	4	5	6	7	8	9	10
15	2.60	2.91	3.08	3.20	3.29	3.36	3.42	3.47	3.51
∞	2.33	2.56	2.68	2.77	2.84	2.89	2.93	2.97	3.00

As it can be seen from Table 1, as k becomes larger, the confidence lower bound in (2.11) becomes more favorable to the confidence lower bound in (2.12) deduced from Dunnett's simultaneous lower confidence bounds. This is not surprising at all because the confidence bound in (2.11) is designed only for the comparison of θ_1 with $\max_{2\le i\le k} \theta_i$ while Dunnett's confidence bound is designed to get the information about the comparison of θ_1 with others. It shows, however, that the confidence bound in (2.11) or the likelihood ratio test in (2.9) is more useful when the interest is in getting an evidence for a specific treatment 1 to be the best.

Comparisons of the confidence bound in (2.11) with other multiple comparisons procedures become more favorable to the bound in (2.11) since Dunnett's MCC interval uses the smallest design constants among all multiple comparisons procedures when there is a control.

3. NONPARAMETRIC PROCEDURE

Consider the usual nonparametric balanced one-way model

$$X_{ij} = \theta_i + \varepsilon_{ij},\ j=1, \cdots, n; i=1, \cdots, k \tag{3.1}$$

where ε_{ij}'s are independent and identically distributed with pdf $f(\cdot)$. Let $R_{1i}(i=2,\cdots,k)$ denote the sum of the ranks of $X_{11}, \cdots, X_{1n}$ in $\{X_{11}, \cdots, X_{1n}; X_{i1}, \cdots, X_{in}\}$ $(i=2, \cdots, k)$.

For testing the hypotheses in (2.2), we consider a rank-sum test with rejection region

$$\min_{2\le i\le k} R_{1i} \ge c \tag{3.2}$$

where c is to be chosen to satisfy the level.

Since $(R_{12}, \ldots, R_{1k})$ is stochastically non-decreasing in θ_1 and non-increasing in $\theta_2, \ldots, \theta_k$, the type I error probability of the rejection region (3.2) is maximized when $\theta_1 = \theta_{[k]}$ and $\theta_{[2]} = \cdots = \theta_{[k-1]} = -\infty$. Therefore the maximum of the type I error proability is given by

$$\max_{\theta_1 \le \theta_{[k]}} P_{\theta}(\min_{2\le i\le k} R_{1i} \ge c) = P_{\theta_1 = \theta_{[k]}}(R_{1[k]} \ge c).$$

Furthermore, when $\theta_1 = \theta_{[k]}$, the distribution of $R_{1[k]}$ is the null distribution of the two-sample Wilcoxon's rank-sum test. Therefore, by taking the critical value of two-sample Wilcoxon's test as c, the test with rejection region (3.2) is a level α test for the hypotheses in (2.2).

To compute the asymptotic power of the test in (3.2) for large n, we consider an alternative

$$H_1(\triangle): \theta_1 = \theta_2 + \triangle, \ \cdots, \ \theta_1 = \theta_k + \triangle \tag{3.3}$$

for a fixed $\triangle > 0$. First, we note that under the alternative $H_1(\triangle)$ in (3.3) the moments of $R_{1i} (i = 2, \ldots, k)$ are given as follows:

$$\begin{cases} E(R_{1i}) = n^2 p_1 \\ \mathrm{Var}(R_{1i}) = n^2 p_1(1-p_1) + 2n^2(n-1)(p_2 - p_1^2) \\ \mathrm{Cov}(R_{1i}, R_{1j}) = n^3(p_2 - p_1^2) \quad (i \ne j) \end{cases} \tag{3.4}$$

where

$$\begin{aligned} p_1 &= p_1(\triangle) = \int_{-\infty}^{\infty} F(x+\triangle) f(x)\, dx, \\ p_2 &= p_2(\triangle) = \int_{-\infty}^{\infty} F^2(x+\triangle) f(x)\, dx \end{aligned} \tag{3.5}$$

and F is the cdf corresponding to f.

