

埋入構造物과 層狀地盤上 構造物에 대한 地盤-構造物 相互作用의 單純解析

A Simplified Soil-Structure Interaction Analytical Technique of Embedded Structure and Structure on Layered Soil Sites

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要 旨

地震荷重에 대한 構造物의 動的 舉動은 地盤의 特性에 따라 현저한 差異를 나타내게 되는데 이러한 現象을 動的 地盤-構造物 相互作用이라고 한다. 地盤-構造物 相互作用의 解析方法은 크게 直接法과 部分構造法으로 구분되며, 이 중 部分構造法은 直接法에 비하여 解析方法은 간단하지만 埋入構造物이나 層狀地盤上 構造物에 대한 해석시 많은 制約을 받게 된다. 본 論文에서는 원천적으로 半無限彈性體地盤上 構造物에만 효과적으로 適用할 수 있는 部分構造法을 적절히 應用하여 埋入構造物 혹은 層狀地盤上 構造物에도 적용할 수 있는 方法을 提示하였으며, 直接法에 의한 해석프로그램인 FLUSH의 해석결과와 比較 檢討하여 그 妥當性を 立證하였다

Abstract

The dynamic behavior of a structure by earthquake is considerably affected by the flexibility of the base soil. This phenomenon is called dynamic soil-structure interaction effect. There are two broad categories of soil-structure interaction analytical technique: direct method and substructure method. Substructure method, in contrast to direct method, has many limitations in applying to embedded structures or structures on layered soil sites, while it is relatively simple. In this paper, a simplified soil-structure interaction analytical procedure using substructure method is proposed to eliminate its original limitations. The proposed method is well applicable to embedded structures or structures on layered soil sites with as good results as FLUSH.

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1. Introduction

The dynamic behavior of a structure by earthquake is considerably changed by the flexibility of the base soil. This phenomenon, called 'soil-structure interaction', takes the most important and sophisticated portion in seismic analysis of structures, specifically for critical structures such as nuclear power plants, offshore structures, hazardous gas storage facilities, etc.

There are two broad categories of soil structure interaction analytical technique: direct method and substructure method. The direct method analyzes the soil-structure system in a single step, while the substructure method treats the problem in a series of steps and the base soil is idealized as equivalent springs and dampers. Both methods have merits and demerits. The direct method theoretically can handle truly nonlinear problems, such as complicated basemat geometries and irregular base soil configurations, while it has many limitations in considering energy transmission and scattering due to its limited model boundaries. Specifically, practical application of direct method is very limited by the available memory size and execution time of computers. On the other hand, the substructure method can easily and reasonably treat 3-dimensional infinite characteristics of the ground, but it has many limitations and needs special treatments, which are not usually so simple, in order to effectively consider the effects of embedment, non-linearity and non-homogeneity of the base soil, and flexibility and geometrical irregularity of the basemat.

This paper proposes a simplified soil-structure interaction analytical procedure through parametric studies using a modified substructure method. Though the conventional substructure method has been effectively applied only

to surface structures on elastic half space ground because of aforementioned limitations and special treatments, the proposed method can also provide as good results as FLUSH⁽¹⁾ even for embedded structures or structures on layered soil sites.

2. Analytical Procedure

The analytical procedure of soil-structure interaction by substructure method may be divided into three basic steps as follows:

- Determination of the foundation input motion
- Determination of the impedance(or compliance) function, representing the force-displacement relationship of the base soil
- Analysis of the coupled soil-structure system by solving the appropriate equations of motion

2.1 Foundation Input Motion

When a ground motion is specified on free surface of the soil deposit, its corresponding motion at a point below the ground surface decreases in magnitude, unless the soil depth is unusually thick^(2,3). Thus the ground motion specified on a free surface can conservatively be applied at the bottom of embedded foundation without any modification⁽⁴⁾. This is a standard in engineering practice and also allowed by the nuclear regulatory agencies in America⁽⁵⁾.

In this study, without having to perform a complicated analysis to determine the foundation input motion, the specified free surface ground motion is directly used as the foundation input motion or partially modified, when needed, by incorporating the results of one-dimensional free field analysis by FLUSH.

2.2 Impedance Function

Impedance functions, representing the force-displacement relationship of base soil, have the values depending on the soil configuration and

material behavior, frequency characteristics of the excitation, and geometry of foundation⁽⁶⁾.

In this study, however, the frequency-independent impedance function⁽⁷⁾ which is derived for massless rigid foundation on elastic half space was basically used, whereas in such cases of embedded foundation and layered soil condition, they (spring constants and damping factors) were properly adjusted to give more reasonable analytical results.

2.3 Analysis of Equations of Motion

The equations of motion of a coupled soil-structure system are established by combining the results from the above first two steps. In general, the solutions for these equations of motion are efficiently obtained by frequency domain analysis, which accounts for frequency dependence of foundation impedance and seismic input motion.

