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Estimation of $P_r(X > Y)$ in the case of Exponential X and Normal Y

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ABSTRACT

In life testing problem, many authors obtained the minimum variance unbiased estimator of $P_r[X>Y]$ for the exponential family generally and conceptually. In this paper, we study the maximum likelihood estimator and minimum variance unbiased estimator of $P_r[X>Y]$ in exponential X and normal Y.

1. Introduction.

Let X and Y be independent random variables with cumulative distribution functions $F(x)$ and $G(y)$, respectively. We assume that the distributions of X and Y are known.

To estimate $\theta = P_r[X > Y]$ has been studied by many authors. This problem is important in Life Testing and some physical situation. Suppose that X is the strength of a component which is subjected to a stress Y. Then, the component fails when $X < Y$ and there is no failure when $X > Y$.

Mazumdar and Downtown (1970) obtained minimum variance unbiased estimator (M.V.U.E.) and maximum likelihood estimator (M.L.E.) of θ for a normal model. Enis and Geisser (1971) considered Bayesian estimators of θ for normal and exponential models. Recently, Tong (1977) and Beg (1980) obtained the M.V.U.E. of θ for the exponential family.

However Tong and Beg studied this problem generally and conceptually for the exponential family. In this paper, we study the M.L.E. and the M.V.U.E. of θ in the exponential X and normal Y with the following p.d.f.'s, respectively,

$$f(x) = \alpha \exp(-\alpha x), \quad x > 0, \quad \alpha > 0, \tag{1.1}$$

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and

$$g(y)=\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(y-\mu)^2\right),$$

$$-\infty<y<\infty, \quad -\infty<\mu<\infty, \quad \sigma^2>0, \tag{1.2}$$

Where α, μ and σ^2 are unknown.

2. Evaluation of θ and Maximum Likelihood Estimator of θ .

2.1. Evaluation of $\theta=P_r[X>Y]$

Let X and Y be independent random variables. Then the θ is reexpressed as follows:

$$\begin{aligned}\theta &= P_r[X>Y] \\ &= \int_0^\infty \int_0^x dG(y)\,dF(x) \\ &= 1-\int_0^\infty F(x)\,dG(x) \\ &= \Phi\left(-\frac{\mu}{\sigma}\right)+\left[1-\Phi\left(\frac{\alpha\sigma^2-\mu}{\sigma}\right) \exp\left(\frac{\alpha^2\sigma^2}{2}-\alpha\mu\right)\right]\end{aligned}$$

Where Φ is the c.d.f. of the standard normal distribution.

2.2. The Maximum Likelihood Estimator of θ

Let $X_1, X_2, \cdots\cdots, X_n$ denote the strengths of the n items and $Y_1, Y_2 \cdots\cdots, Y_m$ denote the stresses. Let $X_1, X_2, \cdots\cdots, X_n$ be independently and identically distributed (i.i.d.) random variables from the p.d.f. (1.1) and $Y_1, Y_2, \cdots\cdots, Y_m$ be i.i.d. random variables from the p.d.f. (1.2) and let $(X_1, X_2, \cdots\cdots, X_n)$ and $(Y_1, Y_2, \cdots\cdots, Y_m)$ be independent. Then the join+ p.d.f.'s of $X_1, X_2, \cdots\cdots, X_n$ and $Y_1, Y_2, \cdots\cdots, Y_m$ are

$$L(X_1, X_2, \cdots x_n \mid \alpha)=\alpha^n \exp\left(-\alpha\sum_{i=1}^{n} x_i\right) \tag{2.1}$$

and

$$L(y_1, y_2\cdots\cdots y_m \mid \mu, \sigma)=\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^m \exp\left(-\frac{1}{2\sigma^2}\sum_{j=1}^{m}(y_j-\mu)^2\right) \tag{2.2}$$

respectively.

From (2.1) and (2.2), we can easily find that the M.L.E.'s of α, μ and σ^2 are

$$\hat{\alpha}=\frac{n}{\sum_{i=1}^{m} X_i}$$

$$\hat{\mu}=\frac{1}{m}\sum_{j=1}^{m} Y_j$$

and

$$\hat{\sigma}^2=\frac{1}{m}\sum_{j=1}^{m}(Y_j-\overline{Y})^2$$

respectively.