It follows from (3.4) and the general asymptotic theory regarding the rank statistics that for large n

$$\{R_{1i} - E(R_{1i})\} / \sqrt{\mathrm{Var}(R_{1i})} \quad (i = 2, \cdots, k)$$

has asymptotic multivariate normal distribution with means 0, variances 1 and equal correlation 1/2. Then the standard analysis yields the following asymptotic power of the test in (3.2): Under $H_1(\triangle)$ in (3.3)

$$P_{\theta}(\min_{2\le i\le k} R_{1i} \ge c) \approx \int_{-\infty}^{\infty} \Phi^{k-1}(x + \sqrt{12n}\,\triangle p_1'(0) - \sqrt{2}\, Z_\alpha)\, d\Phi(x) \tag{3.6}$$

Note that by (3.5) $p_1'(0)$ is given by

$$p_1'(0)=\int_{-\infty}^{\infty} f^2(x)\ dx. \tag{3.7}$$

For the likelihood ratio test in Section 2, it can be easily observed that the asymptotic power for large n is given by

$$\begin{aligned} &P_{\theta}(\bar{X}_1 > \max_{2\le i\le k} \bar{X}_i + t_\alpha(\nu) S\sqrt{2/n}\ ) \\ &\approx \int_{-\infty}^{\infty} \Phi^{k-1}(x+\sqrt{n}\,\Delta/\sigma - \sqrt{2}\,Z_\alpha)\ d\Phi(x) \end{aligned} \tag{3.8}$$

under $H_1(\Delta)$ in (3.3). Therefore it follows from (3.6), (3.7) and (3.8) that the Pitman efficiency of the rank-sum test in (3.2) relative to the likelihood ratio test in (2.9) is that of the Wilcoxon's test relative to t-test, i.e.

$$e_{w,t}(F)=12\sigma^2[\int f^2(x)dx]^2.$$

Next, a confidence lower bound on $\theta_1-\max_{2\le i\le k}\theta_i$ is to be derived from the rank-sum test in (3.2). First note that

$$\begin{aligned} 1-\alpha &\le P_{\theta_1=\theta_{[k]}}(\min_{2\le i\le k} R_{1i}<c) \\ &= P_{\theta_1-\sigma=\theta_{[k]}}(\min_{2\le i\le k} R_{1i}(\delta)<c) \end{aligned} \tag{3.9}$$

where $R_{1i}(\delta)$ is the rank sum of $X_{11}-\delta,\cdots,X_{1n}-\delta$ in $\{X_{11}-\delta,\cdots,X_{1n}-\delta;X_{i1},\cdots,X_{in}\}$ $(i=2,\cdots,k)$.

Furthermore it can be easily observed that

$$\begin{aligned} R_{1i}(\delta) &= \sum_{l=1}^{n}\{\sum_{j=1}^{n} I(X_{ij}\le X_{1l}-\delta)+\sum_{k=1}^{n} I(X_{1k}-\delta\le X_{1l}-\delta)\} \\ &= \frac{n(n+1)}{2}+\sum_{l=1}^{n}\sum_{j=1}^{n} I(X_{ij}\le X_{1l}-\delta). \end{aligned}$$

Therefore, $R_{1i}(\delta)<c$ if and only if

$$\delta \ge (X_1-X_i)_{(n'-c+\frac{n(n+1)}{2})} \tag{3.10}$$

where $(X_1-X_i)_{(1)} \le \cdots \le (X_1-X_i)_{(n^2)}$ denote the ordered $X_{1j}-X_{il}(j, l=1, \cdots, n)$.
Thus it follows from (3.9) and (3.10) that $100(1-\alpha)\%$ confidence lower bound on $\theta_1 - \max_{2\le i\le k} \theta_i$ is given by

$$\theta_1 - \max_{2\le i\le k} \theta_i \ge \min_{2\le i\le k} (X_1 - X_i)_{(n^2-c+\frac{n(n+1)}{2})} \tag{3.11}$$

As in Section 2, a lower confidence bound on $\theta_1 - \max_{2\le i\le k} \theta_i$ can be deduced from Steel's (1959) MCC confidence lower bound on $\theta_1 - \theta_i$ ($i=2, \cdots, k$) based on ranks, which is given by

$$\theta_1 - \theta_i \ge (X_1 - X_i)_{(n^2-r+\frac{n(n+1)}{2})} \quad (i=2, \cdots, k)$$

where r is chosen to satisfy the confidence level $(1-\alpha)$. The deduced confidence lower bound on $\theta_1 - \max_{2\le i\le k} \theta_i$ is therefore given by

$$\theta_1 - \max_{2\le i\le k} \theta_i \ge \min_{2\le i\le k} (X_1 - X_i)_{(n^2-r+\frac{n(n+1)}{2})}. \tag{3.12}$$

The design constant $r = r(k, n, \alpha)$ decreases as k becomes larger and in fact $r(2, n, \alpha) = c = c(n, \alpha)$. Table 2 below shows the difference between r and c (see, Miller (1981)).