In this study, time domain analysis by modal superposition technique was adopted, which is relatively simple and it gives reasonable results if modal damping values are appropriately selected. The modal damping values of soil-structure system were determined by Roesset's technique whose validity had already been confirmed^(8,9).

2.4 Verification of the Analytical Results

For the purpose of assuring the validities of the analytical results by the simplified sub-structure method proposed, various comparative studies were performed using the procedure of this study and the well-known computer program FLUSH.

During the comparative study, when remarkably different results between the two methods were found, the analyses were repeated by adjusting various input parameters until the results of this study approached to those of FLUSH within the limits acceptable for engineering purposes.

3. Analytical Model and Input Motion

For the verification study of this proposed method, a typical shear wall structure shown in Fig. 1 was selected as superstructure. Two soil conditions, elastic half space and horizontally layered soil deposits, were considered as representative base soil configurations, and three different material characteristics of the soil, as shown in Table 1, were also assumed for each soil condition. For the input motion of this study, a synthetic time history prepared by Burns & Roe, Inc., which is compatible with the standard response spectrum specified by U.S. NRC Reg. Guide 1.60, was used. As

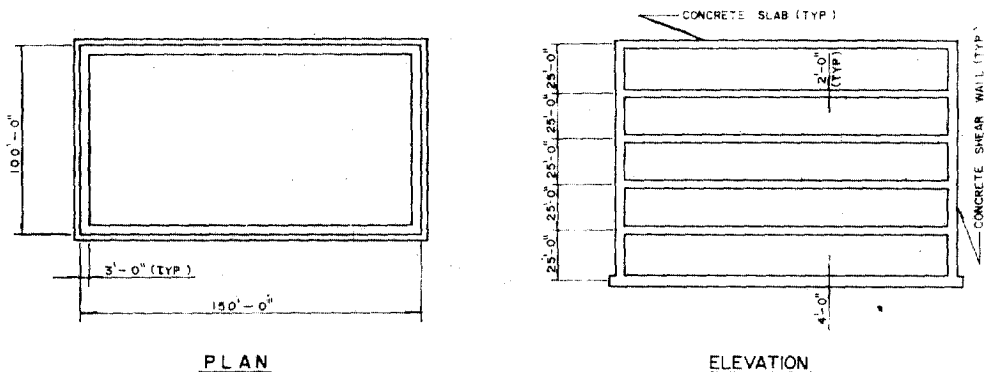
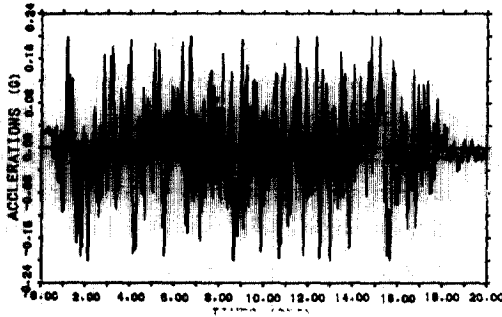


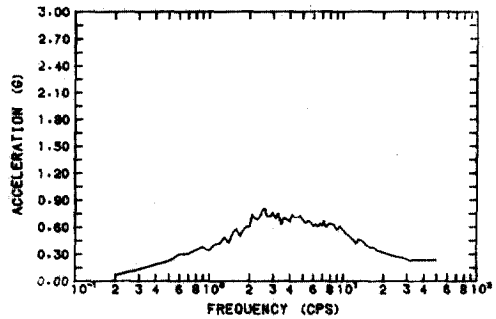
Fig. 1. Configuration of the Typical Shear Wall Structure

Table 1. Material Characteristics of Base Soil

	Soil Type 1	Soil Type 2	Soil Type 3
Unit Weight (pcf)	125.0	135.0	145.0
Poisson's Ratio	0.35	0.35	0.35
Material Damping Factor	0.07	0.05	0.04
Shear Modulus ⁽¹⁰⁾	$G=1,000K_2(\sigma'_m)^{1/2}$ (psf) where, K_2 : Shear Modulus Coefficient —Soil Type 1: 125 —Soil Type 2: 190 —Soil Type 3: 250 σ'_m : Mean Effective Stress(psf)		



(a) Time History



(b) Response Spectrum(4% Damping Value)

Fig. 2. Earthquake Input Motion for Verification Analysis

shown in Fig. 2, duration and peak acceleration are 20 seconds and 0.2g, respectively.

3.1 Modeling of Superstructure

For this study, the superstructure is idealized as a 2-dimensional lumped mass model composed of six mass points and five beam elements as shown in Fig. 3. Table 2 shows the properties of the lumped masses and the connecting beam elements of the model. In addition, a plane strain model composed of ten plane strain elements and seventeen beam elements as shown in Fig. 4 was separately prepared for the verification analysis by FLUSH.

The dynamic characteristics of the two different models are confirmed to be identical by

comparing the results of the modal analysis. As shown in Table 3, the values of the two models are well coincided with each other.

