

# Mathematical Programming Approach for the Multiple Forest Land Use<sup>1</sup>

—Comparison between STEM and Constraint Method—

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## 多目的 山地利用을 위한 數理計劃法의 比較<sup>1</sup>

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### ABSTRACT

The idea of multiple-use of forest land is the one field of economics to improve the efficiency of forest land, and is the famous management technique widely used in the developed forestry country. This paper introduces the STEM and the constraint method, which is one kind of mathematical programming techniques used for multiple forest land use, and discusses the differences between these two methods by using the hypothetical data.

*Key words: multiple forest land use, mathematical programming, STEM, the constraint method.*

### 要 約

多目的 山地利用은 山地利用의 效率性을 提高하기 위한 經濟學의 한 응용분야로서, 外國에서 林業經營에 널리 使用하는 技法이다. 本稿에서는 多目的 經營을 위해 使用되는 數理計劃法의 일종인 STEM과 制約條件法을 林業分野에 도입 적용하여 가상자료에 의거 이들 方法間의 長·短點을 比較 檢討하였다.

#### 1. Introduction

Multiojective mathematical programming (MMP) is one way of considering multiple objectives explicitly and simultaneously in a mathematical programming framework.

The application of mathematical programming to forest land management planning has been limited mainly to the linear programming (LP) or/ and goal programming (GP) for the multiojective

problem.

Up to now, most of the literature dealing with mathematical techniques for multiojective forest land management planning is usually based on the use of GP(Arp and Lavigne<sup>3</sup>, 1982; Bare and Anholt<sup>4</sup>, 1976; Bell<sup>6,7</sup>, 1975, 1976; Dane et al.<sup>11</sup> 1977; Dress,<sup>13</sup> 1976; Dyer et al.<sup>15</sup> 1979; Field<sup>16,17</sup> 1973; 1978; Rustagi,<sup>22</sup> 1976; Walker,<sup>25</sup> 1984; Schuler et al.<sup>23</sup> 1977). GP is the methods for 'a priori' articulation of preference information given. Recently, however, questions about the ability of

<sup>1</sup> 接受 10月 21日 Received on October 21, 1987.

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GP to capture the multiobjective decision methods have been increased. The most intriguing of these questions are those described by Alvord<sup>2</sup> (1983), Cohon and Marks<sup>10</sup> (1975), Dyer et al.<sup>15</sup> (1979), Rosenthal<sup>21</sup> (1983) and Zeleny<sup>26</sup> (1981).

There are some reasons for the increasing interest in MMP. The main reason is that most decision problems are inherently multiobjective and computing technology develop in last ten years. Even though a lot of MMP techniques have been developed in other fields, very limited MMP techniques have been used in forestry. Steur and Schuler<sup>24</sup> (1978), de Kluyver et al.<sup>12</sup> (1980), Allen<sup>1</sup> (1986), Harrison and Rosenthal<sup>18</sup> (1986) and Bare and Mendoza<sup>5</sup> (1987) shows the remarkable application and demonstrates the possibility of multiobjective's use in forestry. They use the interactive technique, two stage technique, NIES and STEM, etc. Especially STEM is preferred because of its convenience as a simple, practical means of finding feasible solutions to multiple objective problem.

The purpose of this paper is to compare STEM method and the constraint method, and to show the difference between them in the multiple forest land use.

## 2. Methods for multiobjective mathematical programming

### 2.1. Multiobjective programming techniques

The MMP techniques for assisting a decision maker in the problem of selecting values for each of  $n$  decision variable,  $x = (x_1, x_2, \dots, x_n)$ , in order to optimize  $p$  ( $p \geq 2$ ) objective functions of these decision variables:  $Z_1(x), Z_2(x), \dots, Z_p(x)$ . We assume that the decision maker want to maximize each of the objective functions simultaneously without loss of generality, subject to numerous constraints on the decision variables, expressed as  $x \in F_d$ . The mathematical expression is:

$$\text{maximize } Z(x) = [Z_1(x), \dots, Z_p(x)] \quad (1)$$

$$\text{st } x \in F_d, x \geq 0 \quad (2)$$

A solution which maximizes each of the objective functions simultaneously.

