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CURVATURE CONDITIONS ON COMPLEX HYPERSURFACES INVOLVING THEIR KAEHLER FORM

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1. Introduction

In this paper we study complex hypersurfaces M^n of a complex space form $\tilde{M}^{n+1}(c)$ of constant holomorphic sectional curvature c with complex dimension $n\geqslant 2$, which satisfy curvature conditions of the from $E\cdot F=\varphi F$, where $E\in\{R, Z, C, B\}$ and $F\in\{R, Z, C, B, Q, G\}$ and where R, Z, C, B, Q and G are respectively the Riemann-Christoffel curvature tensor, the concircular tensor, the Weyl conformal curvature tensor, the Bochner curvature tensor, the Ricci tensor and the Einstein curvature tensor, and φ is the Kaehler form of M.

Curvature conditions of the form $E\cdot F=0$ where the first tensor acts on the second as a derivation, were studied for complex hypersurfaces of complex space forms by P. J. Ryan [Rya], J. Deprez, P. Verheyen and L. Verstraelen [DeVV] and J. Deprez, F. Dillen, M. Petrovic-Torgasev and L. Verstraelen [DDPV], and for Bochner-Kaehler manifolds by H. Takagi and Y. Watanabe [TaWa], J. Deprez, K. Sekigawa and L. Verstraelen [DeSV] and also by two of the present authors [PTV].

For Bochner-Kaehler manifolds we studied previously conditions of the type $E\cdot F=\varphi F$ [LPV].

We recall the following results.

THEOREM A. ([Yano], [DeVV], [MPT]). *Let M^n be a complex hypersurface of a complex space form $\tilde{M}^{n+1}(c)$ of complex dimension $n\geqslant 2$. Then the following statements are equivalent*:

(i) M is flat;

(ii) M is conformally flat;

(iii) M is Ricci-flat;

(iv) $c=0$ and M is a hyperplane in $\mathbf{c}^{n+1}$.

THEOREM B. ([Kon], [YaSa], [DeVV]). *Let M^n be a complex hypersurface of a complex space form $\tilde{M}^{n+1}(c)$, ($n\leqslant 2$). Then M^n is Bochner-flat if and only if M^n is totally geodesic.*

THEOREM C. ([Cher]). *A complex hypersurface M^n of a complex space form $\tilde{M}^{n+1}(c)$ of complex dimension $n \geqslant 2$ is Einstein if and only if M^n is totally geodesic or locally a complex hypersphere in the complex projective space $CP^{n+1}(c)$.*

All results concerning the conditions $E_4 \cdot F_4 = \varphi F_4$, where $E_4 \cdot F_4 \in \{R, Z, C, B\}$ are given in the following two theorems.

THEOREM 1. *Let M^n be a complex hypersurface of a complex space form $\tilde{M}^{n+1}(c)$ of complex dimension $n \geqslant 2$. Then the following conditions are equivalent:*

(*i*) $R \cdot R = \varphi R$;

(*ii*) $C \cdot R = \varphi R$;

(*iii*) $Z \cdot R = \varphi R$;

(*iv*) $B \cdot R = \varphi R$;

(*v*) $R \cdot C = \varphi C$;

(*vi*) $C \cdot C = \varphi C$;

(*vii*) $Z \cdot C = \varphi C$;

(*viii*) $B \cdot C = \varphi C$;

(*ix*) $R \cdot Z = \varphi Z$;

(*x*) $C \cdot Z = \varphi Z$;

(*xi*) $Z \cdot Z = \varphi Z$;

(*xii*) $B \cdot Z = \varphi Z$;

(*xiii*) *M^n is a hyperplane in the complex Euclidean space C^{n+1}.*

THEOREM 2. *Let M^n be a complex hypersurface of a complex space form $\tilde{M}^{n+1}(c)$ of complex dimension $n \geqslant 2$. Then the following conditions are equivalent:*

(*i*) $R \cdot B = \varphi B$;

(*ii*) $Z \cdot B = \varphi B$;

(*iii*) $C \cdot B = \varphi B$;

(*iv*) $B \cdot B = \varphi B$;

(*v*) *M^n is totally geodesic in $\tilde{M}^{n+1}(c)$.*

Next, we consider the consider the conditions $E_4 \cdot D_2 = \varphi D_2$ where $E_4 \in \{R, Z, C, B\}$ and $D_2 \in \{Q, G\}$. All corresponding results are given in the following two theorems.

