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TOTALLY REAL SUBMANIFOLDS WITH HARMONIC CURVATURE

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0. Introduction

Totally real submanifolds in a complex projective space are studied from various points of view [2, 14, 15]. In particular, H. Naitoh [7] has completely classified those submanifolds if the second fundamental form is parallel. On the other hand, compact totally real submanifolds of positive curvature or non-negative curvature are very recently investigated by F. Urbano [12] or Y. Ohnita [9], respectively.

A Riemannian curvature is said to be ***harmonic*** if the Ricci tensor R_{ij} satisfies the so-called Codazzi equation

$$\nabla_k R_{ij} = \nabla_j R_{ki}$$

Riemannian submanifolds with harmonic curvature in a Riemannian manifold of a constant curvature are investigated by E. Ômachi [8], M. Umehara [11] and the authors [3, 4]. In particular, the authors [3] classified completely submanifolds with harmonic curvature in a Riemannian manifold of constant curvature if the normal connection in the normal bundle is flat and the mean curvature vector is parallel. The purpose of this paper is to study totally real submanifolds with harmonic curvature in a Kaehlerian manifold of constant holomorphic curvature.

1. Totally real submanifolds of a Kaehlerian manifold

Let $(\tilde{M}, \tilde{g})$ be a Kaehlerian manifold of a real dimension 2m equipped with an almost complex structure J and a Hermitian metric $\tilde{g}$. Manifolds and submanifolds which are discussed in this paper are assumed to be connected, and all geometric objects are also assumed to be differentiable and of C^∞. Let $\tilde{M}$ be covered by a system of coordinate neighborhoods $\{\tilde{U};\ y^h\}$. Then we have

$$(1.1)\qquad J_j{}^t J_t{}^h = -\delta_j^h,\quad J_j{}^t J_i{}^s g_{ts} = g_{ji},$$

δ_j^h being the Kronecker delta, $J_j{}^h$ and g_{ji} the components of J and $\tilde{g}$, respectively. Here and in the sequel, the following convention on the range of

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indices are used, unless otherwise stated:

$$h, i, j, k, \cdots=1, \cdots, n, n+1, \cdots, 2m,$$
$$a, b, c, d, \cdots=1, \cdots, n,$$
$$x, y, z, u, \cdots, =n+1, \cdots, 2m.$$

The summation convention will be used with respect to those systems on indices. Denoting by ∇_j the operator of covariant differentiation with respect to g_{ji}, we get

(1.2) $$\nabla_j J_i{}^h=0.$$

Let M be an n-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{U; x^a\}$ and immersed isometrically in $\tilde{M}$ by the immersion ϕ: $M\to\tilde{M}$. When the argument is local, M need not be distinguished from $\phi(M)$. We represent the immersion ϕ locally by $y^h=y^h(x^a)$ and put $B_a^h=\partial_b y^h$, $\partial_b=\partial/\partial x^b$, then $B_b=(B_b{}^h)$ are n-linearly independent local tangent vectors of M. We choose $2m$-n mutually orthogonal unit normals $C_x=(C_x)$ to the submanifold. Since the immersion ϕ is isometric, the Riemannian metric g_{cb} induced on M is given by

(1.3) $$g_{cb}=g_{ji}B_c{}^j B_b{}^i.$$

Therefore, denoting by ∇_c the operator of van der Waerden-Bortolotti covariant differentiation with respect to g_{cb}, the equations of Gauss and Weingarten for M are respectively obtained:

(1.4) $$\nabla_c B_b{}^h=h_{cb}^x C_x{}^h,\ \nabla_c C_x{}^h=-h_c{}^a{}_x B_a{}^h,$$

where h_{cb}^x are the second fundamental forms in the direction of C_x and $h_c{}^a{}_x=h_{cbx}g^{ba}=h_{cb}^y g^{ba}g_{yx}$, $g_{yx}=g_{ji}C_y{}^j C_x{}^i$ being the metric tensor of the normal bundle of M and $(g^{ba})=(g_{cb})^{-1}$.