Hwang and Masud<sup>19</sup> (1979) identify solution techniques for multiobjective mathematical programs according to the requirement for preference information versus the optimization. The methods are classified according to the pure approaches of articulation of the decision maker's preference structure:

1. prior to the optimization (prior articulation of preferences)
2. during, or in sequence with, the optimization (progressive articulation of preference), and
3. after the optimization (a posteriori articulation of preferences).

Techniques which employ a prior articulation of preferences can involve the explicit use of a value function as in the direct assessment and maximization of the decision maker's value function, or the implicit use of a value function as in a goal programming. 'A priori' means that the preference information is given to the analyst before he actually solves the problem. And the decision maker provides the information during or after the actual mathematical formulation of the problem. The major disadvantage of such prior approaches is the required preference information. However, the optimization parts of these approaches are relatively trivial.

MMP algorithms which require a progressive articulation of preferences often involve an interactive decisionmaker/computer approach. As in the case of prior articulation of preferences, the emphasis here is on finding one best-compromise solution. As compared to techniques requiring a prior articulation of preferences, these techniques do not require preference information which is quite so difficult for the decision maker to provide. Among these techniques are the STEM (Benayoun et al., 1971), interactive goal programming (Dyer, 1972), the sequential multiobjective programming system (Monarchi et al., 1973) and the approach of Zojnts and Wallenius (1976).

Interactive approaches are based on a formal mechanism by which the decision maker interacts

either with the computer directly or with the analyst as intermediary. The methods operate in an interactive fashion by moving from one noninferior solution to another in direction defined by the decision maker. Termination occurs when the decision maker is satisfied or when further iterations cannot be handled by their solution procedure.

Techniques which require a posteriori articulation of preferences are concerned with finding all (or most) of the efficient solutions to a problem, presenting these solutions to a decision maker, and then having one solution chosen at the best one from this set of efficient solutions through some arbitrary process. This class of methods does not require any information or assumption for the decision maker's utility function. Among these techniques are the parametric method, constraint method, MOLP methods.

2.2 STEP method

The STEP method (STEM) is perhaps one of the first linear multiojective techniques developed to address MMP's via a progressive articulation of preferences. It was first described as the progressive orientation procedure (Benayoun and Tergny,<sup>9</sup> 1970) and later elaborated by Benayoun et al.<sup>8</sup> (1971). Referring to our "model" of progressive algorithms, STEM employs a single objective model which minimizes the maximum weighted distance (out of all of the problem's objective) from the ideal. The set of constraints for this single objective problem are identical to those of the original multi-objective problem at the first iteration. At each subsequent iteration, the decision maker is asked to adjust the feasible region by adjusting his aspiration levels for the objectives.

The methods begins with the construction of a payoff table which is found by optimizing each of the p objectives sequentially, where the solution to the kth such individual optimization, we obtain a solution (X<sub>k</sub>) which maximizes Z<sub>k</sub>. By definition the maximum value for the kth objective, which is called M<sub>k</sub>; i.e. Z<sub>k</sub>(X<sub>k</sub>) = M<sub>k</sub>. The values of the other p-1 objectives implied by X<sub>k</sub> are then evaluate at

X<sub>k</sub>. These values are shown in the kth row of the payoff table. The payoff table is used to develop weights on the distance of a solution from the ideal solution. The diagonal elements represents the ideal solution where the maximum value of each objective is realized. Following this, the STEP methods consists of a calculation and decision making phase.

The STEM have to find a compromise solution which is "nearest" to the ideal solution. This can be got by minimizing the distance (D\*) between the p objective function values and their respective maximum values M<sub>k</sub>. The basic problem in the STEM is

$$\text{minimize } D^* \tag{3}$$

$$\text{s. t.}$$

$$W_k [M_k - Z_k] - D^* \leq 0, k = 1, 2, \dots, p \tag{4}$$

$$X \in F^i, D^* \geq 0 \tag{5}$$

where F<sup>i</sup> is the feasible region at the ith iteration and D\* is used to indicate that the original metric has been modified.