Table 2. Values of design constants $r = r(k, n, \alpha)$

α = 0.05

n \ k	2	3	4	5	6	7	8	9	10
10	128	131	133	135	136	136	137	138	138
30	1026	1026	1056	1062	1067	1071	1075	1077	1080

α = 0.01

n \ k	2	3	4	5	6	7	8	9	10
10	136	140	142	143	144	144	145	145	146
30	1072	1089	1098	1104	1108	1112	1115	1117	1119

4. AN EXAMPLE WITH REAL DATA

The data in Table 3 is given in Watson et al. (1949), and cited in Hsu (1982) to demonstrate how to choose the treatment with the highest coefficient of digestibility on the average.

Table 3. Coefficients of Digestibility for Five Rations for Total Carbohydrates

Treatment = Ration	Block = Steer Group					
	1	2	3	4	5	6
1	86.5	74.5	68.8	79.9	78.2	86.8
2	78.2	76.9	67.8	74.2	72.5	76.5
3	74.7	72.3	72.7	76.3	75.8	76.1
4	72.9	76.9	64.7	73.2	73.2	73.2
5	70.8	73.5	67.2	74.5	71.5	70.4

The data is related to the comparison of the coefficients of digestibility of total carbohydrates of five rations for cattles, which consist of 3 kilograms of hay per animal per day with increasing amounts of linseed oil meal, approximately 1, 2, 3, 4, or 5 kilograms per animal per day. These treatments were assigned random to the animals in each block. The measured coefficients of digestibility, in present, for total carbohydrates are the data in Table 3.

Hsu (1982) applied MCB procedure to this data to detect the best treatment. It seems, however, that the ration with smaller linseed oil meal is likely to mark larger coefficient of digestibility. Thus, we can apply the result in Section 2 to this data set with treatment 1 as a specific candidate for the best.

Assuming the usual balanced two-way no interaction model with normal errors, it can be easily observed that the result in Section 2 can be applied to this data set with obvious modifications. The summary statistics are given as follows:

$$\bar{x}_{1.}=79.1,\ \bar{x}_{2.}=74.4,\ \bar{x}_{3.}=74.7,\ \bar{x}_{4.}=72.4,\ \bar{x}_{5.}=71.3\ ,\ s^2=9.91,\ \nu=20.$$

Note that in this case s^2 is the usual pooled estimate of the error variance with $\nu=(k-1)(n-1)$ degrees of freedom. Therefore, the observed value of the test statistic in (2.9) is given by

$$t=\frac{\bar{x}_{1.}-\max_{2\le j\le 5}\bar{x}_{j.}}{s\sqrt{2/n}}=2.42.$$

Therefore the p-value of this data in testing the hypotheses in (2.2) is given by $\hat{\alpha}=0.01$. It should be noted that the MCB procedure by Hsu (1984) can not detect the treatment 1 as the best treatment at such a level. That is due to the fact that, in MCB procedure, the treatment 1 is designated as the best after observing the data.

REFERENCES

1. Dunnett, C.W. (1955), *"A multiple comparisons procedure for comparing several treatments with a control"*, Journal of the American Statistical Association, 50, 1096-1121.
2. Dunnett, C.W. (1964), *"New tables for multiple comparisons with a control"*, Biometrics, 20, 482-491.
3. Gupta, S.S., Panchapakesan, S., Sohn, J.K. (1985), *"On the distribution of the studentized maximum of equally correlated normal random variables,"* Communications in Statistics, 14, 103-135.
4. Hsu, J.C. (1982), *"Simultaneous inference with respect to the best treatment in block designs,"* Journal of the American Statistical Association, 77, 461-467.
5. Hsu, J.C. (1984), *"Constrained simultaneous confidence intervals for multiple comparisons with the best,"* The Annals of Statistics, 12, 1136-1144.
6. Miller, R.G., Jr. (1981), *"Simultaneous Statistical Inference,"* Springer-Verlag New York Inc.
7. Steel, R.G.D. (1959), *"A multiple comparison rank sum test: treatments versus control,"* Biometrics, 15, 560-572.
8. Watson, G.J., Kennedy, J.W., Davidson, W.M., Robinson, C.H., and Muir, G.W. (1949), *"Digestibility studies with ruminants, XIII, the effect of the plane of nutrition on the digestibility of linseed oil meal,"* Scientific Agriculture, 29, 263-272.