Hence, by invariance property of M.L.E., the M.L.E. of θ is given by

$$\hat{\theta}=\Phi(-\frac{\hat{\mu}}{\hat{\sigma}})+[1-\Phi(\frac{\hat{\alpha}\hat{\sigma}^2-\hat{\mu}}{\sigma})]\ \exp(\frac{\hat{\alpha}^2\hat{\sigma}^2}{2}-\hat{\alpha}\hat{\mu})$$

3. The Minimum Variance Unbiased Estimator of θ

Let us denote $\hat{\alpha}^*=\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$, $\overline{Y}=\sum_{j=1}^{m}Y_j$ and $S^2=\sum_{j=1}^{m}(Y_j-\overline{Y})^2$

Then the $\hat{\alpha}^*$ and $(\overline{Y}, S^2)$ are known to be complete sufficient statistics for α and (μ, σ^2), respectively. [see Lehmann]

Let us define $I(X_1, Y_1)=1$, for $X_1>Y_1$

$=0$, otherwise.

Then we know that $I(X_1, Y_1)$ is clearly unbiased for θ . Since X and Y are independent, we have that $\hat{\alpha}^*$ and $(\overline{Y}, S^2)$ are independent. According to the Blackwell-Rao and Lehmann-Scheffe theorems, the unique M.V.U.E. of θ is given by

$$\begin{aligned}\theta&=E[I(X_1, Y_1)\mid\hat{\alpha}^*, \bar{y}, s^2]\\&=\int\int_R I(x_1, y_1)\,h(x_1, y_1\mid\hat{\alpha}^*, \bar{y}, s^2)\,dx_1\,dy_1\\&=\int\int_R I(x_1, y_1)\,f(x_1\mid\hat{\alpha}^*)\,g(y_1\mid\bar{y}, s^2)\,dx_1\,dy_1\end{aligned}\tag{3.1}$$

where R is the sample space.

Let $U_1=\sum_{i=2}^{n}X_i$. Then the p.d.f. of U_1 is given by

$$f_{U_1}(u_1\mid\alpha)=\frac{1}{\Gamma(n-1)}\alpha^{n-1}u_1^{n-2}\exp(-\alpha u_1),\quad u_1>0,\quad \alpha>0.$$

Since U_1 and X_1 are independent, the joint p.d.f. of U_1 and X_1 is given by

$$f_{X_1, U_1}(x_1, u_1\mid\alpha)=\frac{1}{\Gamma(n-1)}\alpha^n u_1^{n-2}\exp(-\alpha(u_1+x_1)),\quad x_1>0,\quad u_1>0,\quad \alpha>0.$$

Let $U=U_1+X_1$. By simple transformation, the joint p.d.f. of U and X_1 is given by

$$f_{X_1, U}(x_1, u\mid\alpha)=\frac{1}{\Gamma(n-1)}\alpha^n(u-x_1)^{n-2}\exp(-\alpha u),\qquad 0<x_1\leqq u,\quad \alpha>0.\tag{3.2}$$

Hence the joint p.d.f. of X_1 and $\hat{\alpha}^*$ is given by

$$f_{x_1,\hat{\alpha}^*}(x_1,\ \hat{\alpha}^* \mid \alpha) = \frac{1}{\Gamma(n-1)}\, n\alpha^n (n\hat{\alpha}^* - x_1)^{n-2}\ \exp(-n\alpha\hat{\alpha}^*), \tag{3.3}$$

$$0 < x_1 \leqq n\hat{\alpha}^*,\quad \alpha > 0.$$

Also the p.d.f. of $\hat{\alpha}^*$ is given by

$$f_{\hat{\alpha}^*}(\hat{\alpha}^* \mid \alpha) = \frac{1}{\Gamma(n)} (n\alpha)^n (\hat{\alpha}^*)^{n-1}\ \exp(-n\alpha\hat{\alpha}^*),$$

$$0 < \hat{\alpha}^*,\quad \alpha > 0. \tag{3.4}$$

Therefore we have, by (3.3) and (3.4),

$$f(x_1\ \hat{\alpha}^*) = \frac{f_{x_1,\hat{\alpha}^*}(x_1,\ \hat{\alpha}^* \mid \alpha)}{f_{\hat{\alpha}^*}(\hat{\alpha}^* \mid \alpha)}$$

$$= \frac{n-1}{n\hat{\alpha}^*}\left(1 - \frac{x_1}{n\hat{\alpha}^*}\right)^{n-2},\quad 0 < x_1 < n\hat{\alpha}^*. \tag{3.5}$$

