다년간 계속되는 갈수의 크기 및 심도에 관한 빈도분석 방안

An Approach for Frequency Analysis of Multiyear Drought Magnitude and Severity

이 길 성*
Lee, Kil Seong

요 지

부족량의 지속기간에 따른 성질을 관측하여, 다년간 계속되는 가뭄의 크기와 심도에 관한 빈도분석 방안을 개발하였다. 평균 부족량의 변화를 식별하기 위하여, 월 부족량의 통계치에 의한 표준화와 자료의 통합과정이 시행되었다. 가뭄의 크기와 심도에 관한 매개변수를 추정하기 위하여, Gamma 분포함수의 재생산성이 부족량에 대하여 적용되었다. 이들 분포와 지속기간 분포와의 복합화 및 연구 결과가 실시간 예측에 관하여 의미하는 바를 논의하였다.

Abstract

A frequency analysis procedure for the multi-year drought severity/magnitude is developed using observed duration-dependent deficit properties. A standardization of the deficit with the decimated monthly deficit statistics and a data pooling procedure are performed to identify the change of mean deficit. The reproductive properties of the Gamma family of distribution for the deficit are utilized to estimate the parameters of drought magnitude and severity. Compounding of these distributions with the duration distribution and an implication of the results for the realtime forecasting are discussed.

1. Introduction

In the study of multiyear drought one is faced with two important problems; the complexity of the events themselves and the insufficient amount of available data. A drought event is considered to be comprised of three components, namely severity, magnitude and duration. The frequency analysis of droughts is complicated in that each of these components may have its own frequency distribution. The lack of data problem results both from the relative brevity of most available streamflow records and from the multiyear character of dought events.

However, faced with the obstacle of insufficient data, the hydrologist may select from among at least four alternatives: (1) empirical methods or techniques of regional record combination (Dalrymple, 1960 ; Stall and Neill, 1961 ; Whipple, 1966 ; Campbell, 1971), (2) correlation or regression analysis(Huff and

* 정회원 · 서울대학교 공과대학 조교수, 토목공학과

Changnon, 1964 ; Benson and Matalas, 1967 ; Orsbon, 1974 ; Chang and Boyer, 1977), (3) simulation or synthetic streamflow generation (Fiering, 1964 ; Askew, Yeh and Hall, 1971 ; Carrigan, 1971 ; Millan and Yevjevich, 1971 ; Guerrero-Salazar and Yevjevich, 1975 ; Jackson, 1975), and (4) stochastic analysis (Gupta and Duckstein, 1975 ; Todorovic, 1978 ; Sen, 1977 and 1980). While synthetic streamflow generation would attempt to increase the drought sample size by representing the response of the watershed system at future times based upon the historical record, regional record combination attempts to increase the drought sample size by combining the response of different watersheds occuring during the same historical period.

In two previous papers (Dracup, Lee and Paulson, 1980 a, b), we found that the internal dependence of duration and magnitude are the least correlated parameters, whereas duration and severity are the most correlated parameters. We determined that two possible approaches can be taken: (1) determine the frequency of drought magnitude for each given duration, assuming the independence of these characteristics, and (2) determine the frequency of drought severity for each given duration, assuming dependence of these characteristics. But it should be noted that no correlation does not imply independence unless the distributions are Gaussian. In this paper the distribution of magnitude/severity of a drought event is investigated by examining the statistical properties of a surplus-deficit series from the historic inflow into the Oroville reservoir (Feather River, CA., 1906~1960).

2. Theoretical Frameworks

Let $\{X_i\}$ be the annual streamflow records for a given station and $\{Y_i\}$ be the deficit (or surplus) series from a truncation level. Then the severity and the magnitude of a drought (or high flow) event can be defined as the total deviation and the average deviation from a truncation level (say, the mean annual flow), respectively.

$$S_D = Y_1^D + Y_2^D + \cdots\cdots + Y_D^D \quad (1)$$

$$M_D = (Y_1^D + Y_2^D + \cdots\cdots + Y_D^D)/D = S_D/D \quad (2)$$

The duration and severity can be throught of the two primary characteristics of a drought event which depend on the properties of the deficit series and the magnitude can be thought of a secondary parameter which is a function of the severity and the duration. It should be noted that severity or magnitude is not just a random variable but random variable indexed by the random duration. The above definition of drought events eliminates the disadvantage in the running-totals method (Stall and Neill, 1961) for low flow analysis that a drought duration might include above normal years for short recurrence intervals or long durations.