An n-dimensional Riemannian manifold M immersed isometrically into $\tilde{M}$ is called a *totally real* submanifold of $\tilde{M}$ if $JM_x\subset M_x^\perp$ for each point x and $M_x^\perp$ denotes the normal space of M at x [2], [15]. In this case, JX is a normal vector to M, provided that X is a tangent vector. Thus it follows that the dimensions satisfy $m\geqq n$. Let $N_x(M)$ be an orthogonal complement of JM_x in $M_x^\perp$, and then the following decomposition is obtained: $M_x^\perp=JM_x\oplus N_x(M)$. Hence it follows that the space $N_x(M)$ is invariant under the action of J. Accordingly we can put in each coordinate neighborhoods of M,

(1.5) $$J_j^h B_c^j=J_c^x C_x^h,$$

(1.6) $$J_j^h C_x^j=-J_x^a B_a^h+f_x^y C_y^h,$$

where we have put $J_{cx}=\tilde{g}(JB_c, C_x)$, $J_{xc}=-\tilde{g}(JC_x, B_c)$ and $f_{yx}=\tilde{g}(JC_x, C_y)$. From these definitions we easily see that $f_{yx}+f_{xy}=0$ and $J_{cx}=J_{xc}$. By taking account of (1.1) and (1.3), it follows from (1.5) and (1.6) that

$$\begin{cases} J_c{}^x J_x{}^a=\delta_c^a, \ J_c{}^x f_x{}^y=0, \\ f_x{}^z f_z{}^y=-\delta_x^y+J_x{}^e J_e{}^y, \end{cases} \tag{1.7}$$

where $J_c{}^x=J_{cy}g^{yx}$, $f_y{}^x=f_{yz}g^{zx}$. These show that $f^3+f=0$. f being of constant rank, it defines the so-called *f-structure* in the normal bundle [13]. If we apply the operator ∇_c of the covariant differentiation to (1.5) and (1.6) and make use of (1.1), (1.2), (1.4) and these equations, we get

$$h_{cb}{}^x J_{xa}=h_{ca}{}^x J_{xb}, \tag{1.8}$$

$$\nabla_c J_b{}^x=h_{cb}{}^z f_z{}^x, \tag{1.9}$$

$$\nabla_c f_x{}^y=h_c{}^e{}_x J_e{}^y-h_c{}^e{}_x J^{ey}. \tag{1.10}$$

If $\nabla_c f_y{}^x$ vanishes identically, then the f-structure f in the normal bundle is said to be *parallel* [15]. In this case, (1.10) reduces to

$$h_{cey}J^{ex}=h^{cex}J_{ey}. \tag{1.11}$$

Multiplying $h^{cby}J_y{}^a$ to (1.8) and summing up for c, b and a, and using (1.11), we find

$$h^{cby}h_{cb}{}^x J_y{}^a J_{xa}=h^{cby}h_{cay}J^{ax}J_{xb},$$

or, taking account of (1.7), we have

$$h_{cb}{}^x h^{cby}(f_y{}^z f_{zx}+g_{yx})=h_{cb}{}^y h^{cb}{}_y.$$

Therefore it follows that $h_{cb}{}^z f_z{}^x=0$ and hence

$$\nabla_c J_b{}^x=0, \tag{1.12}$$

because of (1.9). We notice from (1.7) that $f_y{}^x$ vanishes identically if $m=n$. Thus an m-dimensional totally real submanifold of a real $2m$-dimensional Kaehlerian manifold has always the equation (1.12).

In the sequel, the ambient Kaehlerian manifold is assumed to be of constant holomorphic sectional curvature $4c$ and of real dimension $2m$, which is denoted by $M^{2m}(c)$. Then the curvature tensor of $M^{2m}(c)$ is given by

$$\tilde{R}_{kjih}=c(g_{kh}g_{ji}-g_{ki}g_{jh}+J_{kh}J_{ji}-J_{ki}J_{jh}-2J_{kj}J_{ih}).$$

Since the submanifold M is totally real, we see, using (1.5), (1.6) and (1.7), that the equations of Gauss, Codazzi and Ricci for M are respectively given by

(1.13) $$R_{dcba}=c(g_{da}g_{cb}-g_{db}g_{ca})+h_{ad}{}^{x}h_{cbx}-h_{db}{}^{x}h_{cax},$$

(1.14) $$\nabla_d h_{cb}{}^{x}-\nabla_c h_{db}{}^{x}=0,$$

(1.15) $$R_{dcyx}=c(J_{dx}J_{cy}-J_{dy}J_{cx})+h_d{}^{e}{}_{x}h_{cey}-h_c{}^{e}{}_{x}h_{dey},$$

where R_{dcba} and R_{dcyx} are the Riemannian curvature tensor and that of the connection induced in the normal bundle of M, respectively.