Also we have

$$g(y_1 \mid \bar{y},\ s^2) = \frac{1}{s} h\left(\frac{y_1 - \bar{y}}{s}\right),$$

where $s = \sqrt{s^2}$

and

$$h(z) = \frac{\Gamma\left(\frac{m-1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{m-2}{2}\right)} \sqrt{\frac{m}{m-1}} \left(1 - \frac{m}{m-1} z^2\right)^{\frac{m}{2}-2} \quad \text{If} \quad 0 < |z| < \sqrt{\frac{m-1}{m}}$$

$$= 0 \qquad \text{, otherwise.}$$

[see Lehmann]
Therefore

$$g(y_1 \mid \bar{y},\ s) = \frac{\Gamma\left(\frac{m-1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{m-2}{2}\right)} \sqrt{\frac{m}{m-1}}\ \frac{1}{s}\left(1 - \frac{m}{m-1} \cdot \frac{(y_1-\bar{y})^2}{s^2}\right)^{\frac{m}{2}-2}$$

$$,\ \text{If } 0 < \left|\frac{y_1 - \bar{y}}{s}\right| < \sqrt{\frac{m-1}{m}}$$

$$= 0,\quad \text{otherwise.} \tag{3.6}$$

By (3.1), (3.5) and (3.6), we obtain the following results: the M.V.U.E. of $\tilde{\theta}$ is

$$\tilde{\theta}=\iint_R I(x_1,\ y_1)\left(1-\frac{x_1}{n\hat{\alpha}^*}\right)^{n-2}\left(1-\frac{m}{m-1}\frac{(y_1-\bar{y})^2}{s^2}\right)^{\frac{m}{2}-2}\ dx_1\,dy_1$$

$$\cdot\frac{n-1}{n\hat{\alpha}^*}\cdot\frac{1}{s}\cdot\frac{m}{m-1}\cdot\frac{\Gamma(\frac{m-1}{2})}{\Gamma(\frac{1}{2})\Gamma(\frac{m-1}{2})} \qquad (3.7)$$

Then we have the followings:

(1) for the case of $\bar{y}-\sqrt{\frac{m-1}{m}}\,s>0$ and $\bar{y}+\sqrt{\frac{m-1}{m}}\,s\leqq n\bar{x}$

(:) when n is odd.

$$\tilde{\theta}=\sum_{K=0}^{(\frac{n-1}{2})}\binom{n-1}{2K}\left(1-\frac{\bar{y}}{n\hat{\alpha}^*}\right)^{2K}\left(\frac{\sqrt{\frac{m-1}{m}}\,s}{n\hat{\alpha}^*}\right)^{n-2K-1}\cdot\frac{\Gamma(\frac{n}{2}-K)\,\Gamma(\frac{m-1}{2})}{\Gamma(\frac{n+m}{2}-K-1)\,\Gamma(\frac{1}{2})}$$

(: :) when n is even

$$\tilde{\theta}=\sum_{K=0}^{n/2}\binom{n-1}{2K}\left(1-\frac{\bar{y}}{n\hat{\alpha}^*}\right)^{2K-1}\left(\frac{\sqrt{\frac{m-1}{m}}\,s}{n\hat{\alpha}^*}\right)^{n-2K}\cdot\frac{\Gamma(\frac{n+1}{2}-K)\,\Gamma(\frac{m-1}{2})}{\Gamma(\frac{n+m-1}{2}-K)\,\Gamma(\frac{1}{2})}$$

(2) For the cases of

$$\bar{y}-\sqrt{\frac{m-1}{m}}\,s<0,\quad 0<\bar{y}+\sqrt{\frac{m-1}{m}}\,s\leqq n\bar{x}\quad\text{and}\quad \bar{y}>0.$$

$$\tilde{\theta}=\frac{1}{2}\Big[\sum_{K=0}^{n-1}\binom{n-1}{K}\left(1-\frac{\bar{y}}{n\hat{\alpha}^*}\right)^{K}\left(\frac{\sqrt{\frac{m-1}{m}}\,s}{n\hat{\alpha}^*}\right)^{n-K-1}$$

$$\cdot\frac{\Gamma(\frac{1}{2}(n-K))\,\Gamma(\frac{1}{2}(m-1))}{\Gamma(\frac{1}{2}(n+m-K)-1)\,\Gamma(\frac{1}{2})}$$

$$\cdot\Big\{(-1)^{n-K-1}+B\Big(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1}\cdot\frac{y^2}{s^2}\Big)\Big\}$$

$$+1-B\Big(\frac{1}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1}\cdot\frac{y^2}{s^2}\Big)\Big].$$

