Cost Analysis of a Stepdown Warranty Policy

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Abstract

This paper proposes a stepdown warranty policy: the warranty period consists of subintervals where the manufacturer's compensation is constant for warranty failures and decreases with the subinterval's number. Manufacturer's unit warranty cost is analyzed for both irrepairable and repairable products. We assume that only minimal repairs are performed for repairable items. Comparison with the free replacement policy indicates that the proposed policy has a longer warranty period if the warranty reserves are the same and that manufacturer's unit warranty cost is smaller if the warranty periods are the same. Some numerical examples are also given.

I. Introduction

Manufacturers use warranties to accomplish a variety of goals such as product promotion, quality assurance, and consumer risk reduction. In some cases warranties are required by law; in other cases warranties are viewed as essential to the producer's competitive position. Consumers, on the other hand, must decide between products with differing warranties or between warranty options for the same product.

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In planning a warranty program for a given product, the manufacturers must deal with two major related problems: First, one must determine both the conditions for compensation and the period of coverage. Second, the amount of money that must be allocated to cover future expenses for failures during a specified warranty period. If the allocation for warranty expense is too large, there is a lost opportunity due to committed funds that could be invested for greater returns. On the other hand, underestimating these expenses will result in hidden losses that must be covered in a future profit plan. Analysis of costs and/or benefits of a warranty policy should be preceded before determining the warranty policy. For an extensive literature survey on warranty policies, see Nguyen and Murthy [4] and references therein.

There are two common types of warranty policies: the free replacement warranty and the prorata warranty. Under a free replacement policy the manufacturers pay the entire costs of repair or replacement if the product fails before the warranty expires and supplies as many repairs or replacements as needed during the warranty period. Under a prorata warranty, the cost of repairing or replacing an item depends on the age of the product at the time of failure.

The length of the period of a free replacement warranty policy is thus shorter than that of a prorata policy if the manufacturer's reserves for warranty costs are the same. Consumers, although most of the repairing or replacing costs will be charged to them at the end of the warranty period, will be likely to prefer the product that has longer warranty period. Under a prorata warranty, however, the exact information of the failure time should be known that may cause practical conflicts between consumers and manufacturers.

This paper proposes a warranty policy that can overcome such shortcomings: The warranty period is partitioned into some subintervals. The manufaturer's compensation for warranty failures is constant in a subinterval and decreases with the subinterval's number. The proposed policy is called a stepdown warranty policy. The well known free replacement policy and the prorata policy are special cases of the proposed stepdown warranty policy.

Costs to manufacturers of stepdown warranty policy for repairable or irrepairable items are analyzed in this paper. Section II defines the stepdown warranty policy. Section III contains the cost analysis for irrepairable items and Section IV for repairable items.

II. Stepdown Warranty Policy

This section describes the proposed stepdown warranty policy. Let W be the length of the warranty period. We devide the warranty interval $[0,W]$ into n subintervals $[0,W_1]$, $\cdots$, $[W_{n-1},W_n]$, where $W_n = W$. If an item fails in the subinterval $[W_{i-1},W_i]$, manufacturer compensates to customers c_i for repair or replacement, $1 \leq i \leq n$. We assume that $c_1 > c_2 > \cdots > c_n > 0$. Manufacturer's warranty cost for an item can be illustrasted as Fig. 1.

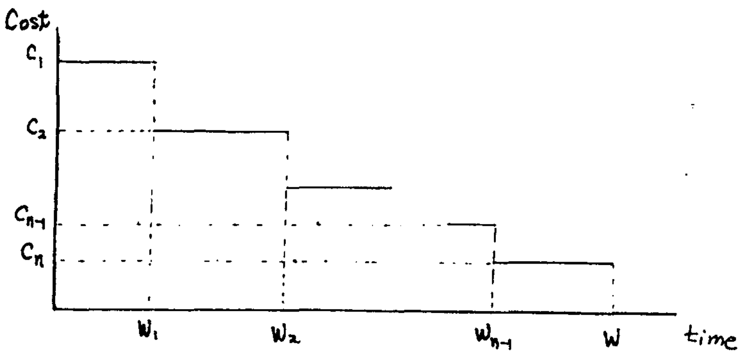

Fig.1. Unit Warranty Cost

If we let n = 1, stepdown warranty policy reduces to the well known free replacement policy. And if n becomes sufficiently large with W being fixed, it becomes the very prorata warranty policy. It is easy to see that stepdown warranty period will be longer than that of the free replacement policy if the manufacturer's reserve for warranty failures are the same. And stepdown warranty policy is easier to apply in practical situations than prorata warranty policy since the latter needs more exact failure time informations than the former.