The weights W<sub>k</sub> in (4) are defined as

$$W_k = N_k / \sum N_k \tag{6}$$

$$\text{where } N_k = \frac{[M_k - n_k]}{M_k} * [\sum (C_{kj})^2]^{-1/2} \tag{7}$$

Term 1                  Term 2

where n<sub>k</sub> is the minimum value of the kth objective found by finding the smallest cell in the kth column of the pay-off table and C<sub>kj</sub> is the coefficient for the jth decision variable in the kth objective function.

A serious drawback of the foregoing interactive procedure is that the weights W<sub>k</sub> are determined in a rather mechanical way on the basis of percentage discrepancies between minimum and maximum values of. A more satisfactory approach would have been accomplished, if these weights would reflect the decision maker's actual relative preferences.

From first term in (7) observe that if the minimum value n<sub>k</sub> does not vary much from the maximum value M<sub>k</sub>, the corresponding objective is not sensitive to a variation in the weighting values. The second term in (7) normalizes the values taken by the objective function so that the effect of scale is

mitigated. Therefore the  $N_k$  represent normalized weights for  $k$ th objectives which depend upon the variation of the minimum value of the objective from the ideal solution.

The solution of (3) – (5) with  $F^i$  in (5) yields a noninferior solution  $X(0)$ , which is closet, given the modified metric in (4), to the ideal solution. The decision maker is asked to evaluate this solution. If it is satisfactory, the methods terminates; and if it is not satisfactory, the decision maker specifies an amount by which objective  $k^*$  is at a more than satisfactory level. A problem with a new feasible region in decision space is then solved. A solution is feasible to the new problem, if and only if the following three conditions are satisfied.

$$Z_k^*(X) \geq Z_k^*(X^i) - Z_k^* \quad (8)$$

for all  $k^*$  satisfactory objectives

$$Z_k(X) \geq Z_k(X^i) \quad (9)$$

for all  $p-k^*$  unsatisfactory objectives

$$X \in F^i \quad (10)$$

For the new problem  $N_{k^*} = 0$ ,  $W_{k^*} = 0$ , and the other  $W_k$ s are recomputed from (6). The problem in (3)-(5) is then resolved with  $i = i+1$ , and since  $W_{k^*} = 0$ , (4) includes constraints for  $k = k^*$  only. The solution to the new problem yields a new noninferior solution, which the decision maker evaluates. The method continues until the decision maker is satisfied. If, at any iteration the decision maker feels that none of the objectives are satisfactory achieved, the algorithm stops with the conclusion that no best compromise solution can be found unless the decision maker is willing to make additional compromises. At most,  $p$  iterations are performed after which the decision maker is satisfied or it is concluded that no compromise solution exists.

### 2.3. The constraint method

The constraint method is perhaps the most intuitively appealing generating technique. It operates by optimizing one objective while all of the others are constrained to some value. Marglin<sup>20</sup> (1967) appears to be the first to have suggested such an ap-

proach to multiobjective problems.

Given a multiobjective problem with  $p$  objectives

$$\begin{aligned} &\text{maximize } Z(x_1, \dots, x_n) \\ &= \{Z_1(x_1, \dots, x_n), Z_2(x_1, \dots, x_n), \\ &\quad \dots, Z_p(x_1, \dots, x_n)\} \end{aligned} \quad (11)$$

$$\text{st } (x_1, \dots, x_n) \in F_d \quad (12)$$

The constrained problem is

$$\text{maximize } Z_h(x_1, \dots, x_n) \quad (13)$$

$$\text{st } (x_1, \dots, x_n) \in F_d \quad (14)$$

$$Z_k(x_1, \dots, x_n) \geq L_k \quad (15)$$

$$k = 1, \dots, h-1, h+1, \dots, p$$

where the  $h$ th objective was arbitrarily chosen for maximization. This formulation is single-objective problem, so it can be solved by the simplex method for the linear problem. The optimal solution to this problem is a noninferior solution to the original multiobjective problem if some conditions are satisfied. The first is the value of  $L_k$  that used in (15) should be satisfied. The second condition for the choice of  $L_k$  is that all constraints on objectives should be binding at the optimal solution to the constrained problem.

