Variable Threshold Detection with Weighted BPSK/PCM Speech Signal Transmitted over Gaussian Channels

(가우시안 채널에 있어 가중치를 부여한 BPSK/PCM 음성신호의 비트검출 한계치 변화에 의한 신호재생)

安 承 春* 徐 廷 旭,* 李 門 浩**

(Seung Choon Ahn, Jung Uck Seo and Moon Ho Lee)

要 約

이 연구에서는 펄스 부호 변조된 신호의 비트에 가중치를 부여하여 가우시안 채널에 송출한 후 수신측에서는 비트 검출한계치를 변화하여 신호를 재생하는 방법을 제시하였다. μ-법칙 펄수부호 변조에서 각 비트는 중요도에 따라 가중치를 두어 변조하게 된다.

또한 수신측에서는 양부호화 음부호의 중간에 소거대역을 설정하여 비트가 이 대역에 떨어지면 그 비트가 속한 신호는 인접신호에 의하여 예측하여 대치된다.

이 통신방식에 의한 BPSK/PCM 음성신호에 대해 시스팀 전체의 신호대 잡음비를 구하였다. 입력신호 수준이 -17dB일 때 가중펄스 부호변조나 비트 검출한계 변화법 보다 이 방법은 각각 5dB, 3dB의 신호대 잡음비의 이득이 있었다. 컴퓨터에 의한 시뮬레이션을 수행하여 이론적으로 구한 신호대 잡음비와 거의 일치하는 결과를 얻었다.

Abstract

In this paper, variable threshold detection with weighted Pulse Code Modulation-encoded signals transmitted over Gaussian channels has been investigated. Each bit in the μ-law PCM word is weighted according to its significance in the transmitter. If the output falls into the erasure zone, the regenerated sample replaced by interpolation or prediction.

The overall system signal to noise ratio for BPSK/PCM speech signals of this technique has been found.

When the input signal level was -17 db, the gains in overall signal s/n compared to weighted PCM and variable threshold detection were 5 db and 3 db, respectively. Computer simulation was performed generating signals by computer. The simulation was in resonable agreement with our theoretical prediction.

*正會員, 韓國電氣通信公社

(Korea Telecommunication Authority)

**正會員, 全北大學校 電子工學科

(Dept. of Elec. Eng., Chonbuk Nat'l Univ.)

接受日字 : 1987年 2月 19日

I. Introduction

The bits in any encoded PCM word are of different importance to the recovered analog signal [3] [6] [7]. The most significant magnitude bit is the most influential in determining the accuracy of the recovered signal, and the importance of the subsequent bits declines rapidly until the least significant bit is reached.

Bit error rate in digital transmission is related with bit energy transmitted so that the more energy for transmission is provided, the lower the bit error rate [3]. Therefore the effect of the errors can be reduced by matching the energy of each bit prior to transmission according to their contribution to the accuracy of the recovered decoded sample. Thus the energies assigned to the most significant magnitude bits are enhanced at the expense of the least significant bits, whose energies are decreased.

Secondly, another approach can be considered with respect to demodulation at the receiver to reduce error, instead of error reduction at the transmitter. The detector identifies the binary signal by making a decision on the analog signal. The output of the matched filter is a logical one or a logical zero. If, however, the output is close to the decision boundary, this decision is unreliable. The digital errors due to unreliability of the most significant bits are more serious than those of the least significant bits. An erasure zone may be established for this unreliable region. If one or more bits in the first M most significant bits of an N bit PCM word are deemed to be unreliable, the word is rejected. The resulting missing decoded sample is replaced by means of interpolation or prediction from neighboring decoded samples. Since the effect of the predictor error or the interpolator error is smaller than that of the digital error, the overall transmission mean square error is improved by this procedure.