3.2 Modeling of Base Soil

For the analysis of this study, base soil was idealized as six frequency-independent springs and dampers. For the verification analysis by FLUSH, base soil was modelled as numbers of plane strain elements. Material nonlinearity of the elements was approximated by defining strain-dependent properties of shear modulus and damping ratio for each element as shown in Fig. 5. Lateral infinity of the base soil was simulated by applying the transmitting boundaries at the both sides of the model. 3-dimensional characteristics of the soil was also con-

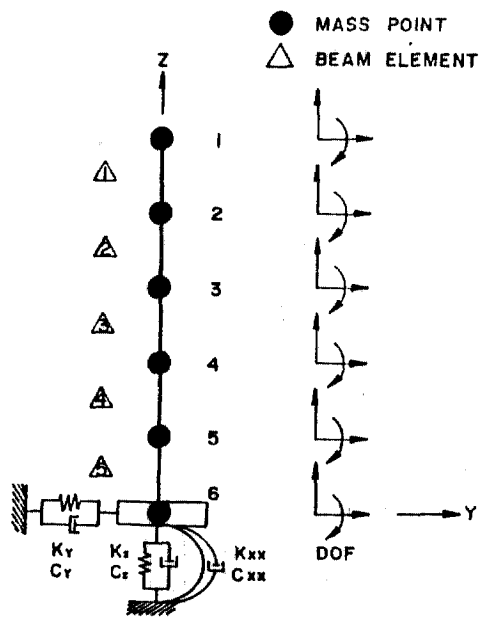


Fig. 3. 2-D Lumped Mass Model

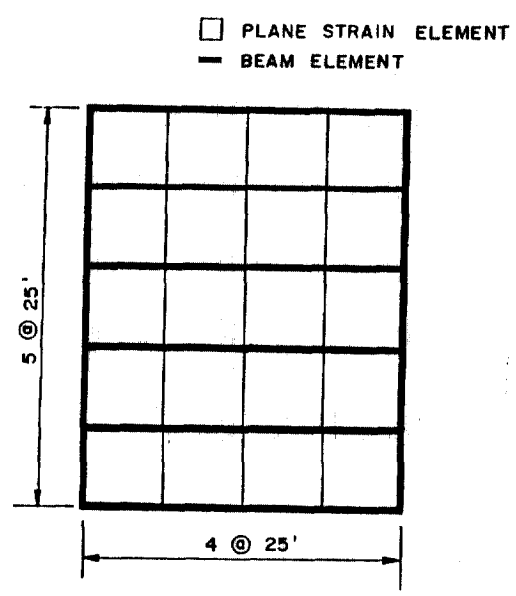


Fig. 4. 2-D Plane Strain Model

Table 2. Properties of 2-D Lumped Mass Model

Translational Mass(K-sec ² /ft)	Bottom Node	366.86
	Intermediate Nodes	314.44
	Top Node	227.11
Rotational Mass(K-ft-sec ²)	Bottom Node	3.98 × 10 ⁵
	Intermediate Nodes	4.46 × 10 ⁵
	Top Node	2.81 × 10 ⁵
Material Properties	Young's Modulus(ksf)	6.90 × 10 ⁵
	Poisson's Ratio	0.2
	Structural Damping Facotr	0.07
Sectional Properties	Axial Area(ft ²)	1,464.0
	Effective Shear Area(ft ²)	600.0
	Area Moment of Inertia(ft ⁴)	2.75 × 10 ⁶

Table 3. Comparison of the Results of Model Analysis for Two Different Superstructure Model

Mode Type	Lumped Mass Model			Plane Strain Model		
	Mode No.	Freq.(hz)	MP F	Mode No.	Freq.(hz)	MP F
Horizontal	1	6.14	-2.864	1	6.16	-2.848
	2	16.90	-1.173	3	16.93	-1.170
	4	26.66	0.377	4	25.84	-0.366
	5	31.93	-0.462	6	31.43	0.479
Vertical	3	17.30	-2.958	2	16.90	-2.879
				5	31.16	-0.662

* Note; MPF stands for Modal Participation Factor.