THEOREM 3. *Let M^n be a complex hypersurface of a complex space form $\tilde{M}^{n+1}(c)$ of complex dimension $n \geqslant 2$. Then the following conditions are equivalent:*

(*i*) $R \cdot Q = \varphi Q$;

(*ii*) $Z\cdot Q=\varphi Q$;

(*iii*) $C\cdot Q=\varphi Q$;

(*iv*) $B\cdot Q=\varphi Q$;

(*v*) *M^n is a hyperplane in C^{n+1}.*

THEOREM 4. *Let M^n be a complex hypersurface of a complex space form $\tilde{M}^{n+1}(c)$ of complex dimension $n\geqslant 2$. Then the following conditions are equivalent:*

(*i*) $R\cdot G=\varphi G$;

(*ii*) $Z\cdot G=\varphi G$;

(*iii*) $C\cdot G=\varphi G$;

(*iv*) $B\cdot G=\varphi G$;

(*v*) *M^n is Einstein hypersurface, i.e. totally geodesic in $M^{n+1}(c)$ or a complex hypersphere in CP^{n+1}.*

2. Basic formulas

2.1. Let $\tilde{M}^{n+1}(c)$ be a complex space form of complex dimension $n+1$ and of holomorphic sectional curvature c, with metric g, complex structure J and Levi Civita connection $\tilde{\nabla}$. Then the curvature tensor $\tilde{R}$ of $\tilde{M}^{n+1}(c)$ has the following form:

$$\tilde{R}(X,Y)=\frac{c}{4}(X\wedge Y+JX\wedge JY+2g(X,\ JY)J),$$

for each $X,Y\in T_p\tilde{M}$, $p\in\tilde{M}$, and where $X\wedge Y$ is the endomorphism of T_pM defined by

$$(X\wedge Y)U=g(U,\ Y)X-g(U,\ X)Y.$$

It is known that a complete simply connected complex space form $\tilde{M}^{n-1}(c)$ is holomorphically isometric with $CP^{n+1}(c)$, C^{n+1} or $D^{n+1}(c)$, according to c being positive, zero or negative. $CP^{n+1}(c)$ is the complex projective space with the Study-Fubini metric of holomorphic sectional curvature c, C^{n+1} the complex Euclidean space, $D^{n+1}(c)$ the unit ball in C^{n+1} with the Bergman metric of holomorphic sectional curvature c.

Let M^n be a complex hypersurface of $\tilde{M}^{n+1}(c)$, $(n\geqslant 2)$. We denote the induced metric, complex structure and connection respectively by g, J and ∇. Then, in a tangent space $T_p\tilde{M}$, at each point $p\in\tilde{M}$ we can choose an orthonormal frame $e_1, e_2, \cdots, e_n, e_{1^*}=Je_1, e_{2^*}=Je_2, \cdots, e_{n^*}=Je_n, \xi, J\xi$, such that the Vectors $e_1, \cdots, e_n, e_{1^*}, \cdots, e_{n^*}$ are tangent to M^n and ξ, $J\xi$ are normal to M^n, and such that the second fundamental tensor $A=A_\xi$ in the direction ξ satisfies.

$$\begin{cases} Ae_i=\lambda_i e_i, \\ Ae_{i^*}=-\lambda_i e_{i^*}, \end{cases}$$

with $\lambda_i \in \boldsymbol{R}^+$, and $i\in\{1, \cdots, n\}$.
Note that if M^n is locally isometric with the complex hypersphere Q^n in $CP^{n+1}(c)$, then $A^2=\frac{c}{4}I$.

The *Ricci curvature tensor* Q of M is defined by

$$Q=\frac{(n+1)cI}{2}-2A^2, \tag{1}$$

and the scalar curvature τ is given by

$$\tau=n(n+1)c-2\mathrm{Tr}(A^2)=n(n+1)c-4\sum_{i=1}^{n}\lambda_i^2. \tag{2}$$

The *Einstein tensor* G of M is defined by

$$G(X,Y)=Q(X,Y)-\frac{\tau g(X, Y)}{2n}, \tag{3}$$

i.e. $G=Q-\frac{\tau I}{2n}$, where $I(X,Y)=g(X,Y)$ is the identity operator.