Combining two preceding concepts one can expect better overall performance. The procedure consists of two steps;

1. The encoded signal bits are weighted according to their significance at the transmitter.

2. A variable threshold detection scheme is used at the receiver, such that if the output falls into the erasure zone, the regenerated sample is replaced by interpolation or prediction

II. Errors in PCM System

1. *Digital Errors in PCM System*

Fig. 1. shows a schematic block diagram for

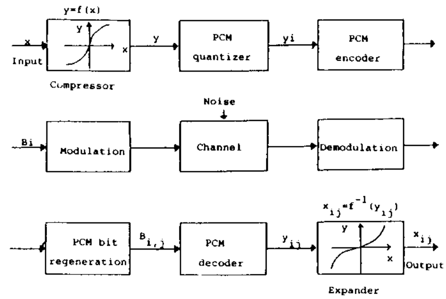

Fig. 1. The Companded PCM System Block Diagram.

the typical companded PCM system. The input signals x are compressed by the compression function f(x). As shown in Fig. 1. finally expansion is performed to regenerate the signal $x_{ij} = f^{-1}(Y_{ij})$. The recovered signal x_{ij} at the detector is different from the initial input signal x at the transmitter for the typical companded PCM system. This difference is called the overall system noise. The average noise power is

$$e^2 = E\{(x-xij)^2\} \quad (1)$$

where E{(.)} stands for the expected value of (.) and is performed over both source and channel statistics.

This digital error can be divided into two classes, the source error and the channel error. Since the source errors and the channel errors are independent, the average noise power is

$$e^2 = e_s^2 + e_c^2 \quad (2)$$

where e_s^2 is the source error, or the quantization error and e_c^2 is the channel error, or the transmission noise power component. Source error includes only the quantization error in this study, since the others are very small in a properly designed PCM system [1].

The quantization error can be written [2],

$$e_s^2 = \frac{s^2\,[\,\ln(1+\mu)\,]^2}{3.2^{2N}}\left(1+\frac{1}{\mu^2 s^2}+\frac{\sqrt{2}}{\mu s}\right) \quad (3)$$

where $s^2 \cong \int_{-1}^{1} x^2\, p(x)\, dx$
is the variance of the signal distribution to be quantized and the probability function p(x) is

assumed symmetrical about zero and concentrated in the range [-1,1]. And μ is the constant of the μ – law PCM system. This quantization noise is the function of the variance of the probability density function of the input signal.

Digital transmission error is due to the digital noise power that occurs in channel transmission and this power may be expressed

$$e_c^2 = E_{ij} [(x_i - x_{ij})^2] \tag{4}$$

where the notation E_{ij} denotes that the average is performed over all the quantization levels and all possible error sequences e_j, x_i is the quantized signal. The source is independent of the channel. Thus (4) can be written,

$$e_c^2 = \sum_{j=1}^{2^N-1} p_j E_i [(x_i - x_{ij})^2] \tag{5}$$

The average E_i is formed over all the possible 2^N quantization levels, and p_j is the probability of an occurrence of the specific error sequence e_j. It is now convenient to define A-factor [3], [4]

$$A_j \equiv E_i [(x_i - x_{ij})^2] \tag{6}$$

The A-factor A_j is the average noise power at the output of the PCM decoder due to the presence of an error sequence e_j. Thus (5) can be expressed.

$$e_c^2 = \sum_{j=1}^{2^N-1} p_j A_j \tag{7}$$

This equation shows that the noise power due to the transmission errors can be calculated with the aid of a set of A-factors. For the input sequence $\{x\}$ with its PDF p(x), the A-factors can be determined using (6),

$$A_j = \sum_{i=0}^{2^N-1} p(x_i)\, d_i\, (x_i - x_{ij})^2 \tag{8}$$

where d_i is the stap of quantization and the sample x is quantized to x_i if it resides in the range from x_i-(d_i/2) to x_i + (d_i/2).

The probability p_j in (7) stands for an occurrance of the error sequence. p_1 to p_8 represents the probability of one bit error of the 8-bit word PCM. p_9 to p_{36} stands for two bit errors, and p_{37} to p_{92} is three bit errors and so on. Therefore a good approximation of (7) [4] for low probability of bit error is

$$e_c^2 \cong \sum_{j=1}^{N} p_j A_j \tag{9}$$

To calculate the A-factors for a single bit error in any of the N-bit positions of a PCM word, (8) is expressed as

$$A_j = \sum_{i=0}^{2^N-1} p(x_i)\, d_i\, a_j(x_i) \tag{10}$$

where $a_j(x_i) = (x_i - x_{ij})^2$; j=1,2, . . . , N

Thus, the $a_j(x_i)$ terms are related to a specific single bit error in the j-th bit position, and a specific input signal amplitude x_i.