For the frequency analysis of severity or magnitude based on an annual streamflow record, the following assumptions might be introduced to compute the central tendency and the variability under the sample size limitation. Suppose $Y_1^D, Y_2^D, \cdots, Y_D^D$ are independent and identically distributed (i.i.d.) random variables. Let D be a nonnegative integer valued random variable which is independent of $\{Y_D^1, Y_2^D, \cdots, Y_D^D\}$. Then, we can derive the following expressions.

$$E(S_D) = E[(D \cdot E(Y^D)] \text{ if identically distributed}$$
$$= E(D) \cdot E(Y) \text{ if independent with duration} \quad (3)$$

$$\mathrm{Var}(S_D) = E[\mathrm{Var}(S_D|D)] + \mathrm{Var}[E(S_D|D)]$$
$$= E[\mathrm{Var}(S_D|D)] + \mathrm{Var}[D \cdot E(Y^D)] \text{ if identically distributed}$$

$=E[D\cdot \mathrm{Var}(Y^D)]+\mathrm{Var}[D\cdot E(Y^D)]$
if i.i.d.
$=E(D)\cdot \mathrm{Var}(Y)+\mathrm{Var}(D)\cdot \{E(Y)\}^2$
if independent with duration (4)

Similarly, $E(M_D)=E[E(Y^D)]$ if identically distributed
$=E(Y)$ if independent with duration (5)

$\mathrm{Var}(M_D)=E[\mathrm{Var}(M_D|D)]+\mathrm{Var}[E(Y^D)]$
if identically distributed
$=E[\mathrm{Var}(Y^D)/D]+\mathrm{Var}[E(Y^D)]$ if i.i.d.
$=\mathrm{Var}(Y)\cdot E(1/D)$ if independent with duration (6)

Based on the above derivations, three observations follow. First, the frequency analysis of severity will require longer record than that of magnitude due to the variance term of duration in equation (4). For a reliable estimate of $\mathrm{Var}(D)$, more than 120 years of streamflow record will be needed in a typical situation unless the geometric or the exponential distribution is specified for the duration. Second, the assumptions in the derivation imply that duration and magnitude are not correlated since

$$\mathrm{Cov}(D,\ M_D)=E(S_D)-E(D)\cdot E(M_D)=0$$

but they are dependent. Therefore in any analysis which includes an assumption of independence between magnitude and duration is too restrictive. Third, the deviation Y^D interpreted as a random variable from the stratified random sampling (i.e. i.i.d. but dependent on duration) gives

$$E(Y^D)=\sum_d P_r\{Y^D\varepsilon Y^d\}\ E(Y^d), \qquad (7)$$

$$\mathrm{Var}(Y^D)=\sum_d P_r\{Y^D\varepsilon Y^d\}\{\mathrm{Var}(Y^d)+[E(Y^d)-E(Y^D)]^2\} \qquad (8)$$

For a given duration d, the above equations (3) to (6) reduce to the following equations since $E(d)=d$ and $\mathrm{Var}(d)=0$.

$E(S_d)=d\ E(M_d)$ by definition
$=d\ E(Y^d)$ if i.i.d. (9)

$\mathrm{Var}(S_d)=d^2\mathrm{Var}(M_d)$ by definition
$=d\ \mathrm{Var}(Y^d)$ if i.i.d. (10)

The last relationship in the equation (9) as well as the equation (8) hold without any assumption when the interpretation of both results is purely computational. The severity S_d has stationary and independent increments with the following property: as far as predicting the value of a random variable depending on the future S_d, $S_{d+1}, \cdots$ of the process (S_d) is concerned, all past information concerning $S_0=0, \cdots, S_{d-1}$ becomes worthless once the present value S_d is given. The magnitude M_d (as a sample mean) and the severity become approximately normal for any distribution of the deviation with a finite variance for long duration. The duration needed to achieve an approximately normal distribution depends on how nonnormal the deviation is.