The *Riemann-Christoffel curvature tensor* R of M is given by

$$R(X,Y)=\tilde{R}(X,Y)+AX\wedge AY+JAX\wedge JAY. \tag{4}$$

The *concircular curvature tensor* Z of M is defined by

$$Z(X, Y)=R(X, Y)-\frac{\tau}{2n(2n-1)}(X\wedge Y). \tag{5}$$

The *Weyl conformal curvature tensor* C of M is defined by

$$C(X, Y)=R(X, Y)-\frac{1}{2(n-1)}(QX\wedge Y+X\wedge QY) + \frac{\tau}{2(n-1)(2n-1)}(X\wedge Y), \tag{6}$$

and the *Bochner curvature tensor* B of M is defined by

$$\begin{aligned} B(X, Y)=R(X, Y)-\frac{1}{2(n+2)}(&QX\wedge Y+X\wedge QY+QJX\wedge JY \\ &+JX\wedge QJY-2g(QJX,Y)J-2g(JX,Y)QJ) \\ &+\frac{\tau}{4(n+1)(n+2)}(X\wedge Y+JX\wedge JY+2g(X,JY)J), \end{aligned} \tag{7}$$

for all vectors X and Y tangent to M at the same point.

These curvature tensors satisfy the following relations:

$$\begin{aligned} &R(X, Y)=R(JX, JY), \\ &R(X, Y)J=JR(X, Y), \\ &QJ=JQ, \\ &GJ=JG, \end{aligned}$$

$$Z(JX,\ JY)=-JZ(X,\ Y)J,$$
$$C(JX,\ JY)=-JC(X,\ Y)J,$$
$$B(JX,\ JY)=B(X,\ Y),$$
$$B(X,\ Y)J=JB(X,\ Y).$$

Then, with respect to an orthonormal basis $\{e_1,\ \cdots,\ e_n,\ e_{1^*},\ \cdots,\ e_{n^*}\}$ of the tangent space TM^n as chosen above, we have the following formulas:

$$(8)\qquad \begin{cases} Qe_i=\mu_i e_i, \\ Qe_{i^*}=\mu_i e_{i^*}, \end{cases}$$

where $\mu_i=\frac{n+1}{2}c-2\lambda_i^2$;

$$(9)\qquad \begin{cases} Ge_i=g_i e_i, \\ Ge_{i^*}=g_i e_{i^*}, \end{cases}$$

where $g_i=\mu_i-\frac{\tau}{2n}$;

$$(10)\qquad \begin{cases} R(e_i,\ e_j)=\nu_{ij}(e_i\wedge e_j+e_{i^*}\wedge e_{j^*}),\ (i\neq j), \\ R(e_i,\ e_{j^*})=\nu_{ij}(e_i\wedge e_{j^*}\wedge e_j)-\frac{c}{2}\delta_{ij}J, \end{cases}$$

where $\nu_{ij}=\frac{c}{4}+\lambda_i\lambda_j$ and $\bar{\nu}_{ij}=\frac{c}{4}-\lambda_i\lambda_j$;

$$(11)\qquad \begin{cases} Z(e_i,e_j)=(\nu_{ij}-\tau_0)e_i\wedge e_j+\nu_{ij}e_{i^*}\wedge e_{j^*},\ (i\neq j), \\ Z(e_i,\ e_{j^*})=(\bar{\nu}_{ij}-\tau_0)e_i\wedge e_{j^*}-\bar{\nu}_{ij}e_{i^*}\wedge e_j-\frac{c}{2}\delta_{ij}J, \end{cases}$$

where $\tau_0=\frac{\tau}{2n(2n-1)}$;

$$(12)\qquad \begin{cases} C(e_i,\ e_j)=(\nu_{ij}+\alpha_{ij})e_i\wedge e_j+\nu_{ij}e_{i^*}\wedge e_{j^*},\ (i\neq j), \\ C(e_i,\ e_{j^*})=(\bar{\nu}_{ij}+\alpha_{ij})e_i\wedge e_{j^*}-\bar{\nu}_{ij}e_{i^*}\wedge e_j-\frac{c}{2}\delta_{ij}J, \end{cases}$$

whee

$$\alpha_{ij}=\frac{\tau}{2(n-1)(2n-1)}-\frac{1}{2(n-1)}(\mu_i+\mu_j);$$

$$(13)\qquad \begin{cases} B(e_i,\ e_j)=\beta_{ij}(e_i\wedge e_j+e_{i^*}\wedge e_{j^*}), \\ B(e_i,\ e_{j^*})=\bar{\beta}_{ij}(e_i\wedge e_{j^*}-e_{i^*}\wedge e_j)+\delta_{ij}(k_i+\frac{1}{n+2}Q)J, \end{cases}$$