We see, from (1.13) that the Ricci tensor R_{cb} and the scalar curvature r of M can be expressed as follows:

(1.16) $$R_{cb}=c(n-1)g_{cb}+h^{x}h_{cbx}-h_{ce}{}^{x}h_b{}^{e}{}_{x},$$

(1.17) $$r=cn(n-1)+h^{x}h_{x}-h_{cb}{}^{x}h^{cb}{}_{x},$$

where $h^{x}=g^{cb}h_{cb}{}^{x}$.

A Riemannian curvature tensor is said to be ***harmonic*** if the Ricci tensor satisfies the Codazzi equation, namely, $\nabla_c R_{ba}$ is symmetric with respect to all indices c, b and a. Differentiating (1.16) covariantly along M, we find

$$\nabla_d R_{cb}=(\nabla_d h^{x})h_{cbx}+h^{x}\nabla_d h_{cbx}-(\nabla_d h_c{}^{ex})h_{bex}-h_c{}^{ex}\nabla_d h_{bex}.$$

By means of (1.4), it follows that it is necessary and sufficient for M to be of harmonic curvature that it satisfies

(1.18) $$(\nabla_d h^{x})h_{cbx}-(\nabla_c h^{x})h_{dbx}-h_c{}^{ex}\nabla_d h_{bex}+h_d{}^{ex}\nabla_c h_{bex}=0.$$

2. Totally real submanifolds with parallel mean curvature vector

Let M be an n-dimensional totally real submanifold with harmonic curvature in $M^{2m}(c)$ such that the f-structure in the normal bundle is parallel. This section is devoted to the investigation of a totally real submanifold with parallel mean curvature vector. The covariant derivative of the Ricci tensor satisfies

(2.1) $$\nabla_d R_{cb}=\nabla_c R_{db}.$$

Let $\mathscr{I}$ be a mean curvature vector field. Namely, it is defined by

$$\mathscr{I}=g^{cb}h_{cb}{}^{x}C_x/n=h^{x}C_x/n,$$

which is independent of the choice of the local field of orthonormal frames $\{C_x\}$. The fact that the mean curvature vector is parallel is assnumed, and we may choose a local field $\{C_x\}$ in such a way that $\mathscr{I}=a\ C_{n+1}$, where $a=\|\mathscr{I}\|$ is constant. Because of the choice of the local field, the parallelism of $\mathscr{I}$ yields

(2.22) $$\begin{cases} h^{x}=0, & x\geqq n+2,\\ h^{n+1}=n\|\mathscr{I}\|. \end{cases}$$

Thus, (1.18) turns out to be

(2.3) $$h_c^{ex}\nabla_d h_{bex} - h_d^{ex}\nabla_c h_{bex} = 0.$$

$\mathscr{J}$ being a normal vector field on M, the curvature tensor R_{dcyx} of the connection in the normal bundle shows $R_{dcn+1x}=0$ for any index x. Thus (1.15) yields

(2.4) $$c(-J_d J_{cx} + J_c J_{dx}) + h_{dex}h_c{}^e - h_{cex}h_d{}^e = 0,$$

where we have put $h_{cb}=h_{cb}^{n+1}$ and $J_c=J_c{}^{n+1}$.

On the other hand, since the f-structure in the normal bundle is parallel, if we apply $J_y{}^a$ to (1.8) and sum for a, then we obtain

$$h_{cby} = h_{ca}{}^x J_y{}^a J_{xb},$$

where we have used (1.7) and (1.11), from which, taking the skew-symmetric part of this with respect to indices c and b,

$$(h_{ce}{}^x J_y{}^e) J_{bx} - (h_{be}{}^x J_y{}^e) J_{cx} = 0.$$

Thus we see, using (1.7) and (1.9), that

(2.5) $$h_{ce}{}^x J_y{}^e = P_{yz}{}^x J_c{}^z,$$

where we have put $P_{yz}{}^x = h_{cb}{}^x J_y{}^c J_z{}^b$. Denoting $P_{xyw}=g_{zw}P_{xy}{}^z$, we see that P_{xyz} is symmetric for all indices, because of (1.11). It follows from (1.7), (1.9) and (1.12) that

(2.6) $$P_{yzx} f_w{}^y = 0$$

and hence $h_{cb}{}^x h^{cb}{}_x = P_{xyz}P^{xyz}$. Thus the square of the length of $h_{cb}{}^x - P_{yz}{}^x J_c{}^y J_b{}^z$ for each x vanishes identically, which shows

(2.7) $$h_{cb}{}^x = P_{yz}{}^x J_c{}^y J_b{}^z.$$

If we differentiate this equation covariantly and take account of (1.12), we obtain

(2.8) $$\nabla_d h_{cb}{}^x = (\nabla_d P_{yz}{}^x) J_c{}^y J_b{}^z,$$

which together with the Codazzi equation (1.14) gives

$$J_c{}^y \nabla_d P_{yz}{}^x - J_d{}^y \nabla_c P_{yz}{}^x = 0,$$

and hence

(2.9) $$\nabla_d P_{yz}{}^x = (J_z{}^e \nabla_e P_{yw}{}^x) J_d{}^w,$$

where we have used (1.7), (1.10), (1.11) and (2.6).