Among the families of distributions which possess the reproductive property, the Gamma distribution is of particular use to describe the distributions; severity S_d, magnitude M_d and surplus Y^d. Let the surplus Y^d be the Gamma distributed random variable with the probability density function

$$f_y(y\ ;\ \lambda, k)=\frac{1}{\Gamma(k)}\lambda(\lambda y)^{k-1}e^{-\lambda y},\ \ y\geq 0 \qquad (11)$$

$$E(Y^d)=k/\lambda,\ \ \mathrm{Var}(Y^d)=k/\lambda^2, \gamma_1=2/\sqrt{k} \qquad (12)$$

Then, $\lambda_d=E(Y^d)/\mathrm{Var}(Y^d)$, $k_d=E^2(Y^d)/\mathrm{Var}(Y^d)$ and the γ_1 are the scaling, shape parameters and the skewness coefficient, respectively. Thus, the distribution of the severity S_d becomes $f_s(s_d\ ;\ \lambda, d\cdot k)=f_s(s\ ;\ \lambda, d\cdot k)$ and the distribution of the magnitude M_d becomes $f_M(m_d\ ;\ d\cdot\lambda,\ d\cdot k)=f_M(m\ ;\ d\cdot\lambda,\ d\cdot k)$ where the index d for the parameters λ and k is dropped for convenience. It should be noted that the exponential distribution for the surplus Y^d with $k=1$ and the scaling of the original series with λ will greatly simplify the distributions, especially for the right-skewed streamflow. The reproductive property applies to the distribution of sums of independent random variables whose distributions are in the same parametric family whether or not the distributions are identically distributed.

In the above theoretical development, several assumptions had been made for the analytical tractability. Specific questions about these assumptions are: (1) Are the random variables $\{Y_i^d\}$ mutually independent or uncorrelated? (2) Are the statistical properties of the deficit Y^d independent with the duration d? Or do not the parameters of the Gamma for the deficit vary with the duration? How can the sample size limitation be reduced without a regionalization or a simulation to test the assumptions?

3. Data Analysis

The proposed method of frequency analysis used the monthly flows instead of the annual flows for a study of multi-year durations to reduce the frequency curve irregularity resulting from the sample size limitation. It consists of a decimation of the monthly series to obtain twelve flow sequences by taking the monthly flow in successive years for each month on the premises that the average of the persistence(or the dependence) for each decimated monthly flow series is that of the annual flows. The operation simulates the regionalization obtained by combining concurrent annual streamflow records of twelve hypothetical stations located within a homogeneous region under a cyclically changing climate.

To devise an effective approach, the monthly variations of drought characteristics should be examined carefully. Monthly deficit statistics relative to those of the annual flow using the mean as a truncation level are shown in the Figure 1. The mean deficit has a cyclic variation but the C_v of deficit has a completely different structure. It not only underestimates the C_v of the annual flow deficit but also has two distinct peaks corresponding to the peak and the trough of the mean monthly streamflow. Figure 2 and Figure 3 show the proportion of deficit for the monthly streamflow and the three mean drought characteristics. The means of duration, magnitude and severity have cyclic variations but the proportion(or the probability) of deficit years overestimates that of the annual streamflow.