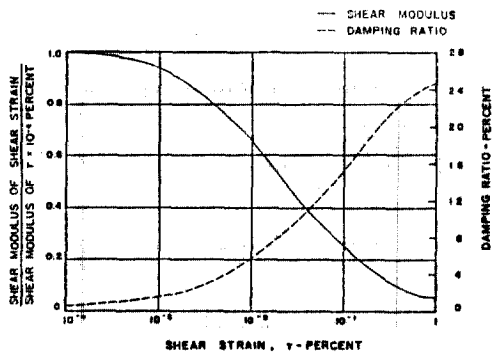
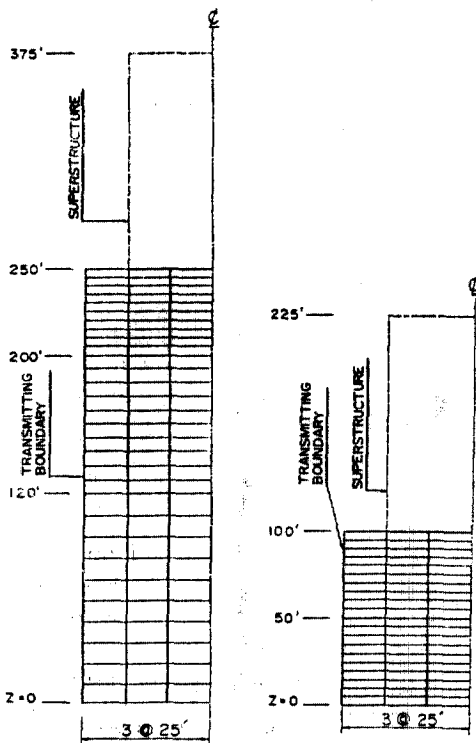


Fig. 5. Variation of Shear Modulus and Damping Ratio with Shear Modulus



(a) Elastic half space (b) Layered soil

Fig. 6. Typical FLUSH Model of the Base Soil

sidered by connecting the so-called viscous boundaries to the model which can simulate the radiational damping effect in the third direction. Fig. 6 shows the typical soil model for FLUSH analysis, which has been slightly

modified from case to case to represent the specific base soil characteristics or geometries.

4. Response Analysis

This section describes the details on the comparative study for the verification of the proposed analytical procedure. The study was performed for three different types of soil-structure system, surface structure, embedded structure, and structure on layered soil. The analytical models and input motion for the study have already been described in previous section.

4.1 Surface Structure

The first comparative study between the proposed method and FLUSH was performed for the surface structure on elastic half space which is the simplest case and most commonly encountered in practice. Table 4 and Fig. 7 show the comparison of in-structure responses for various soil types whose characteristics are tabulated in Table 1. As can be seen, the responses of this study show higher values than those of FLUSH, about 10% and 20% for peak accelerations and peak spectral accelerations, respectively. But the frequency characteristics of two results are very similar. It can be seen that these trends are common to all soil types considered.

4.2 Embedded Structure

For the embedded structure, spring constants and damping factors representing the surrounding soil characteristics become greater than those for surface structure. In general, they can be determined by multiplying the surface structure's values by some modification factors. In this study, modified spring constants and damping factors suggested by Novak^(11,12,13) are used, and the springs and dampers are assumed to be connected to structure at one-third point of the embedment depth from the foundation level⁽¹⁴⁾.

Table 4. Comparison of In-Structure Responses of Surface Structure

Soil Type	Location	Peak Acceleration(g)			Peak Spectral Acceleration				
		Proposed Procedure	FLUSH	Diff. (%)	Proposed Procedure		FLUSH		Diff. (%)
					Sa(g)	f(hz)	Sa(g)	f(hz)	
1	Roof	0.41	0.39	5.1	2.35	2.10	2.01	2.08	16.9
	3rd Floor	0.26	0.26	0	1.44	2.10	1.22	2.08	18.0
	Basemat	0.23	0.23	0	0.91	2.10	0.73	2.08	24.7
2	Roof	0.50	0.49	2.0	2.81	2.60	2.55	2.60	10.2
	3rd Floor	0.28	0.28	0	1.70	2.60	1.52	2.60	11.8
	Basemat	0.23	0.21	9.5	1.04	1.50	0.88	2.40	18.2
3	Roof	0.53	0.51	3.9	2.89	3.00	2.59	3.07	11.6
	3rd Floor	0.33	0.31	6.5	1.66	3.00	1.48	2.90	12.2
	Basemat	0.24	0.21	14.3	1.04	2.60	0.93	2.60	11.8

Table 5. Comparison of In-Structure Responses of 25ft Embedded Structure

Soil Type	Location	Peak Acceleration(g)			Peak Spectral Acceleration				
		Proposed Procedure	FLUSH	Diff. (%)	Proposed Procedure		FLUSH		Diff. (%)
					Sa(g)	f(hz)	Sa(g)	f(hz)	
1	Roof	0.33	0.33	0	1.80	2.59	1.36	1.60	32.4
	3rd Floor	0.23	0.22	4.5	1.09	2.50	0.94	2.60	16.0
	Basemat	0.19	0.19	0	0.81	2.60	0.68	2.60	19.1
2	Roof	0.43	0.34	26.5	2.06	3.00	1.35	3.07	52.6
	3rd Floor	0.28	0.23	21.7	1.25	3.00	0.97	2.60	28.9
	Basemat	0.22	0.19	15.8	0.89	2.60	0.76	2.60	17.1
3	Roof	0.42	0.35	20.0	2.16	3.80	1.50	4.80	44.0
	3rd Floor	0.28	0.25	12.0	1.28	3.00	0.95	2.60	34.7
	Basemat	0.22	0.19	15.8	0.89	2.60	0.78	2.60	14.1