By the way, the equations (1.7), (1.11), (2.6) and (2.7) give $h^x = P_y{}^{yx}$. Since the mean curvature vector is parallel in the normal bundle, it follows that $\nabla_d P_y{}^{yx}=0$. By contracting z and y in (2.9), we easily see that

$$J^{ze}\nabla_e P_{zwx} = 0,$$

which together with (2.8) yields

(2.10) $$J_x{}^d \nabla_d h_{cb}{}^x = 0.$$

Combining (2.8) and (2.9), we verify that

$$J_y^{\ e}\nabla_e h_{cbx} - J_x^{\ e}\nabla_e h_{cby} = 0, \tag{2.11}$$

because P_{xyz} is symmetric for all indices. By making use of (1.12), the covariant derivative of (2.4) gives

$$(\nabla_b h_{ce})h_d^{\ ex} + h_{ce}\nabla_b h_d^{\ ex} = (\nabla_b h_{de})h_c^{\ ex} + h_{de}\nabla_b h_c^{\ ex}, \tag{2.12}$$

which implies

$$(\nabla_b h_{ce})h_{dx}^{\ e}\, h_a^{\ dx} - (\nabla_b h_d^{\ e})h_{cex}h_a^{\ dx} = (\nabla_b h_{cex})h_d^{\ e}h_a^{\ dx} - (\nabla_b h_{dex})h_c^{\ e}h_a^{\ xd}. \tag{2.13}$$

By the properties of (1.18), (2.1) and (2.4) the second term in the right hand side is deformed as follows:

$$-(\nabla_b h_{dex})h_c^{\ e}h_a^{\ dx} = -(\nabla_b h_{da}^{\ \ x})h_c^{\ e}h_{ed}^{\ \ x} = -(\nabla_b h_{da}^{\ \ x})h^{de}h_{ecx} + c(J_c J_{dx} - J_d J_{cx})\nabla_b h_a^{\ dx}.$$

Hence the last two equations give

$$(\nabla^b h^{ca})(\nabla_b h_{ce})h_{de}^{\ \ x}h_a^{\ dx} - (\nabla^b h^{ca})(\nabla_b h_d^{\ e})h_{cex}h_a^{\ dx} = c(J_c J_{dx} - J_d J_{cx})(\nabla^b h^{ca})(\nabla_b h_{da}^{\ \ x}),$$

which together with (2.10) and (2.11) yields

$$(\nabla_e h_{cb})(\nabla_a h^{cb})h_d^{\ ex}h^{ad}_{\ \ x} - (\nabla_b h^{ca})(\nabla^b h^{de})h_{cex}h_{ad}^{\ \ x} = -c(\nabla_c h_{ba})(\nabla^c h^{ba}), \tag{2.14}$$

where we have used (1.7).

On the other hand, for any fixed indices c and x, $(\nabla_b h_{ce})h_{de}^{\ \ x} - (\nabla_d h_{ce})h_{be}^{\ \ x}$ can be regarded as a square matrix of degree n. By taking the norm of this matrix and by using the equation (2.14), the following equation

$$\|(\nabla_b h_{ce})h_{de}^{\ \ x} - (\nabla_d h_{ce})h_{be}^{\ \ x}\|^2 + 2c\|\nabla_c h_{ba}\|^2 = 0 \tag{2.15}$$

can be obtained.

3. Main theorem

This section is devoted to the study of a totally real submanifold on which the f-structure in the normal bundle is parallel. First of all, the following property can be verified.

