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Detectability of Subsurface Thin Layer by Electromagnetic Sounding Systems

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Abstract: An analysis is made for the relative effectiveness in detecting a subsurface thin layer by four electromagnetic depth sounding systems; horizontal coplanar loops, perpendicular loops, vertical coplanar loops and vertical coaxial loops. The moduli and phases of mutual coupling ratios over a three-layered earth for the four systems are evaluated rapidly by the related convolution technique. Root mean square differences between the responses from the three-layered and the homogeneous earths are used to compare the relative effectiveness of the systems quantitatively. Comparing the all systems, it is found that the perpendicular loop system appears to be the most superior to the other systems.

INTRODUCTION

Electromagnetic (EM) depth sounding method is now an important tool in investigating the layered earth structures. Major applications are in foundation engineering and in explorations of petroleum, ground water and geothermal resources. In the EM sounding, loops of wire are used most frequently for generating the EM fields. A small current-carrying loop of wire generates the magnetic field which cannot be distinguished from that caused by a dipole magnet if the field is observed at a distance of about five times the diameter of loop. Theoretical curves and field data are usually given in terms of the mutual coupling ratio (Z/Z_0). Considerable works have been done for determining the EM fields over a layered earth (e.g., Frishknecht, 1967; Koefoed et al., 1972).

The detection of subsurface thin layers is a major problem in sounding methods, whether it would be DC resistivity, EM, magnetotelluric or even seismic method. Primarily, the low-frequency portion of primary excitation will favour the detectability of deeper layers, while top layer will be resolved by the high-frequency energy. In EM soundings, the thickness of target layer, its resistivity, the frequency content in the primary source, the source-receiver separation and the source-receiver system are equally important as detectability parameters (Patra and Mallick, 1980).

Verma (1977) used the linear digital filter method for computing Z/Z_0 on the surface of a three-layered earth to study the detectability effect for the EM sounding systems. However, he showed only the modulus of Z/Z_0 for the three-layered earth to compare the relative effectivencess of the EM sounding systems. In this paper, the root mean square (RMS) difference proposed by Patra and Mallick (1980) and the phase information of Z/Z_0 are introduced to make clear the relative effectiveness of the systems. The systems considered here are horizontal coplanar loops (Ⅰ), perpendicular loops (Ⅱ), vertical coplanar loops (Ⅲ) and vertical coaxial loops (Ⅳ), respectively, as shown Fig. 1. The Z/Z_0 for a three-layered earth, which contains a intermediate thin conductive or resistive layer, is evaluated by the Anderson's related convolution method (Anderson, 1979).

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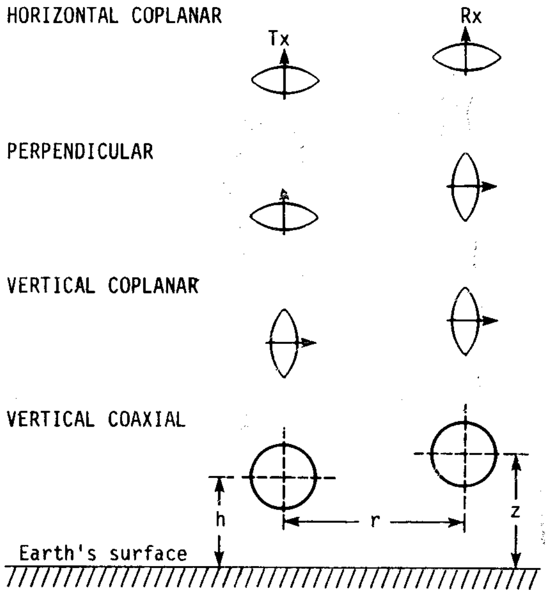

Fig. 1 Common loop configurations considered in this study.

MUTUAL COUPLING RATIO

The mutual coupling ratios for the four systems are given as (Frischknecht, 1967)

1) horizontal coplanar loops:

$$(Z/Z_0)_{\mathrm{I}}=1+B^3T_0,$$

2) perpendicular loops:

$$(Z/Z_0)_{\mathrm{II}}=B^3T_1,$$

3) vertical coplanar loops:

$$(Z/Z_0)_{\mathrm{III}}=1+B^2T_2,$$

and

4) vertical coaxial loops:

$$(Z/Z_0)_{\mathrm{IV}}=1+B^2(T_2-BT_0)/2. \qquad (1)$$

Here

$$T_0(A,B)=\int_0^\infty[R(d,g)g^2e^{-gA}]J_0(gB)dg,$$

$$T_1(A,B)=\int_0^\infty[R(d,g)g^2e^{-gA}]J_1(gB)dg,$$

and

$$T_2(A,B)=\int_0^\infty[R(d,g)ge^{-gA}]J_1(gB)dg, \qquad (2)$$

where J_0 and J_1 denote Bessel functions of order 0 and 1, respectively, and $R(d,g)$ is a recursively defined complex function, expressed in terms of the M-layered ($M>1$) model parameters, of the form