III. Costs for Irrepairable Products

We now analyze the unit warranty costs to the manufacturer for irrepairable items. We assume that all failures are independent and identically distributed. It is further assumed that at each failure the item is replaced and the replacement item has the original warranty.

Let X_i be the time interval between failure i and i-1, $i \geq 1$. Then the cost of a failure to the manufacturer is

$$I(X_i) = c_i, \quad w_{i-1} < X_i \leq W_i. \tag{1}$$

The total unit warranty expense accumulated for a particular item during a warranty cycle of length W is

$$C(t) = \sum_{i=1}^{N(t)} I(X_i), \qquad t \leq W, \tag{2}$$

where $\{N(t),\ t>0\}$ is a counting process denoting the number of failures in $[0,t]$. The process is renewed once W is reached.

Proposition 1.

Let F and f be the failure time distribution function and the probability density function, respectively. The expected unit warranty cost for an item under stepdown policy is

$$A(W) = \frac{1}{1-F(W)} \sum_{i=1}^{n} c_i(F(W_i)-F(W_{i-1})). \tag{3}$$

Proof. Since N(W) is a stopping time, we have

$$\begin{aligned} A(W) &= E(C(W)) \\ &= E\left(\sum_{i=1}^{N(W)} I(X_i)\right) \\ &= E(N(W))E(I(X_1)). \end{aligned}$$

The random variable N(W) is geometrically distributed with

$$P(N(W) = k) = (1-F(W))F(W)^{k-1}, \qquad k=1,2,\cdots.$$

Thus we have

$$E(N(W)) = 1/(1-F(W)). \tag{4}$$

We also have

$$\begin{aligned} E(I(X_1)) &= \sum_{i=1}^{n} c_i \int_{W_{i-1}}^{W_i} f(x)dx \\ &= \sum_{i=1}^{n} c_i(F(W_i)-F(W_{i-1})). \end{aligned} \tag{5}$$

From (4) and (5), we easily get (3).

Proposition 1 enables one to examine the economic effect of various values of the parameters c_i, W_i and those associated with the failure time distribution. In particular, if we let $c_i = ic/n$, $1 \le i \le n$, where c is the unit replacement cost, we have the following proposition.

Proposition 2. Let W' be the free replacement warranty period under which the manufacturer compensates the whole replacement cost c for warranty failures. If the manufacturer's reserve for warranty costs are the same, W' should satisfy

$$(1-F(W'))^{-1} = \sum_{i=1}^{n} \overline{F}(W_{i-1}/n)/n(1-F(W)), \qquad (6)$$

where $\overline{F}(W) = 1 - F(W)$. Let $B(W')$ be the free replacement warranty costs to the manufacturer. If the warranty periods are the same for two policies, i.e. $W = W'$, the cost ratio $B(W)/A(W)$ is

$$B(W)/A(W) = 1/(1 - \sum_{i=1}^{n-1} F(W_i/n)/nF(W)). \qquad (7)$$

Proof. Under a free replacement policy, we have

$$B(W') = E(\sum_{i=1}^{N(W')} \int_0^{W'} cf(x)dx)$$

$$= cF(W')/(1-F(W')). \qquad (8)$$

Equating (8) and (3) with $c_i = ci/n$ we get (6). And $B(W)/A(W)$ easily follows if we let $W' = W$ in (8).

It is easy to see that the denominator in (7) is smaller than 1, which implies that the free replacement policy needs more reserves than stepdown warranty. Conversely, we see that the free replacement warranty period is shorter than that of stepdown warranty.

Example 1. A tire manufacturer offers a 36 month warranty that provides total replacement for failures occurring within the first 6 months of sale and half thereafter. At each failure, the tire is replaced at a total cost of 100. It is assumed that the customer is willing to pay the other half of the necessary money in order to get a new tire after the first 6 months. Failures are assumed to be distributed exponentially with a mean time 50 months, i.e. $f(t) = e^{-t/50}/50$, $t > 0$.