where

$$\beta_{ij}=\nu_{ij}-\frac{1}{2(n+2)}(\mu_i+\mu_j)+\frac{\tau}{4(n+1)(n+2)},$$

$$\beta_{ij}=\nu_{ij}-\frac{1}{2(n+2)}(\mu_i+\mu_j)+\frac{\tau}{4(n+1)(n+2)},$$

and

$$k_i=-\frac{c}{2}-\frac{\tau}{2(n+1)(n+2)}+\frac{1}{n+2}\mu_i,$$

for all $i,j\in\{1,\ \cdots,\ n\}$.

2.2. If E_4, F_4 are two curvature tensors of the 4th order of the space M, then the operation $E_4 \cdot F_4$ is defined by

$$(14) \qquad (E_4(X, Y)\cdot F_4)(U, V)W = E_4(X, Y)(F_4(U, V)W) - F_4(U, V)(E_4(X, Y)W) - F_4(E_4(X, Y)U, V)W - F_4(U, E_4(X, Y)V)W,$$

for tangent vectors X, Y, U, V, W on M.

If D_2 is a symmetric 2nd order curvature tensor on M and if D is the endomorphism in $T_p(M)$ defined by $D_2(X, Y)=g(DX, Y)=g(X, DY)$, then the operation $E_4 \cdot D_2$ is defined by

$$(15) \qquad (E_4(X, Y)\cdot D_2)U = E_4(X, Y)(DU) - D(E_4(X, Y)U),$$

for all tangent vectors X, Y, U on M.

If φ is the Kaehler form of M, then φD_2 and φE_4 are defined by

$$(16) \qquad (\varphi D_2)(X, Y)U = (\varphi(X, Y)\cdot D_2)U = \varphi(X, Y)DU,$$

and

$$(17) \qquad (\varphi E_4)(X, Y)(U, V)W = (\varphi(X, Y)\cdot E_4)(U, V)W = \varphi(X, Y)E_4(U, V)W.$$

By the condition $E_4 \cdot F_4 = \varphi F_4$, we mean that $(E_4(X, Y)\cdot F_4)(U, V)W - (\varphi(X, Y)\cdot F_4)(U, V)W = 0$, for all $X, Y, U, V, W \in T_p(M)$ and for all $p \in M$. The condition $E_4 \cdot D_2 = \varphi D_2$ has a similar meaning.

We know that each of the tensors R, Z, C, B of the space M^n $(n \geqslant 2)$ can be expressed in the form given by the following formulas

$$(18) \qquad \begin{cases} E_4(e_i, e_j) = a_{ij} e_i \wedge e_j + b_{ij} e_{i^*} \wedge e_{j^*}, \\ E_4(e_i, e_{j^*}) = a'_{ij} e_i \wedge e_{j^*} - b'_{ij} e_{i^*} \wedge e_j + c'_{ij} J, \end{cases}$$

and satisfies

$$(19.\text{a}) \qquad E_4(Je_i, Je_j) = E_4(e_i, e_j)$$

or

$$(19.\text{b}) \qquad E_4(Je_i, Je_j) = -JE_4(e_i, e_j)J.$$

Besides, any of the tensors Q and G can be expressed as

$$(20) \qquad \begin{cases} D_2 e_i = d_i e_i, \\ D_2 e_{i^*} = d_i e_{i^*}. \end{cases}$$

Tensors E_4, F_4, D_2 appearing in the sequel will not be necessarilly some of the tensors R, Z, C, B, but they all are assumed to satisfy the conditions (18)—(20).

3. Proofs of theorems

PROOF OF THEOREM 1. To prove Theorem 1 we first prove the following three lemmas.