$$R(d,g)=(V_1F_1-g)/(V_1F_1+g). \qquad (3)$$

The V_1 and F_1 in (3) given by backward recurrence using

$$V_m=\sqrt{g^2+2i\sigma_m/\sigma_1}, \qquad (4)$$

$$m=M,M-1,\cdots,1;\ i=\sqrt{-1},$$

$$\begin{aligned}F_{m-1}=&[(V_{m-1}+V_mF_m)\\&-(V_{m-1}-V_mF_m)\exp(-V_{m-1}d_{m-1})]\\&/[(V_{m-1}+V_mF_m)\\&+(V_{m-1}-V_mF_m)\exp(-V_{m-1}d_{m-1})], \qquad (5)\end{aligned}$$

$$m=M,M-1,\ldots,2;\ F_M=1,$$

where

σ_m=conductivity (mho/m) of layer m,
$d_m=2t_m/\delta$,
t_m=thickness (m) of layer m ($t_M=\infty$),
$\delta=\sqrt{2/(\sigma_1\mu_0\omega)}$=skin depth,
$\mu_0=4\pi\times10^{-7}$(H/m), $\omega=2\pi f$, $f>0$ (Hz),
$B=r/\delta$, $r^2=x^2+y^2$,
$A=(z+h)/\delta$,
(x,y,z)=receiver loop center,

and

$(0,0,h)$=source loop center.

DETECTABILITY EFFECT

The EM response curves are computed with the following parameters:

$\rho_1=100\Omega\cdot$m, $\rho_2=5$ and $20,000\Omega\cdot$m,
$\rho_3=100\Omega\cdot$m, $d_1=100$m, and $d_2=10$m.

The EM curves for three-layered cases have been compared with that of homogeneous earth when the medium is considered to be homogenous with a resistivity of the top layer ($100\Omega\cdot$m).

Fig. 2 shows a complex representation of Z/Z_0 for the four systems with $A/B=1/5$, and all curves are drawn for 10^1 to 10^5 Hz. The solid curves indicate for the homogeneous earth and the broken curve is used as a reference to compare the three-layered earth with intermediate conductive layer. The EM responses for vertical coplanar loop and vertical coaxial loop systems

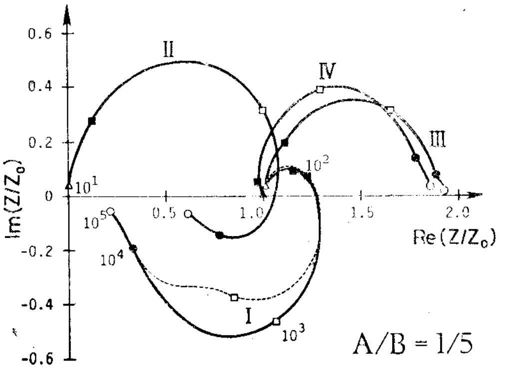

Fig. 2 Phasor diagrams of Z/Z_0 for the four sounding system with $A/B=1/5$. The solid and broken lines indicate the EM responses for the homogeneous earth and for the three-layer earth with intermediate conductive layer, respectively. The numeral values represent frequencies of primary source (Hz).

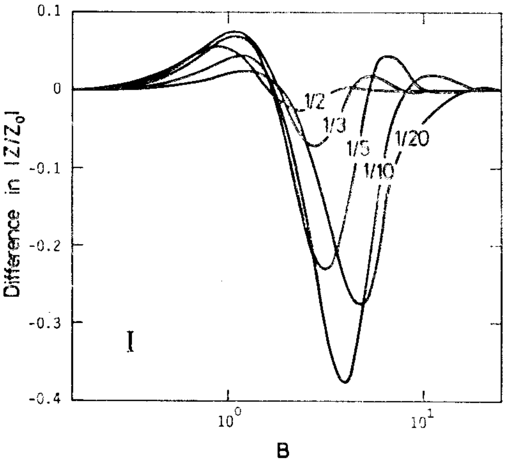

Fig. 3 Differences in $|Z/Z_0|$ between the conductive layer and the homogeneous cases for the horizontal coplanar loop system. The numeral values represent A/B.

have no negative value in $\mathrm{Im}(Z/Z_0)$ for the homogeneous case. This mean that these systems always have the phase range of 0° to 90°. Comparing the homogeneous and the three-layered cases in the horizontal coplanar loop system, an appreciable difference can be recognized in the frequency range of 10^2 to 10^4 Hz. Such difference can be regarded as "detectability effect".