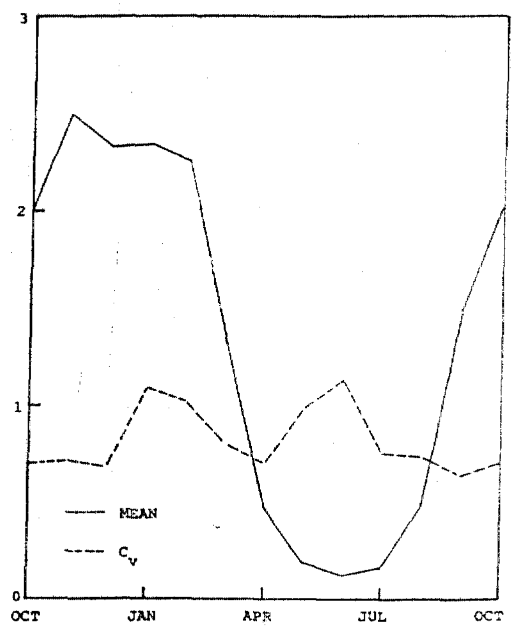

Fig. 1. Monthly Deficit Statistics Relative to those of the Annual Streamflow

These observations suggest that the standardization(or the linear cyclic transformation) should be based on the deficit statistics. The standardization with the deficit statistics not only normalizes the mean and the standard deviation of deficit but also normalizes the mean severity to zero due to the equation

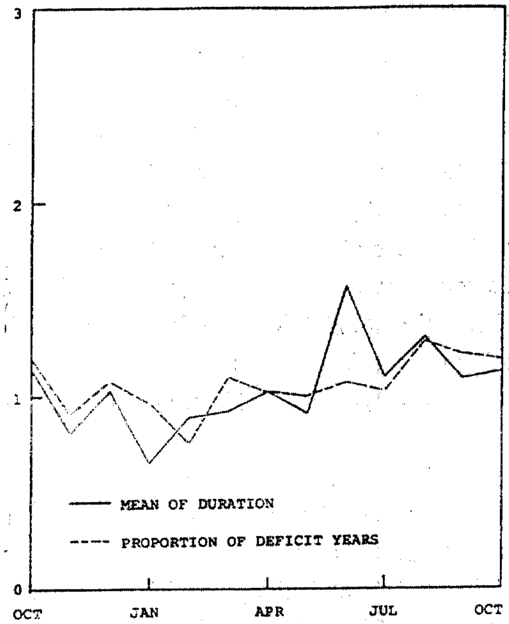

Fig. 2. Monthly Duration Statistics Relative to those of the Annual Streamflow

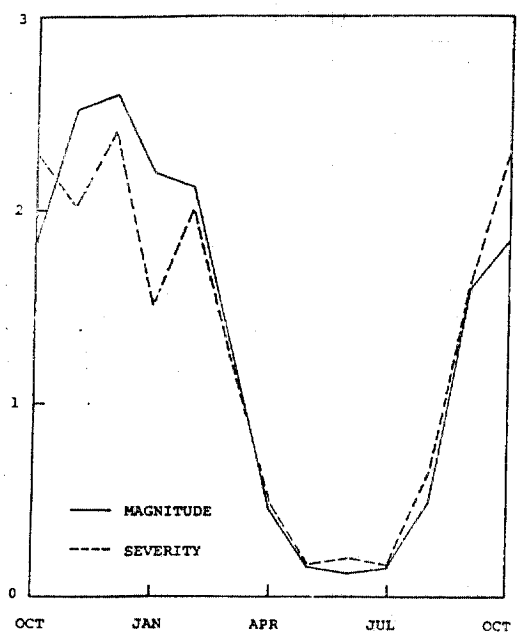

Fig. 3. Monthly Mean Drought Statistics Relative to those of the Annual Streamflow

(3). Thus, in order to transform the monthly decimated surplus-deficit series to the annual surplus-deficit series, the following operation is suggested:

$$\frac{Y^m - \mu_y^m}{\hat{\sigma}_y^m} \cdot \hat{\sigma}_y + \mu_y \tag{13}$$

where μ_y^m, $\hat{\sigma}_y^m$ = mean and standard deviation of the monthly surplus or deficit,

μ_y, $\hat{\sigma}_y$ = mean and standard deviation of the annual surplus or deficit.

For the transformed deficit series, the samples which violate the physical limitation of the range might be discarded before any further drought event study.