2. *Performance Criterion*

The objective performance criterion to be used here is the overall signal to noise ratio(s/n)

$$s/n = \frac{E(x^2)}{e_s^2 + e_c^2} \tag{11}$$

where $E(x^2)$ is the average power of the input signal sequence.

III. Variable Threshold Detection with Weighted PCM Signal

Variable threshold detection with weighted PCM is a digital transmission technique in which each encoded PCM bit is transmitted with different energy according to its importance and the detection process uses variable thresholds depending on the bit position.

1. *Weighted PCM System*

In PCM communication, analog signals are sampled and encoded into binary. At this time, each bit in any word has a different contribution. Therefore, when the signal is reconstructed in the receiver, the most significant magnitude bit is the most influential in determining the accuracy of the recovered speech sample.

We can consider binary amplitude modulation as a means of introducing weighted PCM

[8] [9]. In this case each bit b_j in every PCM word has its magnitude multiplied by $\sqrt{w_j}$.

$$B_j = \sqrt{w_j}\, b_j \tag{12}$$

where b_j is the original (unweighted) PCM bit (± 1).

For the general modulation case, let the energy of the j-th bit be

$$E_j = W_j E \tag{13}$$

where E is the average energy per bit of the PCM signal without weighting. The total energy used to transmit a PCM word is kept unchanged

$$N = \sum_{j=1}^{N} w_j \tag{14}$$

From (12), the channel S/N for the j-th bit is

$$H_j = W_j H$$

where H is the average channel S/N, namely,

$$H = E / No$$

and No is the one-sided spectral density function of the additive white Gaussian noise.

2. *Variable Threshold Detection with Weighted Signal*

When conventional detection is employed, the output is a logical one or a logical zero, the state being dependent upon which side of the decision boundary the output of the processing circuits resides. By contrast, variable threshold detection has more than two output states. There are three states corresponding to three zones. The middle zone ranges from -z to +z and is known as the crasure zone. If one or more bits in the first M most significant bits of an N-bit PCM word are deemed to be unreliable, the entire word is rejected. The resulting missing decoded sample is replaced by means of interpolation or prediction from neighboring decoded samples.

2.1. *Digital Errors for the Least Significant Bits*

Now the probability of the least significant bits which are not subjected to variable threshold detection shall be considered. For binary signaling in white Gaussian noise, matched filter reception, and using the optimum threshold setting, the bit error rate [1] is

$$p_e = Q\left(\sqrt{\frac{Ed}{2\,No}}\right) \tag{17}$$

where Q(x) is the error function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dt$$

and E_d is the difference signal energy at the receiver input. This result can be used to evaluate the p_e of binary signaling schemes where matched filter reception is used. The j-th weighted binary signal [1] of an N-bit PCM word is, using (12),

$$s_{1j}(t) = A\sqrt{w_j}\cos w_c t \quad 0 < t \leqslant T \text{ (binary 1)}$$

$$s_{2j}(t) = -A\sqrt{w_j}\cos w_c t \quad 0 < t \leqslant T \text{ (binary 0)} \tag{18}$$

Thus, the BER for the weighted PCM is

$$p_{1j} = Q\left(\sqrt{\frac{2\, w_j E}{No}}\right) \tag{19}$$

where the average energy per bit is $E = A^2 T/2$.

From (16) and (19), the probability of error for each bit can be written as

$$p_{1j} = Q\left(\sqrt{2\, w_j H}\right) \tag{20}$$

where H is the average channel s/n.

Using this error probability and (9), digital errors for the least significant bits (N-M) are

$$e_{c1}^{\ 2} = \sum_{j=M+1}^{N} Q\left(\sqrt{2\, w_j H}\right) A_j \tag{21}$$

where these least significant bits are from the (M+1)-th bit to the N-th bit of an N-bit word.