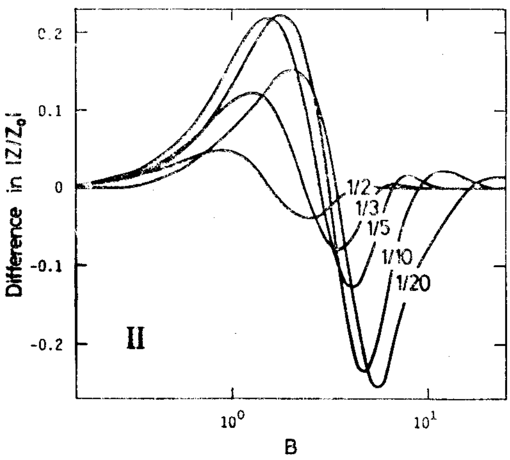

Fig. 4 Differences in $|Z/Z_0|$ between the conductive layer and the homogeneous cases for the perpendicular loop system. The numeral values represesent A/B.

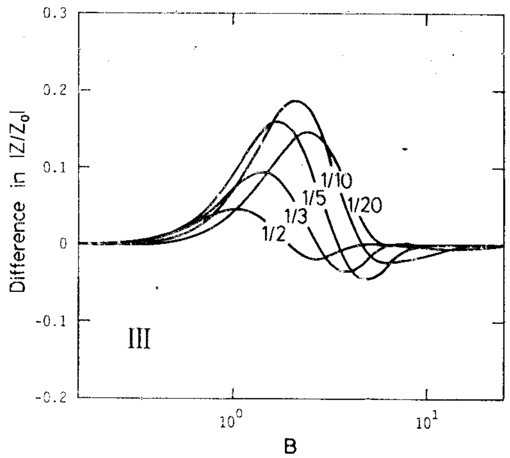

Fig. 5 Differences in $|Z/Z_0|$ between the conductive layer and the homogeneous cases for the vertical coplanar loop system. The numeral values represent A/B.

Fig. 3 to 6 show differences in $|Z/Z_0|$ between the conductive layer and the homogeneous cases for the four systems. In all systems, the detectability of thin conductive layer is most apparent when $A/B=1/10$. From Fig. 3 one can find that the portion of large negative values mainly contributes to the detectability of the horizontal coplanar loop system. The detectability effect of the perpendicular loop system is, however,

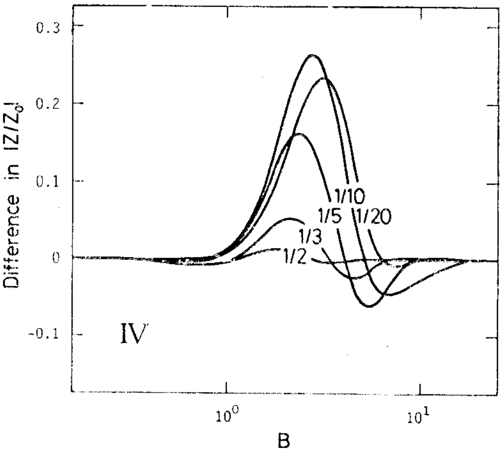

Fig. 6 Differences in $|Z/Z_0|$ between the conductive layer and the homogeneous cases for the vertical coaxial loop system. The numeral values represent A/B.

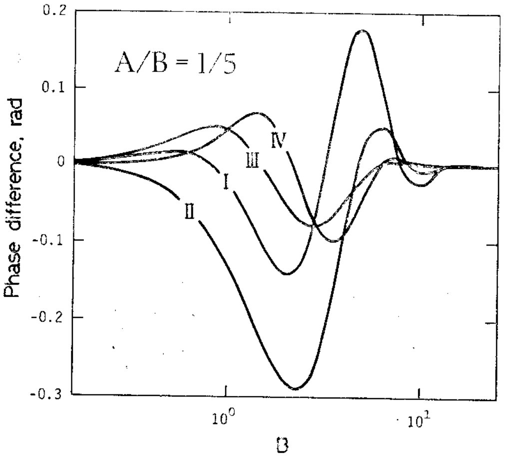

Fig. 7 Differences in phase of Z/Z_0 between the conductive layer and the homogeneous cases for the four system when $A/B=1/5$.

recognized by the both parts of large positive and large negative values (Fig. 4). When vertical coplanar loop and vertical coaxial loop systems are compared (Figs 5 and 6), it is relatively weaker in the coplanar loop system for larger coil separations (see curves of A/B= 1/10 and 1/20). The two vertical systems generally have weaker detectability effects than the horizontal coplanar loop and perpendcular loop systems.

Table 1 RMS differences for four EM sounding systems.