The most sensitive parameters in the seismic responses of soil-structure systems are shear modulus of base soil and ground input motion. For shear modulus, since it varies with soil depth, it was necessary to determine a representative shear modulus to be used for this study. Through iterative parametric study, it was found that the shear modulus at the depth of one-tenth of foundation width below the foundation bottom line gives the most reasonable analytical results regardless of embedment depth. For ground input motion, it is also necessary to determine the specific ground

motion at the foundation level by an independent analysis, because the ground motion varies with soil depth. However, for simplifying the analysis, the foundation input motion was assumed to have the same frequency characteristics as the response spectrum of the ground surface motion, while its peak acceleration was the value at the foundation level obtained by the simplified one-dimensional free field analysis.

The comparisons of the two analytical results are shown in Table 5 and Fig. 8 for a 25ft embedded structure, in Table 6 and Fig.

Table 6. Comparison of In-Structure Responses of 50ft Embedded Structure

Soil Type	Location	Peak Acceleration(g)			Peak Spectral Acceleration				
		Proposed Procedure	FLUSH	Diff. (%)	Proposed Procedure		FLUSH		Diff. (%)
					Sa(g)	f(hz)	Sa(g)	f(hz)	
1	Roof	0.32	0.26	23.1	1.34	3.00	0.95	2.60	41.1
	3rd Floor	0.24	0.18	33.3	1.00	2.60	0.75	2.60	33.3
	Basemat	0.20	0.18	11.1	0.82	2.60	0.58	2.60	41.4
2	Roof	0.32	0.27	18.5	1.49	4.15	1.07	4.80	39.3
	3rd Floor	0.24	0.20	20.0	0.94	3.00	0.79	2.60	19.0
	Basemat	0.20	0.17	17.6	0.78	2.60	0.66	2.60	18.2
3	Roof	0.33	0.29	13.8	1.84	4.80	1.13	6.71	62.8
	3rd Floor	0.24	0.22	9.1	1.09	4.20	0.80	2.60	36.3
	Basemat	0.20	0.18	11.1	0.77	2.60	0.69	2.60	11.6

Table 7. Comparison of In-Structure Responses of 75ft Embedded Structure

Soil Type	Location	Peak Acceleration(g)			Peak Spectral Acceleration				
		Proposed Procedure	FLUSH	Diff. (%)	Proposed Procedure		FLUSH		Diff. (%)
					Sa(g)	f(hz)	Sa(g)	f(hz)	
1	Roof	0.29	0.21	38.1	1.10	2.60	0.79	2.60	39.2
	3rd Floor	0.23	0.17	35.3	0.91	1.60	0.64	2.60	42.2
	Basemat	0.21	0.18	16.7	0.82	2.60	0.50	2.08	64.0
2	Roof	0.31	0.21	47.6	1.38	4.80	0.79	2.60	74.7
	3rd Floor	0.23	0.18	27.8	0.91	4.20	0.71	2.60	28.2
	Basemat	0.20	0.17	17.6	0.79	2.60	0.56	2.08	41.1
3	Roof	0.34	0.23	47.8	1.65	4.80	0.82	4.80	101.2
	3rd Floor	0.24	0.19	26.3	1.01	4.80	0.74	2.60	36.5
	Basemat	0.20	0.18	11.1	0.78	2.60	0.60	2.60	30.0

9 for a 50ft embedded structure, in Table 7 and Fig. 10 for a 75ft embedded structure. Judging from the results above, the proposed procedure of this study gives 0~48% higher values than those of FLUSH analyses for the peak accelerations and 14~101% higher values for the peak spectral accelerations, which means that the results of the proposed procedures are always conservative though their ranges are variable. The fact that the differences of the two analytical results become larger at the higher locations of the structure is also observed.

Fig. 11 shows the ratios of in-structure peak spectral acceleration between the proposed analytical procedure and FLUSH at various levels of the structure for each embedment depth and each type of soil properties. As shown, the ratio becomes larger as the soil condition becomes stiffer and the embedment depth becomes larger. However, the case of 25ft embedded structure of soil type 2 shows a little different trend from the others. It also can be seen that in the softer soil, the differences at the lower location of Structure are larger than those at the higher location, whereas the trends are