Figure 4 shows the duration variations of deficit statistics after the standardization and data pooling along with the corresponding numbers of event n_d and the sample sizes N_d where $N_d = d \cdot n_d$. Both mean and standard deviation are increasing but the coefficient of variation remains relatively constant, especially for short durations where the sample statistics are more reliable. If the Gamma distribution has been chosen for the deficit, this observation implies that the skewness (or the shape parameter k_d) to which the distribution is

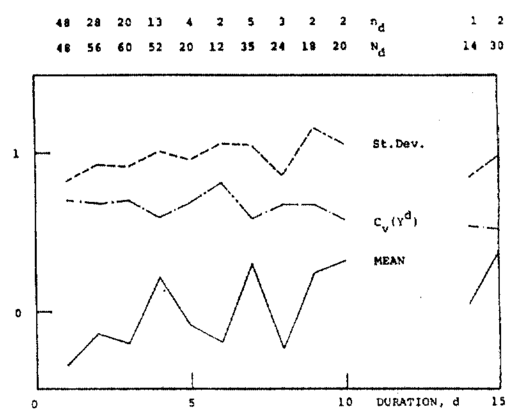

Fig. 4. Duration-dependent Deficit Statistics after Standardization and Data Pooling

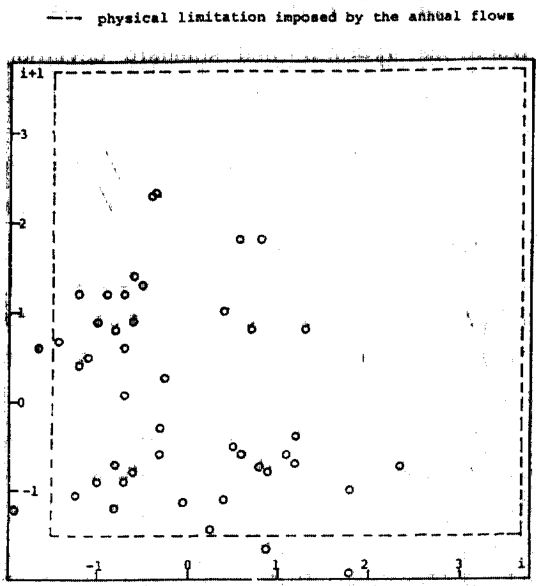

Fig. 5. Scattergram of the Standardized Mean Deficit of Drought

reproductive remains constant.

In order to see the serial dependence between successive deficits in a given event, the following standardization of the mean deficit is performed:

$$[E(Z_i^d)-\mu_z^d]\frac{\sqrt{n_d}}{\hat{\sigma}_z^d} \qquad (14)$$

where μ_z^d, $\hat{\sigma}_z^d$=mean and standard deviation of the standardized surplus or deficit Z^d. Figure 5 shows the scattergram of these standardized mean deficits and the physical limitation posed by the range of the annual flow deficit. First, the maximum mean deficit can not attain the limit because of the well sustained minimum dry year streamflow. Second, a severe drought year is unlikely to follow after a previous severe drought year. On the whole, there is no significant lag-one serial dependence between successive mean deficits.

4. Frequency Analysis

For the frequency analysis of drought severity or magnitude, a frequency of duration needs to be studied in addition to the analysis of deficits for a complete probabilistic description of multi-year drought events. Let the total number of drought event be $n=\sum_d n_d$ and the total number of drought years be $N=\sum_d N_d$ where $N_d=d\cdot n_d$. Then the following sample estimates can be defined:

$$P_r\{D=d\}=n_d/n,\ P_r\{Y^D\varepsilon Y^d\}=N_d/N, \qquad (15)$$

and $E(D)=N/n$.