When the constrained problem is solved, if (15) is satisfied as an equality, then surplus variable is not in the basis. Its reduced cost ( $W_k$ ) must be non-negative since we are at the optimal solution of maximization problem.  $W_k$  is the amount by which  $Z_k(x_1, x_2, \dots, x_n)$ . Therefore, this shadow price is equivalent to the trade-off between the  $h$ th and  $k$ th objectives.

The major step is as follows:

The first step is to construct a payoff table explained in STEM. The pay-off table provides a systematic way of finding a range values for each of the  $L_k$ . This approach come from the belief that the optima for each problem represent endpoint of the noninferior set. This approach guarantees the feasibility and noninferiority of the constrained problem for two-objective problems. However higher-dimensional problems will usually lead to some infeasible constrained problems. Without the use of the pay-

off table the number of infeasible constrained problems might be much greater.

The second step is to convert a multiobjective programming problem such as (11) and (12) to its corresponding constrained problem as in (13) – (15).

In third step, the  $n_k$  and  $M_k$  from step 1 represent a range for objective  $k$  in the noninferior set:  $n_k \leq Z_k \leq M_k$ . This range applies as well to  $L_k$ , the right-hand side of the constraint on objective  $k$ . Choose the number of different values of  $L_k$  that will be used in the generation of noninferior solutions. Call this  $r$ .

Step 4 is to solve the constrained problem set up in step 2 for every combination of values for the  $L_k$ ,  $k = 1, \dots, h-1, h+1, \dots, p$ , where

$$L_k = n_k + [t/(r-1)] (M_k - n_k),$$

$$t = 0, \dots, (r-1) \tag{16}$$

Since  $r$  values of each of the objectives will be used in step 4, there are  $r^{(p-1)}$  combinations of values of the  $L_k$ . Each of the  $r^{(p-1)}$  constrained problems that is feasible will yield a noninferior solution. These solutions are desired approximation of the noninferior set.

Each of the  $r^{(p-1)}$  constrained problems requires the solution of a linear program.

### 3. Case Study and Results

#### 3.1. Modeling of Case Study

In order to demonstrate the role of MMP in the forest land use planning process and to compare the difference, a very simplified case study is considered. An integration of timber and wildlife are considered in the model, but the basic framework can be applied to other resource outputs and values.

The problem concerns a forest area consisting of Pine as classed by the age class distribution shown in Table 1. Even though numerous wildlife species inhabited in the forest area, two species have been selected as the major species inhabiting in the area.

Table 1. Age class of case study area

Age class (years)	Pine (ha)
0 – 10	2,700
11 – 20	2,160
21 – 30	1,440
31 – 40	1,440
41 – 50	1,080
50 +	1,180
	10,000

Table 2. Maximum number of animals that feed or reproduce (per ha per year) at various stages of forest development in case study area

Wildlife species	Seedling (0-10)	Young (11-40)	Mature (41+)
1. Squirrel	1	2	3.5
2. Woodpecker	–	0.2	0.5

Table 2 shows the maximum number of animals per year that utilize the forest for either feeding and/or reproduction purposes during successional stages of forest.

For instance, in the acre of young Pine (i.e. 11-40 years), about 2 squirrels feed and/or reproduce. Hence, the numbers in Table 2 represent the maximum number of animals that make use of an hectare of forest land for feeding and reproduction. The data in Table 1 and 2 are hypothetical data.

The objectives of forest management over the 50 years are to: (a) maximize the timber harvest plus ending inventory volume, (b) maximize the number of squirrel (species 1), and (c) minimize the number of woodpecker (species 2). Resource constraints which must be satisfied included: (a) area of forest land by age class and (b) nondeclining timber harvest flow policy on a decadal basis. The major concern facing the land manager is how to schedule timber harvesting activities over the 50-yr. planning horizon to best attain the stated objectives.