LEMMA 5. *Let M^n be a complex hypersurface of a complex space $\tilde{M}^{n+1}(c)$, $(n\geqslant 2)$. If E_4, F_4 are 4th order covariant curvature tensors satisfying conditions* (18) *and* (19), *then $E_4\cdot F_4=\varphi F_4$ if and only if $F_4=0$.*

PROOF. Denoting

$$S_{\bar{i}\bar{j}\bar{k}\bar{l}\bar{m}}=(E_4(e_{\bar{i}},\ e_{\bar{j}})\cdot F_4)(e_{\bar{k}},\ e_{\bar{l}})e_{\bar{m}}-\varphi(e_{\bar{i}},\ e_{\bar{j}})F_4(e_{\bar{k}},\ e_{\bar{l}})e_{\bar{m}},$$

where $\bar{i}$ is an index i or an index i^*, we have mutually distinct indices i, j that $S_{ii^*ii^*j}=c'_{ii}e_{j^*}=0$, and hence

(21) $$c'_{ii}=0,\ (\forall i).$$

Similarly, for mutually distinct indices i, j, we have that

$S_{ii^*ii^*i}=(-a'_{ii}-b'_{ii}+c'_{ii})e_{i^*}=0$, $(\forall i)$, which by relation (21) becomes

(22) $$a'_{ii}+b'_{ii}=0,\ (\forall i).$$

Next, since $S_{ii^*ijj}=0$, it follows for different indices i and j that

(23) $$a_{ij}=0.$$

Next, for distinct indices i and j, $S_{ii^*iji^*}=0$ implies that

(24) $$b_{ij}=0.$$

Also, we have that $S_{ii^*ij^*i}=0$ implies for different i and j that

(25) $$a'_{ij}=c'_{ij}=0.$$

Finally, from $S_{ii\,ij\,i}=0$, it follows for different i and j that

(26) $$b'_{ij}=c'_{ij}=0.$$

Relations (21)—(26) show that $F_4=0$ on M^n.

LEMMA 6. *Let M^n be a complex hypersurface of a complex space form $M^{n+1}(c)$, $(n\geqslant 2)$. If E_4 is a 4th order covariant curvature tensor which satisfies* (18) *and* (19), *and B is the Bochner curvature tensor, then $E_4\cdot B=\varphi B$ if and only if $B=0$.*

PROOF. Denoting

$$S_{\bar{i}\bar{j}\bar{k}\bar{j}\bar{m}}=(E_4(e_{\bar{i}},\ e_{\bar{j}})\cdot B)(e_{\bar{k}},\ e_{\bar{l}})e_{\bar{m}}-(e_{\bar{i}},\ e_{\bar{j}})B(e_{\bar{k}},\ e_{\bar{l}})e_{\bar{m}},$$

we have for different indices i and j that $S_{ii^*ii^*j}=(k_i+\omega\mu_j)e_{j^*}=0$, and thus:

(27) $$k_i+\omega\mu_j=0.$$

Similarly, for mutually distinct indices i and j, from $S_{ii^*ijj}=0$, it follows that

(28) $$\beta_{ij}=0.$$

and moreover $S_{ii^*ij^*i}=0$ implies that

(29) $$\bar{\beta}_{ij}=0.$$

Finally, from $S_{ii^*ii^*i}=(2\overline{\beta}_{ii}-k_i-\omega\mu_i)e_{i^*}=0$, it follows for different indices i and j that

$$2\overline{\beta}_{ii}-k_i-\omega\mu_i=0. \tag{30}$$

From relations (27)—(30) we obtain that $B=0$.

LEMMA 7. *Let M^n be a complex hypersurface of a complex space form $\widetilde{M}^{n+1}(c)$, $(n\geqslant 2)$. If E_4 is a 4th order covariant curvature tensor which satisfies* (18) *and* (19) *and B is the Bochner curvature tensor, then $B\cdot E_4=\varphi E_4$ if and only if $E_4=0$.*

PROOF. Denoting

$$S_{\bar{i}\bar{j}\bar{k}\bar{l}\bar{m}}=(E_4(e_{\bar{i}},\ e_{\bar{j}})\cdot B)(e_{\bar{k}},\ e_{\bar{l}})e_{\bar{m}}-\varphi(e_{\bar{i}},\ e_{\bar{j}})B(e_{\bar{k}},\ e_{\bar{l}})e_{\bar{m}},$$

we have for different indices i and j that $S_{ii^*ii^*j}=c'_{ii}e_{j^*}=0$, and hence

$$c'_{ii}=0. \tag{31}$$

Also, from $S_{ii^*ii^*i}=(-\tilde{a}'_{ii}-\tilde{b}'_{ii}+\tilde{c}'_{ii})e_{i^*}=0$, using relation (31) we have