Assuming the geometric distribution for the duration yields a constant failure probability $n/N=1/E(D)$ and gives explicit expressions for n_d and N_d in the equations (15).

$$P_r\{D=d\}=\frac{n}{N}\left(1-\frac{n}{N}\right)^d,\ P_r\{Y^D\varepsilon Y^d\}=\left(\frac{n}{N}\right)^2\left(1-\frac{n}{N}\right)^d\cdot d \qquad (16)$$

Thus, the geometric distribution replaces less reliable proportions with more reliable proportions but assumes a specific dependence between first two moments:

$$\mathrm{Var}(D)=E^2(D)-E(D).$$

To apply the previous data analysis procedure for the drought duration, several precautions should be exercised (Figure 6). First, the standardization and data pooling is a deficit orientated procedure in that the variance of duration has not received appropriate accounts. Thus, the 14 year and the 15 year duration events need to be deleted as outliers (if we use the Tschebycheff's estimate of maximum duration; Lee, Sadeghipour and Dracup, 1986) just like what has been done to the deficit which lies outside of the physical limitation. Second, the computation of the probability $P_r\{Y^D\varepsilon Y^d\}$ shows some abnormality at 5 and 6 year duration events which is not so apparent in the probability $P_r\{D=d\}$. The hazard function of duration is rather sensitive to these abnormalities and the frequency averaging procedure is preferred to the data pooling for the frequency analysis of drought duration. The Weibull distribution is probably the most widely used distribution for the hazard function.

$$F(d\,;\alpha,\beta,\delta)\doteq 1-\exp -\left(\frac{d-\delta}{\alpha-\delta}\right)^{\beta} \quad \alpha-\delta>0,\ \beta>0 \tag{17}$$

where $(\alpha-\delta)$ is called the scale parameter of the characteristic life, and the β is called the shape parameter or the Weibull slope. Then, the hazard function $h(d)$ becomes

$$h(d)=\frac{f(d)}{1-F(d)}=\frac{\beta(d-\delta)^{\beta-1}}{(\alpha-\delta)^{\beta}}, \quad d\geq\delta\geq 0 \tag{18}$$

where $E(D)=(\alpha-\delta)\Gamma(1+1/\beta)+1$, $\mathrm{Var}(D)=(\alpha-\delta)^2[\Gamma(1+2/\beta)-\Gamma^2(1+1/\beta)]$. This distribution gives a better approximation or an extension to the conventional approach in that it can have a nonconstant failure rate and it yields to the exponential distribution when $\beta=1$.

For a given records of the annual streamflow, how the conditional distribution of the random variables S_d and M_d can be modeled? Assuming i.i.d. deficits based on the scattergram(Figure 5), the equations(9) and (10) can be utilized for the distributions of severity and magnitude but the problem is how to estimate the variance of Y^d under the sample size limitation. Using the Gamma distribution with a constant shape parameter k based on the previous observation(Figure 4), the distribution of deficit Y^d for each duration can be scaled (divided) with $E(Y^d)$ to yield one common Gamma distribution $f_y(y\,;k,k)$ since

$$E(Y^d)=1,\quad \mathrm{Var}(Y^d)=1/k \tag{19}$$

Then the distribution of magnitude standardized(multiplied) with $d\cdot k/E(M_d)$ becomes the Gamma distribution $f_M(m\,;1,\ d\cdot k)$ such that

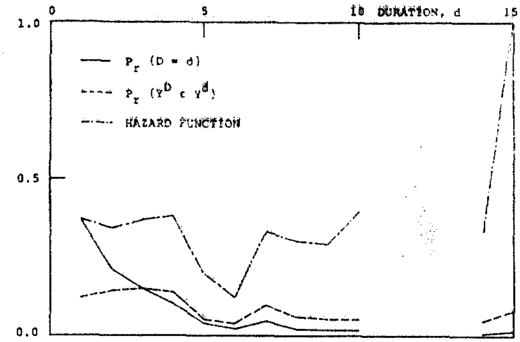

Fig. 6. Drought Duration Statistics after Standardization and Data Pooling

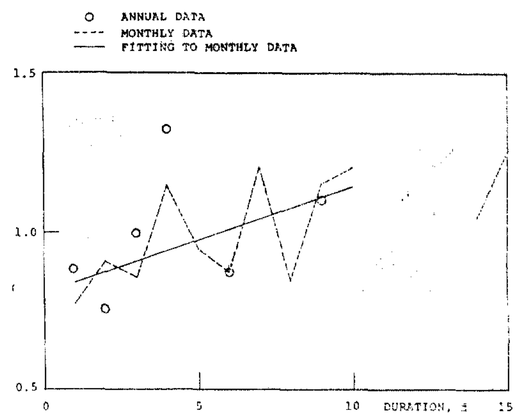

Fig. 7. Least-squares Fitting to the Scaled Mean Deficit of Drought

$$E(M_d)=\mathrm{Var}(M_d)=d\cdot k \tag{20}$$

and the distribution of severity standardized with $d\cdot k/E(S_d)$ becomes the Gamma distribution $f_s(s\,;1,\ d\cdot k)$ such that