To keep the case study management simple, only one timber harvest alternatives are defined for age class and other timber management activities

**Table 3.** Maximum numbers of wildlife and timber volumes produced over 50-yr. planning horizon

Initial Age Class	1	2	3	4	5	6	7	8	9	10	11
	-10	-20	-30	-40	-50	Maximum 1	No. 2	Harvest 1st	Vol. 2nd	Ending Inventory	Total
0	-	-	-	-	H	105	11	18	0	0	18
10	-	-	-	H	-	105	11	18	0	0	18
20	-	-	H	-	-	105	11	18	0	1	19
30	-	H	-	-	-	105	11	18	0	4	22
40	H	-	-	H	-	95	9	15	7	5	27
50	H	-	-	-	H	105	11	15	10	0	25

H; Harvest activity, -; No harvest activity

such as thinning, fertilization and prescribed burning are not considered.

The land manager's problem is to decide how many acres in each age class should be harvested to attain the stated objectives while satisfying the above constraints. The large number of possible combinations of assignments of acres to harvest suggest the use of a MMP approach to this kind of problem. This problem involves 5 objective functions which are: (a) maximize timber harvest plus ending inventory volume, (b) maximize the number of species 1 and minimize the number of the species 2. In addition to the area constraints, a timber harvest flow constraint requires that the total timber harvest volume not decline from period to period and that the first period harvest be at least 25,000m<sup>3</sup>.

Using data in Table 2, Table 3 are constructed showing how timber harvest alternatives affect the wildlife and the volume of timber produced. Table 3 defines the various harvest alternatives by planning period. Columns 1 through 5 of table are explicit descriptions of the harvest alternatives for each existing age class. Column 6 and 7 shows the maximum number of species that utilize the forest for harvesting.

For example, for the first age class in Table 3, it shows that a maximum of 105 squirrels make use of an hectare of the Pine over the 50-yr. planning horizon.

The decision maker is willing to settle for an output level for each objectives which is 20% in

min/max values.

### 3.2. Solution by STEP method

STEP method begins with the construction of the pay-off table. The table is founded by optimizing each of the objectives individually, where the simultaneous optimization of three objectives is not possible. Therefore, decision maker should find a compromise solution.

Table 4 describes a summary of three compromise solution obtained using the STEP method.

Using the pay-off table and (6-7), the initial set of weights are shown in the 2nd column (1st W<sub>k</sub>) in Table 5. The weight(W<sub>k</sub>) represents the relative variation in the value of the objectives.

**Table 4.** Pay-off table for case study

Solution	Timber Har. Vol. (m <sup>3</sup> )	No. of Wildlife	
		squirrel	woodpecker
1	191,880*	983,340	101,988
2	190,816	988,660*	103,052
3	137,360	669,285	69,079*

\*represent the ideal solution.

**Table 5.** The value of weight

Objective function	N <sub>k</sub>	1st W <sub>k</sub>	2nd W <sub>k</sub>
Timber harvest	0.0053	0.2760	0
Squirrel	0.0013	0.0677	0
Woodpecker	0.0126	0.6563	1
	0.0192	1.0000	1.0000

**Table 6.** Summary of two compromise solutions for the case study

Objective Function	Max/Min Value	First Compromise	% Diff. from Max/Min	Second Compromise	% Diff. from Max/Min
Timber	191,880	164,632	14.2	158,210	17.5
Species 1	988,660	828,088	16.2	790,929	20.0
Species 2	69,079	85,724	24.1*	81,831	18.5

\*Objectives exceed satisfactory levels.

In calculating the first compromise solution in Table 6, the value of species 2 exceed the satisfactory level. Therefore decision maker hope to adjust the value of output to attain better solution. In modeling of this case study, decision maker willing to settle all objective value in the range of 20% of min/max values. (8-9) are appended to (3-5) to redefine  $F^{i+1}$  and a second compromise solution is obtained. To get second compromise solution a new set of weights should be used. In this model timber harvest and species 1 are now satisfied, so  $w_1$  and  $w_2$  are 0, and the weight for  $w_3 = 1$ . The second compromise solution shown in Table 6.