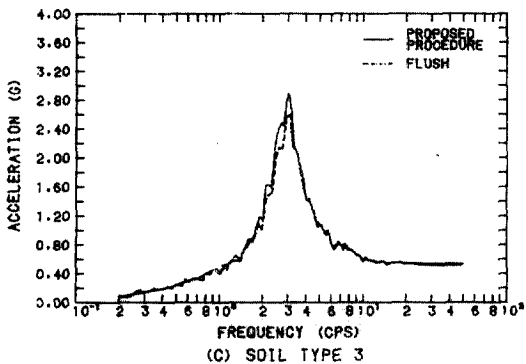
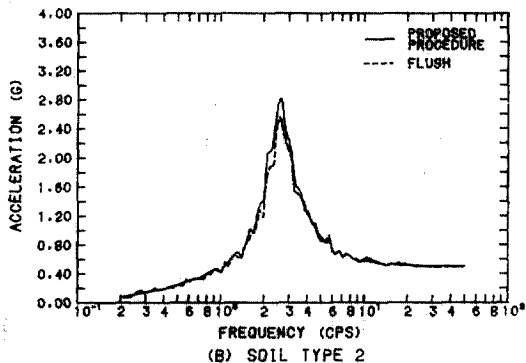
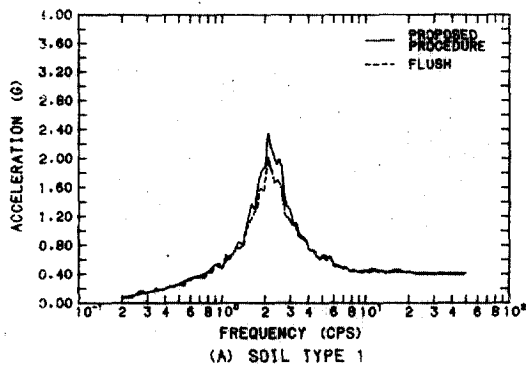


Fig. 7. Comparison of In-Structure Response Spectra of Surface Structure(Roof, 4% Damping Value)

vice versa in the stiffer soil.

4.3 Structure on Layered Soil Sites

In the case of layered soil sites, geometrical damping representing the radiational energy dissipation is reduced in comparison with the elastic half space because of the confinement effect of earthquake energy within each layer.

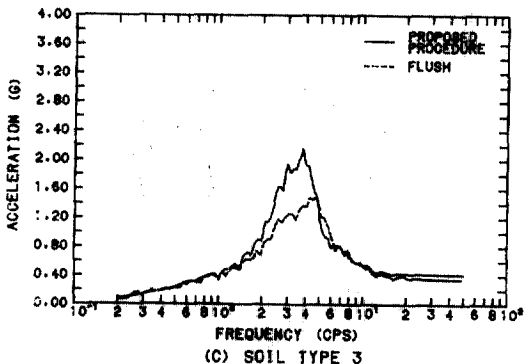
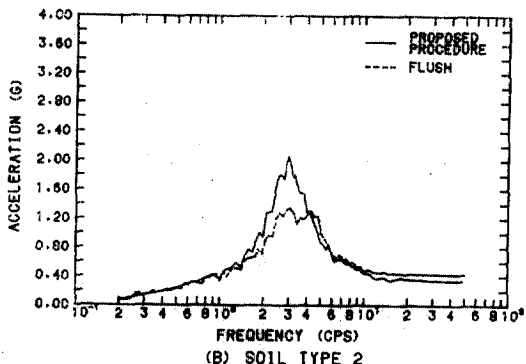
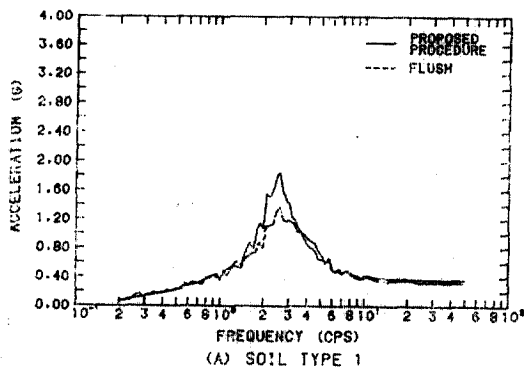
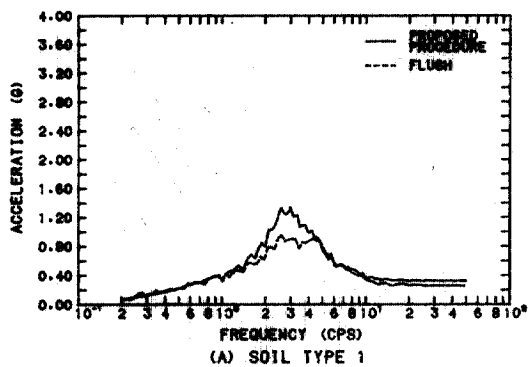
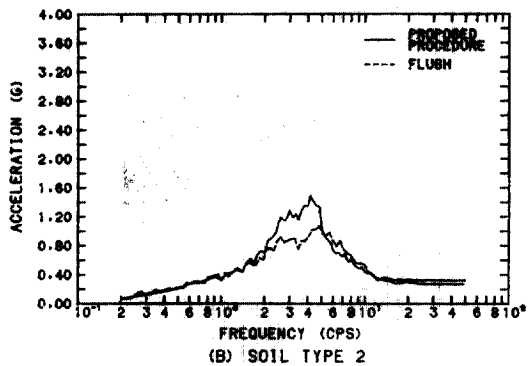