(3) For the case of $\bar{y}-\sqrt{\frac{m-1}{m}}\,s<0,\ 0<\bar{y}+\sqrt{\frac{m-1}{s}}\,s\leqq n\bar{x}$ and $\bar{y}<0.$

$$\tilde{\theta}=\tfrac{1}{2}\Big[\sum_{K=0}^{n-1}\binom{n-1}{K}\left(1-\frac{\bar{y}}{n\hat{\alpha}^*}\right)^{K}\left(\frac{\sqrt{\frac{m-1}{m}}\,s}{n\hat{\alpha}^*}\right)^{n-K-1}\frac{\Gamma(\frac{n-K}{2})\,\Gamma(\frac{m-1}{2})}{\Gamma(\frac{n+m-K}{2}-1)\,\Gamma(\frac{1}{2})}$$

$$\cdot \{1-B(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1}\cdot\frac{\bar{y}^2}{s^2}\}$$

$$+1+B(\frac{1}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1}\cdot\frac{\bar{y}^2}{s^2})\}$$

(4) For the case of $\bar{y}+\sqrt{\frac{m-1}{m}}s<0.$

$$\tilde{\theta}=1.$$

(5) For the case of $0<n\bar{x}\leqq\bar{y}-\sqrt{\frac{m-1}{m}}s.$

$$\tilde{\theta}=0.$$

(6) For the case of $0<\bar{y}-\sqrt{\frac{m-1}{m}}s$ and $\bar{y}<n\bar{x}<\bar{y}+\sqrt{\frac{m-1}{m}}s.$

$$\tilde{\theta}=\frac{1}{2}[\sum_{K=0}^{n-1}\binom{n-1}{K}(1-\frac{\bar{y}}{n\tilde{\alpha}^*})^K\left(\frac{\sqrt{\frac{m-1}{m}}s}{n\tilde{\alpha}^*}\right)^{n-K-1}$$

$$\cdot\frac{\Gamma(\frac{n-K}{2})\Gamma(\frac{m-1}{2})}{\Gamma(\frac{n+m-K}{2}-1)\Gamma(\frac{1}{2})}$$

$$\cdot\{(-1)^{n-K-1}\cdot B(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1}\cdot\frac{(n\bar{x}-\bar{y})^2}{s^2})+1\}]$$

(7) For the case of and $0<\bar{y}-\sqrt{\frac{m-1}{m}}s$ and $0<\bar{y}<n\bar{x}<\bar{y}+\sqrt{\frac{m-1}{m}}s.$

$$\tilde{\theta}=\frac{1}{2}[\sum_{K=0}^{n-1}\binom{n-1}{K}(1-\frac{\bar{y}}{n\hat{\alpha}^*})^K\left(\frac{\sqrt{\frac{m-1}{m}}s}{n\hat{\alpha}^*}\right)^{n-K-1}$$

$$\cdot\frac{\Gamma(\frac{n-K}{2})\Gamma(\frac{m}{2}-1)}{\Gamma(\frac{n+m-K}{2}-1)\Gamma(\frac{1}{2})}\cdot\{1-B(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1}\cdot\frac{(n\bar{x}-\bar{y})^2}{s^2})\}]$$

(8) For the case of $\bar{y}-\sqrt{\frac{m-1}{m}}s>0$ and $0<\bar{y}<n\bar{x}<\bar{y}+\sqrt{\frac{m-1}{m}}s.$

$$\tilde{\theta}=\frac{1}{2}[\sum_{K=0}^{n-1}\binom{n-1}{K}(1-\frac{\bar{y}}{n\hat{\alpha}^*})^K\left(\frac{\sqrt{\frac{m-1}{m}}s}{n\hat{\alpha}^*}\right)^{n-K-1}\frac{\Gamma(\frac{1}{2}(n-K))\Gamma(\frac{m-1}{2})}{\Gamma(\frac{1}{2}(n-m-K)-1)\Gamma(\frac{1}{2})}$$

$$\cdot \{(-1)^{n-K-1} \cdot B(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1} \cdot \frac{(n\bar{x}-\bar{y})^2}{s^2})$$

$$+B(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1} \cdot \frac{\bar{y}^2}{s^2})\}$$

$$+1-B(\frac{1}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1} \cdot \frac{\bar{y}^2}{s^2})]$$

(9) For the case of $\bar{y}-\sqrt{\frac{m-1}{m}}s<0,\quad \bar{y}<n\bar{x}<\bar{y}+\sqrt{\frac{m-1}{s}}s$ and $\bar{y}<0.$

$$\tilde{\theta}=\tfrac{1}{2}[\sum_{K=0}^{n-1}\binom{n-1}{K}(1-\frac{\bar{y}}{n\hat{\alpha}^*})^K\left(\frac{\sqrt{\frac{m-1}{m}}s}{n\hat{\alpha}^*}\right)^{n-K-1}\frac{\Gamma(\frac{n-K}{2})\Gamma(\frac{m-1}{2})}{\Gamma(\frac{n+m-K}{2}-1)\Gamma(\frac{1}{2})}$$

$$\cdot \{B(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1} \cdot \frac{\bar{y}^2}{s^2})$$

$$-B(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1} \cdot \frac{(n\bar{x}-\bar{y})^2}{s^2})\}$$

$$+1-B(\frac{1}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1} \cdot \frac{\bar{y}^2}{s^2})].$$