The result of the second compromise solution are shown in Table 6. By decreasing the timber harvest and species 1, species 2 produced less. All objective values are satisfy the decision maker. The compromise solution can be got by the 2nd iteration by  $(x_1, x_2, x_3, x_4, x_5, x_6) = (1642.6, 1548.5, 1388.9, 1388.9, 1080, 586.7)$ . Therefore the decision maker finds a satisfactory solution and terminates the STEM.

**3.3. Solution by the constraint method**

The first step is to construct a payoff table. The payoff table for this case study was previously presented as Table 4.

In step 2 the constrained problem is set up:

$$\text{MAX } Z_1(x_1, \dots, x_6) \tag{17}$$

$$\text{st } (x_1, x_2, x_3, x_4, x_5, x_6) \text{ [ Fd} \tag{18}$$

$$Z_2 (X_1, \dots, X_6) \geq L_2 \tag{19}$$

$$Z_3 (X_1, \dots, X_6) \leq L_3 \tag{20}$$

where objective  $Z_1$  has been chosen arbitrary for maximization.

In step 3 the range for the right-hand side of (19) and (20) is established as  $n_2 \leq L_2 \leq M_2$  or  $669,285 \leq L_2 \leq 988,660$  and  $n_3 \leq L_3 \leq M_3$  or  $69,079 \leq L_3 \leq 103,052$ . The value of  $r$ , the number of different values for  $L_2$  and  $L_3$  to be used in generating noninferior solutions, is set. Choose  $r = 11$ .

In step 4 the problem in (17) – (20) is solved  $11 \times 11 = 121$  times.

The values for  $L_2$  and  $L_3$  can be computed from

$$L_2 = 669,285 + 1/10 * t(988,660 - 669,285),$$

$$t = 0, \dots, 10$$

$$L_3 = 69,079 + 1/10 * t(103,052 - 69,079),$$

$$t = 0, \dots, 10$$

i.e.

$$L_2 = 669,285 + 31,937.5t, t = 0, \dots, 10$$

$$L_3 = 69,079 + 3,397.3t, t = 0, \dots, 10$$

The constraint problem will be solved 121 times with  $L_2$  and  $L_3$ . The noninferior solution can

**Table 7.** Value of  $L_2$  and  $L_3$

t	$L_2$	$L_3$
0	669,285	*69,079
1	701,222.5	*72,476.3
2	733,160	*75,873.6
3	765,097.5	*79,270.9
4	*797,035	*82,668.2
5	*828,972.5	86,065.5
6	*860,910	89,462.8
7	*892,847.5	92,860.1
8	*924,785	96,257.4
9	*956,722.5	99,654.7
10	*988,660	103,052

\*represent the possible value of  $L_2$  and  $L_3$

Table 8. The value of noninferior solution

Objective function	Min/Max Value	20% min/max Value	Noninferior Solution	% diff. from Min/Max
Harvest	191,880	153,504	160,160	16.5
Squirrel	988,660	790,928	798,750.5	19.2
Woodpecker	69,079	82,894.5	82,669	19.7

be get by the combination of  $L_2 = 4$  and  $L_3 = 4$ . The other solutions yields the no feasible solution or not satisfied solution. The noninferior solution is  $Z = (Z_1, Z_2, Z_3) = (160,160, 798,751, 82,669)$  when  $(x_1, x_2, x_3, x_4, x_5, x_6)$  is  $(1570.6, 1529.4, 1440, 1440, 1060, 668)$ .

#### 4. Discussion and Conclusion

In this paper, this STEP method (an interactive MOLP technique) and the Constraint method (technique for a posteriori articulation of preference information given) are compared as an hypothetical forest land management planning procedure.

STEP method is convenient because of the its simple procedure in problem, but it is impossible to generate the explicit trade-off information because this method does not require explicit information from the decision maker. The constraint method is possible to generate the trade-offs among the objectives at the resulting noninferior solution as well as the solution. The disadvantage which has severely limited in practical big problem is that they usually generate a large number of nondominated solution, as has been shown by others (Cohon and Marks, 1975); it is almost impossible for the decision maker to select the best one that is satisfactory. Therefore, it is not likely to be useful in the large-scale mathematical programming problems that arise in multiple forest land use.

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