Fig. 8. Comparison of In-Structure Response Spectra of 25ft Embedded Structure(Roof, 4% Damping Value)

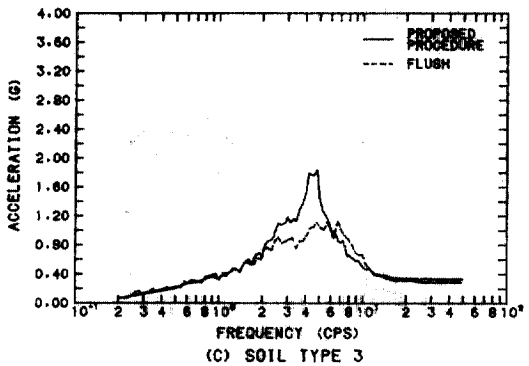
In this study, even for layered soil sites, simplified analysis is continued assuming the layered soil as an elastic half space having the soil properties of the top layer on the condition that modal damping values are appropriately reduced. And the most optimal reducing factors of modal damping values are deter-



(A) SOIL TYPE 1



(B) SOIL TYPE 2

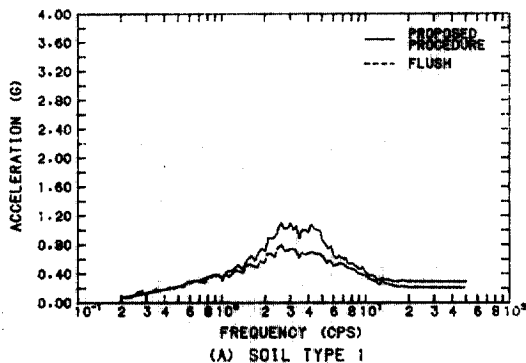


(C) SOIL TYPE 3

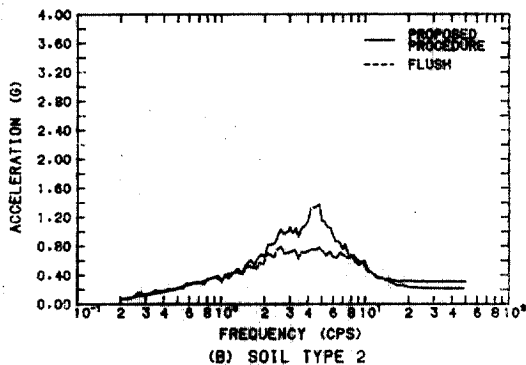
Fig. 9. Comparison of In-Structure Response Spectra of 50ft Embedded Structure(Roof, 4% Damping Value)

mined after several repetitive parametric studies by comparing the proposed analytical results with FLUSH results of the real layered soil conditions.

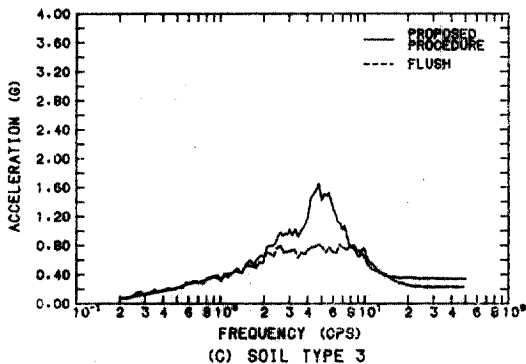
In order to illustrate how the proposed procedure is applicable to layered soil conditions, it was applied to two specific cases as follows:



(A) SOIL TYPE 1



(B) SOIL TYPE 2



(C) SOIL TYPE 3

Fig. 10. Comparison of In-Structure Response Spectra of 75ft Embedded Structure(Roof, 4% Damping Value)

- Single 100ft soil layer overlying bedrock which has the properties of soil type 1
- Two 50ft soil layers overlying bedrock which have the properties of soil type 1 and 2.

Fig. 12 shows the analytical results for the above two cases, and the response spectra of this study were obtained by using the modal

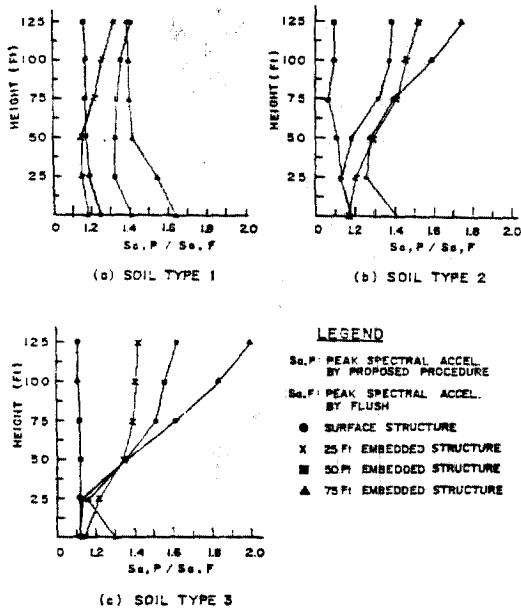


Fig. 11. Ratios of In-Structure Peak Spectral Acceleration Between the Proposed Procedure and FLUSH

damping values reduced by 24% and 33%, respectively. These reducing factors were evaluated by comparing with the FLUSH results.