2.2. *Digital Errors for the Most Significant Bits*

For variable threshold detection, the most significant M bits have an erasure zone. The bit error probability for these bits may be classified into two areas of probability; nondetective error when the signal falls out of the erasure zone, and error of the erasure zone when the signal falls into the erasure zone. First, as shown in Fig. 2, consider that the received signal which is corrupted in the channel falls outside of the erasure zone. For the matched filter from (17),

$$p = Q\left(\sqrt{\frac{Ed}{2\,No}} + \sqrt{\frac{2\,z^2}{No}}\right) \quad (22)$$

Now, we can consider probability of the j-th bit error of the first M significant bits. In this case the BER is

$$p_{mj} = Q\left(\sqrt{2\,w_j\,H} + \sqrt{\frac{2\,z_j^2}{No}}\right) \quad (23)$$

Threshold level z_j can be normalized, then (23) becomes

$$p_{mj} = Q\left(\sqrt{2\,w_j\,H}\,(1 + T_j)\right) \quad (24)$$

This p_{mj} is the probability that the received signal falls outside of the erasure zone.

Therefore, digital errors for the most significant bits are

$$e_{c2}^{\,2} = \sum_{j=1}^{M} Q\left(\sqrt{2\,w_j\,H}\,(1 + T_j)\right) A_j \quad (25)$$

where e_{c2}^2 is a part of digital errors for the most significant bits.

For the most significant M bits, another bit error probability should be considered, as mentioned earlier. When the received signal falls into the erasure zone, error may occur. As illustrated in Fig.3, the probability can be solved similarily as obtained in (24).

$$p_{zj} = Q\left(\sqrt{2\,w_j\,H}\,(1 - T_j)\right). - Q\left(\sqrt{2\,w_j\,H}\,(1 + T_j)\right) \quad (26)$$

This probability p_{zj} is that the j-th bit of the M most significant bits falls into the erasure zone. When any one bit of the M most significant bits falls into the erasure zone, the word is replaced. Since p_{zj} is a very small amount, the probability p_r of this occuring (26) can be approximated as

$$P_r = \sum_{j=1}^{M} p_{zj}$$

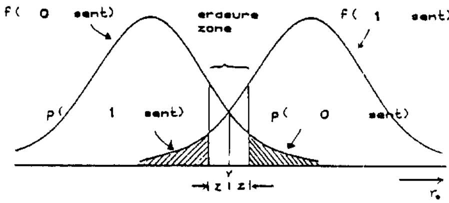

Fig. 2. Nondetective Error of Variable Threshold Demodulation.

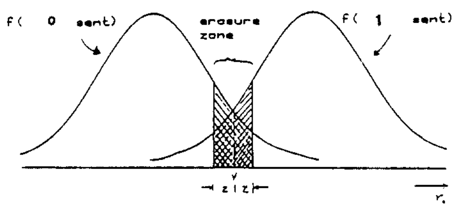

Fig. 3. Error of the Erasure Zone.

We can consider now the error due to this replacement by using interpolation or prediction. This error can be expressed as the relative mean square error δ. That is

$$\delta s^2 = E\left[(x - \hat{x})^2\right] \quad (28)$$

where x is the input sample, $\hat{x}$ is the sample at the output of receiver after the prediction process or interpolation process has been applied, and s^2 is the input signal power. The error due to making up the discarded sample is $\delta\, s^2$ and the digital error power can be obtained by this term multiplied by its probability of occurrence p_r. Therefore, digital error for this case is

$$e_{c3}^{\,2} = \delta s^2 \sum_{j=1}^{M} \left[Q\left(\sqrt{2w_j\,H}\,(1-T_j)\right) - Q\left(\sqrt{2w_j\,H}\,(1+T_j)\right) \right] \quad (29)$$

This equation gives the digital error power when one of the most significant M bits falls into the erasure zone.