	Horizontal coplanar	Perpendicular	Vertical coplanar	Vertical coaxial
Modulus	0.112	0.139	0.094	0.080
Phase(°)	5.329	10.24	2.783	3.206

Fig. 7 shows a difference between the conductive layer and the homogeneous cases in phase of Z/Z_0 when A/B=1.5. The maximum phase difference occur in the horizontal coplanar loop system, and the minimum phase difference appear in the perpendicular loop system. The vertical coplanar loop and vertical coaxial loop systems have relatively weak responses in the phase of Z/Z_0.

In order to compare the detectability effect quantitatively, root mean square (RMS) difference proposed by Patra and Mallick (1980) is introduced in this paper. If $\Delta_1, \Delta_2, \ldots$ are the differences in the responses at frequencies f_1, $f_2, \ldots$, respectively, the RMS difference over N points is

$$\text{RMS difference}=\left[\frac{1}{N}\sum_{n=1}^{N}\Delta_n^2\right]^{1/2}. \qquad (6)$$

Table 1 shows the RMS differences for the four systems when A/B=1/5. In the evaluation of the RMS differences, 21 B values logarithmically spaced from 0.5 to 50 are used. From this table, one can find that the perpendicular loop system has the maximum detectability effect both in modulus and in phase, while the vertical coplanar loop and vertical coaxial loop systems have relatively weak responses.

Fig. 8 shows a difference in $|Z/Z_0|$ between the resistive layer and the homogeneous cases when A/B=1/5. As it would be expected, the resolution of intermediate resistivity layer is much weaker than that of intermediate conductive layer. Also in this case, the maximum difference occur in the horizontal coplanar loop system and the minimum difference appear in

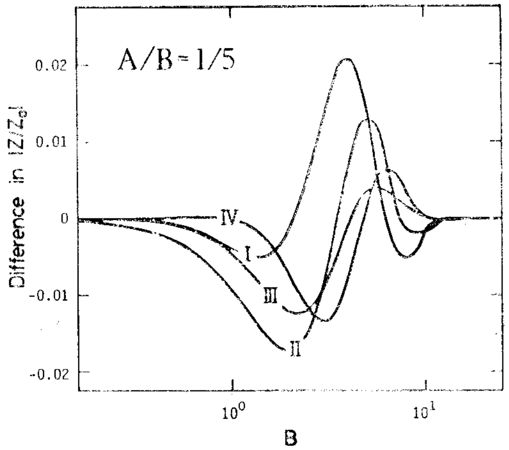

Fig. 8 Differences in $|Z/Z_0|$ between the resistive layer and the homogeneous cases for the four systems when A/B=1/5.

the perpendicular loop system, whereas the vertical loop systems have relatively weak responses.

DISCUSSION AND CONCLUSIONS

The kernel relations between T_0, T_1 and T_2 transforms in (2) are quite simple, except that $R(d, g)$ is involved when $M>1$. Two are identical and one is related by $1/g$. Therefore, the Anderson's related convolution method (Anderson, 1979) is very attractive in computing the EM responses, because all three transforms are rapidly obtained by the Anderson's filter in little more than the time needed to evaluate a single transform. Furthermore, once T_0, T_1 and T_2 are known, the mutual coupling ratio for any system is immediately given by (1).

When all systems are compared, it may be concluded that, for the detection of intermediate thin layer, the perpendicular loop system appears to be superior to the horizontal coplanar loop system; the detectability effects in the vertical coplanar loop and the vertical coaxial loop systems are comparable among themselves, but weaker than in the horizontal coplanar loop system.

The measurement error of EM responses is assumed to be of the order of 3% in amplitude and 3° in phase. Patra and Mallick (1980) defined the RMS difference of 10% in amplitude and 10° in phase as detectability level. This definition is arbitrary, but acceptable. On this basis, it may be concluded that the detectability of subsurface thin layer considered here ($d_1/d_2 = 10$) by EM soundings is generally poor in the conductive case, and hopeless in the resistive case. The perpendicular loop system alone has sufficient detectability of intermediate conductive layer both in amplitude and in phase (Table 1).

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전자탐사법의 각종 루프시스템에 의한 지하박층의 검색능력

김 희 준

요약：전자탐사법에서 대표적인 4가지 루프시스템인 수평공면 루프, 직교루프, 수직공면루프 및 수직공축 루프의 지하박층 검색에 대한 상대적인 효율에 관하여 검토하였다. 3층 구조에 대한 상호전자결합의 절대치와 위상은 디지탈 필터법을 이용한 상관중합법으로 구하였으며, 4가지 시스템의 상대적인 효율은 3층구조와 균질구조에 대한 전자응답간의 표준편차로 정량적으로 비교하였다. 모든 시스템을 종합적으로 비교검토한 결과, 지하박층의 검색능력은 직교루프 시스템이 가장 우수한 것으로 나타났다.