(10) For the case of $\bar{y}-\sqrt{\frac{m-1}{m}}s<0$ and $0<n\bar{x}<\bar{y}.$

$$\tilde{\theta}=\tfrac{1}{2}[\sum_{K=0}^{n-1}\binom{n-1}{K}(1-\frac{\bar{y}}{n\hat{\alpha}^*})^K\left(\frac{\sqrt{\frac{m-1}{m}}s}{n\hat{\alpha}^*}\right)^{n-K-1}$$

$$\cdot\frac{\Gamma(\frac{n-K}{2})\Gamma(\frac{m-1}{2})}{\Gamma(\frac{n+m-K}{2}-1)\Gamma(\frac{1}{2})}$$

$$\cdot \{B(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1} \cdot \frac{\bar{y}^2}{s^2})$$

$$-B(\frac{n-K}{2},\ \frac{m}{2}-1;\ \frac{m}{m-1} \cdot \frac{(n\bar{x}-\bar{y})^2}{s^2})\}$$

$$+1-B(\frac{1}{2},\ \frac{n}{2}-1;\ \frac{m}{m-1} \cdot \frac{\bar{y}^2}{s^2})].$$

In this notation,

$$B(p.\ q;\ x)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}\int_0^x t^{p-1}(1-t)^{q-1}\,dt,\quad 0<t,\quad x<0.$$

4. Empirical Result for small samples.

In section 2 and 3, we derived the estimators of θ. In this section, we investigate their relative performance for a small sized samples through simulation. For fixed $n = 10, 12, 15, 18$ and 20, estimates of the mean squared error (M.S.E.) and bias are obtained.

The estimated M.S.E.'s and biases of M.L.E. and M.V.U.E. appear in the Table. Although $\tilde{\theta}$ is known to be unbiased, its estimated bias is recorded for a check on the computations.

From the Table, we know the followings:

(1) The estimated biases of M.V.U.E. are found to be much smaller in magnitude than those of M.L.E.

(2) In all cases included in the study, the magnitude of $(\text{Bias})^2$ is relatively negligible in contrast with that of the M.S.E.

(3) When $E[X]>E[Y]$, the estimated M.S.E.'s of M.V.U.E. are smaller than those of M.L.E.

(4) When $E[X]<E[Y]$, the estimated M.S.E.'s of M.V.U.E. are larger than those of M.L.E.

TABLES

$\mu=0$ $\sigma=1$ $\alpha=1$ $\theta=.76158$

(:) $n=m=10$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.00186	−.00058	.01080	.01064

(:) $n=m=12$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.00136	−.00022	.00876	.00857

(:) $n=m=15$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.00124	−.00028	.00708	.00690

(:) $n=m=18$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.00224	−.00153	.00676	.00665

(:) $n=m=20$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.00387	−.00325	.00748	.00739

$\mu=5$ $\sigma=2$ $\alpha=.2$ $\theta=.39761$

(:) $n=m=10$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.01603	−.00020	.01340	.01448

(:) $n=m=12$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.01348	−.00009	.01125	.01198

(:) $n=m=15$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.01142	−.00050	.00925	.00972

(:) $n=m=18$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.00962	−.00050	.00760	.00810

(:) $n=m=20$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
−.00925	−00102	.00725	.00750

$\mu=5 \quad \sigma=5 \quad \alpha=.2 \quad \theta=.46192$

(:) $n=m=10$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
–.01302	–.00021	.01582	.01582

(:) $n=m=12$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
-.01090	–.00008	.01311	.01317

(:) $n=m=15$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
–.00894	.00026	.01054	.01048

(:) $n=m=18$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
–.00820	–.00090	.00917	.00914

(:) $n=m=20$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
–.00846	–.00183	.00883	.00881

$\mu=5 \quad \sigma=10 \quad \alpha=.2 \quad \theta=.49014$

(:) $n=m=10$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
–.01144	–.00433	.00863	.01803

(:) $n=m=12$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
–.00423	–.00183	.00775	.01428

(:) $n=m=15$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
–.00415	-.00029	.01149	.01107

(:) $n=m=18$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
–.00415	–.00100	.00995	.00959

(:) $n=m=20$

Bias		M.V.U.E.	
M.L.E.	M.V.U.E.	M.L.E.	M.V.U.E.
–.00486	–.00199	.00959	.00928

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