So far, three kinds of digital transmission errors have been discussed. Hence, total error is as follows

$$e_c^2 = \sum_{j=1}^{M} p_{mj} A_j + \sum_{j=M+1}^{N} p_{1j} A_j + \delta s^2 \sum_{j=1}^{M} p_{zj}$$

$$= \sum_{j=1}^{M} Q(\sqrt{2w_j H}(1+T_j)) A_j +$$

$$\sum_{j=M+1}^{N} Q(\sqrt{2w_j H}) A_j +$$

$$\delta s^2 \sum_{j=1}^{M} [Q(\sqrt{2w_j H}(1-T_j)) -$$

$$Q(\sqrt{2w_j H}(1+T_j))] \qquad (30)$$

where $N = \sum_{j=1}^{N} w_j$

This digital error equation is a function of weighting factor w_j and threshold T_j.

3. Optimum Weighting Profile and Variable Thresholds

In the preceding section an expression for the total transmission error was obtained. To minimize this transmission error, the optimum weighting profile w_j and variable thresholds T_j should be found. These optimum values can be obtained by the constrained the minimization method [5].

4. Theoretical Performance

The companded PCM considered in the theoretical calculation was an 8-bit μ-law PCM, $\mu = 255$. The binary code employed was binary-folded PCM. The input signal was assumed to have an Laplacian PDF, a density function that is known to be representative of the long term PDF of speech signals. Laplacian PDF is

$$p(x) = \frac{1}{\sqrt{2}\,\sigma} e^{-\sqrt{2}|x|/\sigma}$$

where σ = standard deviation

Input level of -17 db which corresponded to the input signal having a standard deviation of $\sqrt{2}/10$, was considered.

Fig. 4. indicates the variation of overall s/n as a function of channel s/n for an input signal level of -17 db. The curves for the PCM using only weighting and for the PCM using only variable threshold detection provide references from which the combined performance can be compared. The curves of Fig. 4 therefore, show the gian in s/n due to using combined strategy, as compared to weighting PCM or variable threshold detection PCM. It is observed that the combined strategy of variable threshold and weighted PCM yielded a gain of 5 dB and 3 dB over wieghted and variable threshold PCM, respectively. This is true over a wide range of channel s/n. The weighting profile of this system is presented in Table 1. as a function of channel s/n. Table 2. shows the values of the individual thresholds T_1, T_2, ..., T_6, where MSBs are 6.

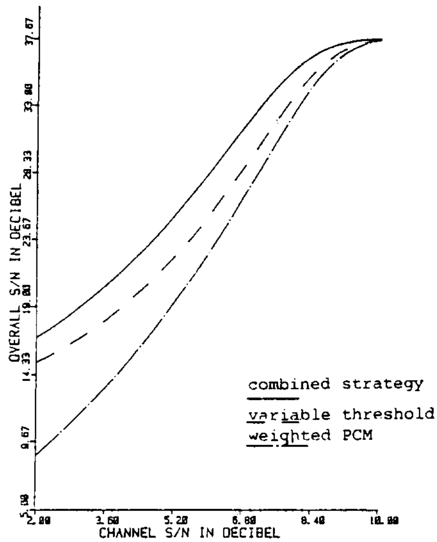

Fig. 4. Theoretical Results as a Function of Channel s/n, for Input Level -17 db.

IV. Computer Simulation

The performance of variable threshold detection with weighted PCM is investigated by the digital computer simulation.

Table 1. Weighting Profiles.

s/n	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8
10	1.1529	1.1764	1.1575	1.0902	1.0146	0.9324	0.8049	0.6710
8	1.2282	1.2626	1.2350	1.1359	1.0239	0.9008	0.7069	0.5067
6	1.3281	1.3772	1.3378	1.1964	1.0356	0.8572	0.5720	0.2958
4	1.4387	1.5054	1.4518	1.2596	1.0408	0.7959	0.4035	0.1042
2	1 5208	1.6011	1.5366	1.3034	1.0344	0.7284	0.2471	0.0284