$$E(S_d)=\mathrm{Var}(S_d)=d\cdot k \tag{21}$$

Thus, one additional advantage of using the Gamma distribution is that magnitude and severity converge to the same standardized Gamma distribution with k computed from the scaled deficit statistics. The coefficient of variation and the skewness coefficient of both scaled magnitude and severity are decreasing as the duration increases since

$$C_v=1/\sqrt{d\cdot k},\ \gamma_1=2/\sqrt{d\cdot k} \qquad (22)$$

The above procedure applied to the annual flow is restricted to the historical durations and gives no variability informations for duration which has only one observation. Figure 7 shows a simple least-squares fitting(which is unbiased but not efficient with the coefficient of determination $r^2=0.83$) to the scaled mean deficit after standardization and data pooilng up to 10 year duration. This duration-dependent procedure explains the reason why one cannot reproduce(underestimate) the historical severity or magnitude for long durations using duration-independent deficit properties(Askew, et. al., 1971 ; Sen, 1977) since the mean(and the standard deviation) of deficit tend to increase as the duration increases. The conditional reliabilities(probabilities of exceedance) for the 3, 4, 6 and 9 year droughts which had only one observation in the annual flow are approximately 0.35, 0.10, 0.65 and 0.50 respectively with the interpolation from the table of standard Gamma variates. It implies that the historical 4-year drought had unrepresentatively extreme severity and magnitude and that the 9-year drought for the Feather River, CA., which corresponds with the famous California six-year drought had extremely long duration(mean recurrence interval$\simeq$360 years) but had the severity a little less than the average.

With these conditional distributions of magnitude M_d and severity S_d from the deficit, one might determine the compound distributions of magnitude M_D and severity S_D if the distribution of duration has been identified. Although the distributions resulting from compounding may be difficult to determine analytically, owing to the integration or summation involved, the moments are more easily found from the equations (3) to (6). Note that the mean of S_D(or M_D) is just the weighted average of the means of S_d(or M_d) for the various values of d but the variance of S_D(or M_D) is not the weighted average of the variances of S_d(or M_d). Also, the durationd-ependent deficit properties have a significant implication for the real-time forecasting in that the mean and the variance of annual deficits tend to increase for an ongoing drought event. Since the probability of terminating that drought tends to decrease up to mean duration and then continues to increase, there exists a unique trade-off between the severity(or the magnitude) and the duration for a given drought event.

5. Conclusions

Instead of specifying a dependence structure for annual streamflows, the duration-dependent deficit characteristics are considered for the frequency analysis of multiyear drought severity and magnitude. The general shape of the time-dependent mean and variance of deficit was generated using a standardization and data pooling of the monthly decimated streamflows due to the severe sample size limitation present in the annual streamflow records. The reproductive property of the Gamma family of distributions is utilized for the deficit to describe the frequencies of multiyear drought magnitude and severity.

In summary, the proposed methodology of severity and magnitude frequency analysis of multiyear drought event consists of the following steps:

(1) Parameter estimation of the Gamma distribution for the ananul deficit by

scaling with the duration-dependent mean deficit for each duration,

(2) Standardization of the deficits for each month with the deficit statistics from the decimated monthly streamflow,

(3) Data pooling for each duration from the standardized data sets to smooth the duration-dependent scaled mean deficit,

(4) Frequency analysis of magnitude/severity from the standardized Gamma dsitribution of magnitude/severity,

(5) Compounding or moments estimation of magnitude/severity distribution with the distribution or moments of drought duration.

감사의 말

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