LEMMA 3.1. *Let M be an n-dimensional totally real submanifold with harmonic curvature in $M^{2m}(c)$, $(c \geqq 0)$ such that the f-structure in the normal bundle is parallel. If the mean curvature vector is parallel, then $h_{cb}^{\ \ n+1}$ is parallel.*

PROOF. It is sufficient to prove the above result in the case where the ambient space is Euclidean. Then the equation (2.15) shows

$$(\nabla_b h_{ce})h_{de}^{\ \ x} - (\nabla_d h_{ce})h_{be}^{\ \ x} = 0 \tag{3.1}$$

for any indices and hences it follows from (2.12) that

$$h_{ce}\nabla_d h_b^{\ ex} = h_{de}\nabla_c h_b^{\ ex}. \tag{3.2}$$

In the case where $x = n+1$ in (3.2), we get

(3.3) $$h_{ce}\nabla_d h_b^{\ e}=h_{de}\nabla_c h_b^{\ e}.$$

By the similar method to the discussion on hypersurfaces with harmonic curvature in a Euclidean space, the result is verified and the proof is therefore sketched briefly. When a function h_m for any integer $m\geqq 1$ is given by

$$h_m=h_{a_2}^{\ a_1}h_{a_3}^{\ a_2}\cdots h_{a_1}^{\ a_m},$$

it satisfies the following eguation

$$\nabla_b h_m=m\, h_{a_2}^{\ a_1}\cdots h_{a_m}^{\ a_{m-1}}\nabla_b h_{a_1}^{\ a_m}.$$

By using (1.14) and by combining (3.3) together with the above equation, it follows that the function h_m is constant for any integer m. Since the matrix $A^{n+1}=(h_c^{\ an+1})$ is diagonalizable, the local field $\{e_a\}$ on M can be specialized so that $h_c^a=\lambda_c\delta_c^{\ a}$, which implies the fact that all functions h_1, h_2, $\cdots$, h_m, $\cdots$ are constant means that each eigenvalue λ_a is constant on M. Then, by taking account of the calculation for the Laplacian of the constant h_2, the following relation

(3.4) $$\nabla_d h_{cb}\nabla^d h^{cb}+R_{cbbc}(\lambda_c-\lambda_b)^2=0$$

can be obtained.

On the other hand, the equation (3.2) gives

(3.5) $$(\lambda_c-\lambda_b)\nabla_d h_{cb}^{\ x}=0$$

for any indices d and x at a point p in M and hence

(3.6) $$\nabla_d h_{cb}^{\ x}=0$$

for any index x provided that $\lambda_c\neq\lambda_b$. By μ_1, $\cdots$, μ_α mutually distinct eigenvalues of A^{n+1} are denoted. Let n_1, $\cdots$, n_α be their multiplicities. By using the notation $[a]=\{b:\ \lambda_a=\lambda_b\}$ we see from (3.6) that

$$\nabla_d h_{cb}^{\ x}=0 \text{ for } c\notin[a],\ b\in[a],$$

which yields

(3.7) $$\nabla_d h_{cb}=0 \text{ for } c\notin[a],\ b\in[a].$$

Therefore, differentiating (3.3) covariantly, we have

$$\nabla_a h_{ce}\nabla_d h_b^{\ e}+h_{ce}\nabla_a\nabla_d h_b^{\ e}=\nabla_a h_{de}\nabla_c h_b^{\ e}+h_{de}\nabla_a\nabla_c h_b^{\ e},$$

from which it follows by using the Ricci formula

$$2(\nabla_a h_{ce}\nabla_d h_b^{\ e}-\nabla_a h_{de}\nabla_c h_b^{\ e})$$
$$=h_{de}(R_{abf}^{\ \ \ e}h_c^{\ f}-R_{abc}^{\ \ \ f}h_f^{\ e})-h_{ce}(R_{abf}^{\ \ \ e}h_d^{\ f}-R_{abd}^{\ \ \ f}h_f^{\ e}).$$

Putting $a=c$ and $d=b$, and diagonalizing the matrix A^{n+1}, we get

$$2(\nabla_e h_c^{\ c}\nabla^e h_b^{\ b}-\nabla_e h_{cb}\nabla^e h^{cb})=R_{cbbc}(\lambda_c-\lambda_b)^2,$$

where the summation convention is not used with respect to c and b. According to (3.7) the above equation means

$$R_{cbbc}(\lambda_c-\lambda_b)^2=0 \text{ for } c\notin[a],\ b\in[a].$$

Consequently, by combining this equation together with (3.4) it turns out that the shape operator A^{n+1} is parallel. Since distinct eigenvalues μ_r $(r=1, \cdots, \alpha)$ are constant, the smooth distributions $\mathscr{D}_r$ which consists of all eigenspaces associated with the eigenvalue can be defined, and they are mutually orthogonal. Furthermore, A^{n+1} being parallel, we see that distributions $\mathscr{D}_r$ are all parallel and hence completely integrable, because all eigenvalues $\mathscr{D}_r$ are constant. Thus, by means of the local decomposition theorem [5] the above discussions are summerized in the following way.