Table 2. Thresholds

s/n	T_1	T_2	T_3	T_4	T_5	T_6
10	0.1296	0.1389	0.1315	0.1035	0.0688	0.0263
8	0.1929	0.2051	0.1953	0.1575	0.1081	0.0432
6	0.2827	0.2979	0.2858	0.2370	0.1693	0.0719
4	0.4136	0.4320	0.4173	0.3568	0.2670	0.1227
2	0.6202	0.6438	0.6249	0.5465	0.4259	0.2126

Fig. 5. shows the variation of overall s/n when B bit μ-law PCM encoded speech was conveyed by two-level PSK modulation over a Gaussian channel. The input speech level was -17 dB. The weighting profile and the threshold values used in these simulations,were the optimal ones. Fig. 5. displays a variable decision threshold, weighting PCM, and the combined techniques, respectively. In this simulation, the combined technique of variable threshold and weighting PCM yielded a gain 5 dB and 3 dB over weighted and variable threshold PCM, respectively, over a wide range of channel s/n. Some differences between the theoretical and simulation results were shown, since signals were generated by computer and many bit errors were produced in the low channel range.

V. Summary and Conclusion

A new communication technique has been presented for the use of variable decision threshold with weighted PCM for μ-law PCM signals transmitted over Gaussian channels.

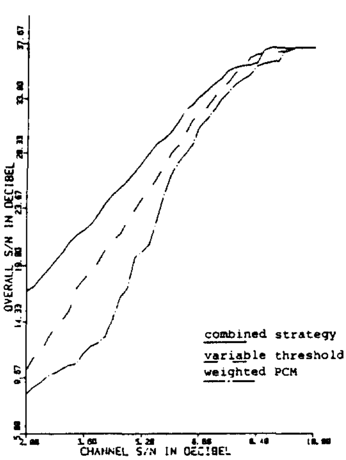

Fig. 5. Simulation Results as a Function of Channel s/n, for Input Level -17 db.

Each bit in the μ-law PCM word has been transmitted with different weighting according to its significance, and regenerated at the receiver according to its individual erasure thresholds.

From our theoretical results and simulations it is concluded that the application of variable threshold detection with weighted PCM, where each bit in the μ-law PCM word is transmitted with different weighting and assigned its unique threshold value, does offer significant advantages in s/n when the signals are transmitted by BPSK modulation over Gaussian channels.

References

[1] L.W. Couch II, *Digital and Analog Communication Systems,* Macmillan, 1983.

[2] M. Schwartz, "*Information Transmission, Modulation and Noise*", McGraw-Hill, pp.117-128, 1980.

[3] N. Rydbeck and C.E. Sundberg, "Analysis of digital errors in non-linear PCM systems", *IEEE Trans. Commun.*, COM-24, no.1, pp.59-65, Jan. 1976.

[4] C.E. Sundberg, "The effect of single bit errors in standard nonlinear PCM systems", *IEEE Trans. Commun.*, COM-24, no.9, pp. 1062-4, Sep. 1976.

[5] J. Vlach and K. Singhal, *Computer Methods for Circuit Analysis and Design,* Van Nostrand Reinhold, pp.436-463, 1983.

[6] E. Bedrosian, "Weighted PCM", *IRE Trans. Inform tion Theory,* IT-4, pp. 45-9, March 1958.

[7] C.E. Suudberg, "Optimum weighted PCM for speech signal," *IEEE Trans. Commun.*, COM-26, no. 6, pp. 872-81, June 1978.

[8] W.E. Wong, C.E. Sundberg and R. Steele, "Weighting Strategies for Companded PCM Transmitted over Rayleigh Fading and Gaussian Channels", *AT&T Bell Lab. Tech. J.*, 63, no.4, pp.587-624, April 1984.

[9] Moon Ho Lee, "Weighted hadamard transform image coding," *Journal of the Korean Institude of Electronics Engineers,* vol.24, Number 2, March 1987.

[10] S.C. Ahn, J.U. Seo and M.H. Lee, "Weighted PCM Signal Transmitting with the Erasure Zone at the Detector," *IEEE Proc. Tencon 1987,* pp. 388-392, Aug. 1987. *