$$a'_{ii}+b'_{ii}=0. \tag{32}$$

Next, for distinct indices i and j, from $S_{ii^*ijj}=0$, it follows that

$$a_{ij}=0. \tag{33}$$

Similarly, for mutually distinct indices i and j, we have from $S_{ii^*iji^*}=0$, that

$$b_{ij}=0. \tag{34}$$

Next, for $S_{ii^*ij^*i}=0$, we have for different indices i and j that

$$a'_{ij}=c'_{ij}=0. \tag{35}$$

Finally, from $S_{ii^*ij^*j}=0$, it follows for different indices i and j, that

$$b'_{ij}=c'_{ij}=0. \tag{36}$$

From relations (31)—(36) we have that $E_4=0$ on M^n.

The proof of Theorem 1 follows at once from Lemma 5 and Lemma 7, using [MPT] (Theorem 3.1 and Theorem 3.2) together with the fact that $Z=0$ characterizes the Riemannian manifolds of constant sectional curvature, which for complex hypersurfaces implies that the hypersurface must be flat.

PROOF OF THEOREM 2. The proofs concerning the cases (i), (ii) and (iii) follow immediately from Lemma 6.

So, it remains to prove that (iv)$\Rightarrow$(v).

Denoting by

$$S_{\bar{i}\bar{j}\bar{k}\bar{l}\bar{m}}=(B(e_{\bar{i}},\ e_{\bar{j}})\cdot B)(e_{\bar{k}},\ e_{\bar{l}})e_{\bar{m}}-\varphi(e_{\bar{i}},\ e_{\bar{j}})B(e_{\bar{k}},\ e_{\bar{l}})e_{\bar{m}},$$

we have for different indices i and j that $S_{ii^*ii^*j}=-(k_i+\omega\mu_j)e_{j^*}=0$, and hence

$$k_i+\omega\mu_j=0. \tag{37}$$

Similarly, for mutually distinct indices i and j, we have that $S_{ii^*ii^*i}=(2\bar{\beta}_{ii}-k_i-\omega\mu_i)e_{i^*}=0$, which implies that

$$2\bar{\beta}_{ii}-k_i-\omega\mu_i=0. \tag{38}$$

Also, from $S_{ii^*ijj}=0$, using relations (37) and (38), we have for different indices i and j that

$$\beta_{ij}=0. \tag{39}$$

Finally, from $S_{ii^*ij^*i}=0$, using relations (37) and (38), we have that for distinct indices i and j

$$\bar{\beta}_{ij}=0. \tag{40}$$

Relations (37)—(40) show that $B=0$ on M^n.

To prove Theorems 3 and 4, we first prove the following two lemmas.

LEMMA 8. *Let M^n be a complex hypersurface of a complex space form $\tilde{M}^{n+1}$* (c), $(n\geqslant 2)$. *If E_4 is a 4th order covariant curvature tensor satisfying conditions* (18) *and* (19), *and D_2 is a 2nd order curvature tensor satisfing conditions* (20), *then $E_4\cdot D_2=\varphi D_2$ if and only if $D_2=0$.*

PROOF. Assume that $S=E_4\cdot D_2-\varphi D_2=0$. Then for mutually distinct indices i and j we have that $S_{ii^*j}=d_je_j=0$, and hence

$$d_j=0, \quad (\forall j). \tag{41}$$

This completes the proof.

LEMMA 9. *Let M^n be a complex hypersurface of a complex space form $\tilde{M}^{n+1}$* (*c*), $(n\geqslant 2)$. *If D_2 is a 2nd order curvature tensor satisfying conditions* (20) *and B is the Bochner curvature tensor, then $B\cdot D_2=\varphi D_2$ if and only if $D_2=0$.*

PROOF. Assume that $S=B\cdot D_2-\varphi D_2=0$. Then for mutually distinct indices i and j we have that $S_{ii^*j}=d_je_j=0$, which implies that $d_j=0$, and thus $D_2=0$.

Now, we are able to prove Theorems 3 and 4.

PROOF OF THEOREM 3. It is clear that by Theorem 3.1 from [MPT] that condition (v) implies all other statements.

Conversely, by Lemmas 8 and 9 and the same result from [MPT], it follows that M^n is a hyperplane in C^{n+1}. This completes the proof of this theorem.

PROOF OF THEOREM 4. If M is Einstein, then clearly all conditions (i)—(iv) are satisfied.

The converse statement is immediate by Lemmas 8 and 9.

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