THEOREM 3.2. *Let $M^{2m}(c)$ be a real 2m-dimensional complex space form of constant holomorphic curvature $4c\geqq 0$, and M an n-dimensional totally real submanifold of $M^{2m}(c)$ with harmonic curvature such that the f-structure in the normal bundle is parallel. If the mean curvature vector of M is parallel, then M is localy a product of Riemannian manifolds.*

COROLLARY. *Let M be an m-dimensional totally real submanifold with harmonic curvature in $M^{2m}(c)$, $c\geqq 0$. If the mean curvature vector of M is parallel, then M is locally a product of Riemannian manifolds.*

REMARK 1. Let $S^n(c)$ be an n-dimensional sphere of constant curvature c. When S^{2n+1} is considered as a hypersurface in an $(n+1)$-dimensional complex Euclidean space C^{n+1}, the product $S^1(c_1)\times\cdots\times S^1(c_{n+1})$ which is denoted by $N(n+1)$ is a totally real submanifold of C^n. Then the mean curvature vector is parallel and the normal connection is flat.

REMARK 2. The product manifold $N(n+1)$ is contained in a unit sphere S^{2n+1}, where $1/c_1+\cdots+1/c_{n+1}=1$. For the isometric immersion ϕ' of $N(n+1)$ into S^{2n+1} and a principal S^1-bundle over an n-dimensional complex projective space P_nC with the projection $\bar{\pi}$: $S^{2n+1}\longrightarrow P_nC$, we assume that there exists a principal S^1-bundle $N(n+1)$ over M with totally geodesic fibers and the projection π such that π is compatible with the fibration $\bar{\pi}$. Then $M=(N(n+1))$ is an n-dimensional totally real submanifold of P_nC and it is easily seen that the mean curvature vector is parallel, the normal connection is flat and moreover the second fundametal form is also parallel. Of course, the condition of the curvature tensor is satified. In the case where $c>0$, under the additional condition that the normal connection is flat in Theorem 3.2, there exits a totally geodesic submanifold P_nC in P_mC such that M satisfies the above situation in P_nC.

REFERENCES

[1] B.Y. Chen, *Geometry of submanifolds*, Marcel Dekker, Inc., 1973.

[2] B.Y. Chen and K. Ogiue, *On totally real submanifolds*, Trans. Amer. Math. Soc., 193(1974), 257—266.

[3] U-H. Ki and H. Nakagawa, *Submanifolds with harmonic curvature*, Tsukuba J. Math. 10(1986), 285—292.

[4] U-H. Ki, H. Nakagawa and M. Umehara, *On complete hypersurfaces with harmonic curvature*, in a Riemannian manifold of constant curvature, Tsukuba J. Math. 11(1987), 61—76.

[5] S. Kobayashi and K. Nomizu, *Foundations of differential geometry*, I, Inter. Pub., New York, 1963.

[6] Y.G. Lumiste and A.D. Chakmazyan, *Normal connection and submanifolds with parallel normal fields in a space of constant curvature*, J. Soviet Math., 21 (1983), 107—127.

[7] H. Naitoh, *Parallel submanifolds of complex space form* I, II, Nagoya Math. J., 90(1983), 85-117 ; 91 (1983), 119—149.

[8] E. Ômachi, *Hypersurfaces with harmonic curvature in a space of constant curvature*, Preprint.

[9] Y. Ohnita, *Totally real submanifolds with nonnegative sectional curvature*, Preprint.

[10] B. Smyth, *Submanifolds of constant mean curvature*, Math. Ann., 205 (1975), 265—280.

[11] M. Umehara, *Hypersurfaces with harmonic curvature*, Tsukuba J. Math. 10(1986), 79—88.

[12] F. Urbano, *Totally real minimal submanfolds of a complex projective space*, Preprint.

[13] K. Yano, *On a structure defined by a tensor field of type* (1,1) *satisfying* $f^3+f=0$, Tensor N.S., 14(1963), 99—109.

[14] K. Yano and M. Kon, *Totally real submanifolds of complex space form*, I, Tôhoku Math. J., 28(1976), 215—225.

[15] K. Yano and M. Kon, *Anti-invariant submanifolds*, Marcel Dekker, Inc., 1976.

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