Thus $n = 2$, $W_1 = 6$, $W_2 = 36 = W$, $c_1 = 100$ and $c_2 = 50$. From Proposition 1, we have

$$A(W) = 64.34.$$

If the reserve for free replacement is the same as that of the stepdown policy, the warranty period $W' = 24.84$ (months). If the warranty periods are the same, we have $B(W) = 105.44$.

Example 1 shows that stepdown warranty policy has a longer period than free replacement policy if the manufacturer's reserve for warranty failures are the same and that has a smaller unit warranty cost if the warranty periods are the same.

IV. Costs for Repairable Products

Section III analyzed warranty costs for irrepairable products. In this section, we analize the warranty costs for repairable items. We assume that only minimal repairs are performed if the product fails during the warranty period. It is also assumed that the warranty expires after W and is not renewed and the failure rate is an IFR.

Since the minimal repairs are performed on failure, we have the following lemmas.

Lemma 1. Let $r(t)$ be the failure rate function and $R(t)$ be the cumulative failure rate function. Then we have

$$P(N(t) = k) = (R(t))^k e^{-R(t)}/k!, \quad t \le W, \ k=0,1,2,\cdots.$$

Proof. See Barlow and Proschan [1].

Lemma 1 shows that the number of failures by time t follows a nonhomogeneous Poisson process with intensity function $r(t)$. From Lemma 1, we easily get the following lemma (see for proof Parzen [6], p139-143).

Lemma 2. Let T_i be the time at which the ith failure occurs and $S_i = R(T_i)$, $i=1,2,\cdots$. Then given $N(t) = k$, the random vector $(S_1,\cdots,S_k)$ is distributed as the order statistics of size k from the uniform distribution $U(0,R(t))$.

From these lemmas, we have the following proposition.

Proposition 3. The expected stepdown unit warranty cost to the manufacturer is

$$A(W) = \sum_{i=1}^{n} c_i(R(W_i) - R(W_{i-1})). \tag{9}$$

Proof. Let $I(\cdot)$ be the same as in (1). Then we have

$$\begin{aligned} A(W) &= \sum_k E(I(T_1) + \cdots + I(T_k) \mid N(W) = k)P(N(W) = k) \\ &= \sum_k E(I(R^{-1}(S_1))+\cdots+I(R^{-1}(S_k)) \mid N(W)=k)P(N(W)=k) \\ &= \sum_k kE(I(R^{-1}(S_1)))P(N(W) = k) \\ &= \sum_k \frac{k}{R(W)} \int_0^{R(W)} I(R^{-1}(t))dt \, \frac{R(W)^k e^{-R(W)}}{k!} \\ &= \int_0^{R(W)} I(R^{-1}(t))dt \\ &= \int_0^{W} I(t)r(t)dt, \end{aligned} \tag{10}$$

where $R^{-1}(\cdot)$ is the inverse function of $R(\cdot)$. From (10), we easily get (9).

Under the same assumptions and notations as in Proposition 2, we have the following.

Proposition 4. If the manufacturer's warranty reserves are the same, the free replacement period W' is

$$W' = R^{-1}(R(W) - \sum_{1}^{n-1} R(W_i/n)/n). \tag{11}$$

And if W' = W, we have

$$B(W)/A(W) = (1 - \sum_{1}^{n-1} R(W_i)/nR(W))^{-1}. \tag{12}$$

Proof. Under the free replacement policy, we have

$$B(W') = c \sum_{k} kP(N(W) = k)$$

$$= cR(W'). \tag{13}$$

From (13), (11) and (12) easily follow.

Example 2. A stepdown warranty is offered for a microcomputer with c_i's and W_i's are the same as in Example 1. The failure time distribution is a Weibull with

$$f(t) = 0.05t \exp(-(t/20)^2).$$

Since $R(t) = (t/20)^2$, we have A(W) = 166.5 and B(W) = 324. If the manufacturèr wants to offer free replacement warranty with the same reserve, the warranty period should be W' = 25.81.

These results implies that the remarks at the end of Example 1 al hold.

V. Conclusions

This paper proposes a stepdown warranty policy. Unit warranty costs for irrepairable and repairable products are analyzed. Comparision with the well known free replacement policy shows that the proposed policy has a longer warranty period if the warranty reserves are the same and has a smaller unit warranty cost if the warranty periods are the same.

This paper treats only the problem of cost analysis for a stepdown warranty policy. The problems of determining optimal stepdown policy, warranty reserves, or maintenance schedules for stepdown warranted items will be interesting for further studies.

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