Since it was confirmed that the responses by the proposed analytical procedure can give the same, at least similar, analytical result as FL

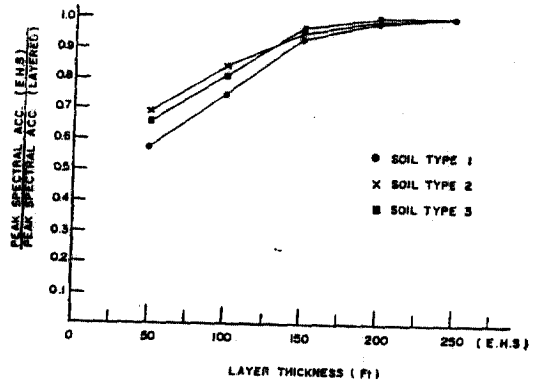


Fig. 13. Ratio of Peak Spectral Acceleration Between the Structure on Elastic Half Space and Structure on a Single Soil Layer

USH, only if modal damping values are appropriately reduced, similar analyses were continued varying both the layer thickness and soil stiffness to determine the corresponding reducing factors of modal damping values. For this purpose, single soil layer overlying bedrock were assumed. The layer thickness was varied from 50ft to 250ft and three soil types tabulated in Table 1 were considered for each case. The case of 250ft soil layer whose thickness was 2.5 times the foundation width was regarded as a elastic half space. Fig. 13 shows the

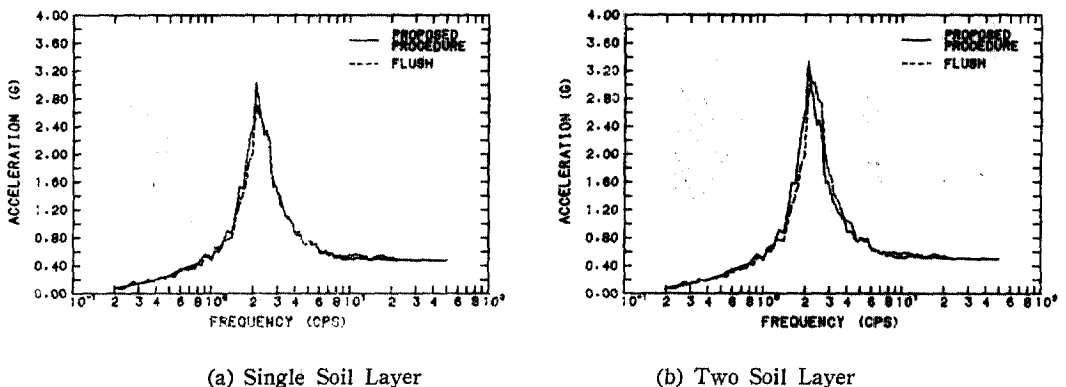


Fig. 12. Comparison of In-Structure Response Spectra of the Structure on Layered Soil Overlying Bedrock (Roof, 4% Damping Value)

ratios between the peak spectral accelerations for the structure on elastic half space and those for the structure on a single soil layer for each soil type. This implies that, when the soil-structure interaction analysis for the structure lying on layered soil sites is performed by the proposed procedure, reasonable results for engineering purposes can be obtained by reducing the modal damping values corresponding to the ratio given in Fig. 13.

5. Conclusion

Substructure method for soil-structure interaction analysis has many limitations in applying to the embedded structure or the structure on layered soil sites. In this paper, a simplified soil-structure interaction analytical procedure using substructure method is proposed, which is well applicable to the embedded structures or the structures on layered soil sites.

Several items to be kept in mind for the effective application of the proposed procedure are as follows:

- 1) For the surface structure on elastic half space, the proposed procedure is satisfactorily applicable without further modification.
- 2) When applying to embedded structures, the input motion which has the reduced peak acceleration corresponding to the foundation level but whose frequency characteristic is same as that of ground surface motion, gives the most reasonable results.
- 3) When applying to embedded structures, the proposed procedure gives too conservative results when the embedment depth becomes larger.
- 4) When applying to layered soil sites, calculated modal damping values shall be appropriately reduced prior to response analysis.

- 5) As for the shear modulus of base soil the value obtained at the depth of one-tenth of the foundation width below the foundation bottom line gives the most reasonable results.

The proposed procedure is validated only for some limited structure and base soil conditions. Therefore, further study is required to confirm that the proposed procedure can also be applicable without any limitations.

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