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Eigenstructure Assignment for Linear Multivariable Systems

(線型 多變數 시스템에 대한 Eigenstructure Assignment)

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要 約

본 논문은 선형 다변수 시스템에 있어서 출력 궤환제어에 의한 폐루프 시스템의 eigenstructure assignment 알고리즘들을 보다 일반화 하는 것이다. 출력 궤환 제어에 의한 폐루프 시스템의 eigenstructure assignment에 대하여 필요충분조건이 제시되며 eigenstructure assignment에 대한 기존의 결과들은 이 결과로 부터 이끌어질 수 있다.

Abstract

This paper generalizes the previous results of the closed-loop eigenstructure assignment via output feedback in linear multivariable systems. Necessary and sufficient conditions for the closed-loop eigenstructure assignment by output feedback are presented. Some known results on entire eigenstructure assignment are deduced from this results.

I. Introduction

One of the popular methods of modifying the dynamic response of a linear multivariable system is the placement, via linear state or output feedback, of the closed-loop eigenvalues at arbitrarily prescribed points in the complex plane. Since Wonham presented the fundamental result [1] on eigenvalue assignment in linear time-invariant systems, this problem has generated a considerable amount of literature. Wonham's result states that the closed-loop eigenvalues of any controllable system may be arbitrarily assigned by state feedback. However, in most practical situations the state is not available directly. It is desirable to find the condition under which the system is eigenvalue assignable with incomplete state observation.

The problem of simultaneous assignment of eigenvalues and eigenvectors (eigenstructure assignment) has received considerable attention [2-8]. Most of the previous results for the eigenstructure assignment have some limitations in the sense that eigenvalues of the closed-loop system are distinct or different from eigenvalues of the open-loop system or there is requirement of state feedback. To avoid a condition which eigenvalues of the closed-loop system are distinct, Klein and Moore [2] has generalized the eigenstructure assignment in [3]. Fahmy and Tantawy [4] has generalized the previous results [5-6] to accommodate the case where the set of closed-loop eigenvalues and the set of the open-loop eigenvalues have elements in common. However, these results can not be used in the case of output

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feedback. Kimura [7] has generalized his previous result in [8]. However, this result can also be utilized only in the case where eigenvalues of the closed-loop system are distinct and different from any eigenvalues of the open loop system.

In this paper, a generalization of eigenstructure assignment by output feedback for linear time-invariant multivariable systems is presented without using assumptions that eigenvalues of the closed-loop system are distinct or different from any eigenvalues of the open-loop system. The whole procedure is attractively simple and provides more insight into the eigenstructure assignment. Furthermore an algorithm for the computation of maximal rank matrices N_i and S_i satisfying $(A-\lambda_i I_n, B)\, N_i = 0$ and $(A-\lambda_i I_n, B)\, S_i = I_n$ is also presented. This algorithm greatly facilitates the synthesis of entire eigenstructure assignment by output feedback. The presented method is illustrated by designing an output feedback regulator for a fourth-order two-input two-output continuous system.

II. Main Results

Consider a controllable and observable linear time-invariant system

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1a)$$
$$y(t) = Cx(t), \qquad (1b)$$

where x, u, y are n, m, r-vectors, respectively, and A, B, C are real constant matrices of appropriate dimensions with B and C of full rank. If an constant real output feedback

$$u(t) = Ky(t) \qquad (2)$$

is applied to (1), the closed-loop system becomes

$$\dot{x}(t) = (A + BKC)x(t). \qquad (3)$$

A set of complex numbers Λ is called symmetric if every nonreal element of Λ is accompanied by its conjugate and a symmetric set Λ is called assignable to the system (1) if there exists a constant real m x r matrix K such that a set of eigenvalues of (A + BKC) is Λ. Let $\Lambda = [\lambda_1, ..., \lambda_s]$ be a symmetric set of complex numbers and let $[d_i \mid i=1, ..., s\, ; s \leq n]$ be a set of positive integer satisfying $\sum_{i=1}^{s} d_i = n$. In [9] it is shown that if the closed-loop system has s blocks of order $d_1, ..., d_s$, in its Jordan canonical form, then there are s corresponding generalized right eigenvector chains defined by

$$(A + BKC - \lambda_i In)v_{i1} = 0 \qquad (4a)$$
$$(A + BKC - \lambda_i I_n)\, v_{ij} = V_{ij-1}\,, \quad j = 2, ..., d_i. \qquad (4b)$$

Then, s corresponding generalized left eigenvector chains are defined as follows;

$$t'_{id_i}(A + BKC - \lambda_i I_n) = 0 \qquad (5a)$$
$$t'_{ij}\ (A + BKC - \lambda_i I_n) = t'_{i\,j+1}, \quad j=1, ..., d_i-1. \qquad (5b)$$

In the following, J, J_r and J_{n-r} are n × n, r × r and (n-r) x (n-r) Jordan canonical matrices, respectively. A matrix V is defined as

$$V = [V_1, V_2, ..., V_s] \qquad (6)$$

in which V_i is an n x d_i submatrix of the form

$$V_i = [v_{i1}, v_{i2}, ..., v_{id_i}]$$

and similarly columns of matrices T, W, Z are also composed of t_{ij}, w_{ij}, z_{ij} (i=1, ... s; j=1, ..., d_i).

The following theorem gives necessary and sufficient conditions for the existence of K which yields prescribed eigenvalues and eigenvectors.

Theorem 1: There exists a real matrix K such that for i=1, ..., s

$$(A + BKC - \lambda_i I_n)v_{i1} = 0 \qquad (7a)$$
$$(A + BKC - \lambda_i I_n)v_{ij} = v_{i\,j-1}, \ j = 2, ..., d_i \qquad (7b)$$

if and only if the following conditions are satisfied.

1) The vectors in $[v_{ij} \mid i=1, ..., s; j=1, ..., d_i]$ are linearly independent in C^n and $\lambda_i = \bar{\lambda}_k$ implies $v_{ij} = \bar{v}_{kj}$ (j=1, ..., d_i).

2) There exists a set of vectors $[w_{ij} \mid i=1, ..., s; j=1, ..., d_i]$ such that

$$[A - \lambda_i I_n, B] \begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} = v_{ij-1} \tag{8}$$

where $v_{i0} = 0$.

3) There exists a matrix Z in $C^{r \times n}$ such that

$$V^{-1}BW = Z'CV. \tag{9}$$

Proof (Sufficiency): From the condition 2) and 3), one can obtain the following form;

$$\begin{aligned} AV - VJ &= -BW \\ &= -VZ'CV \end{aligned} \tag{10}$$

Choose an output feedback gain K by

$$K = W(CV)' [CV(CV)']^{-1} \tag{11}$$

From the condition 1), $v_{ij} = \bar{v}_{kj}$ implies $w_{ij} = \bar{w}_{kj}$, which verifies that the output feedback gain (11) is a real matrix. Here, it can be shown using (9) and (10) that the output feedback gain (11) satisfies (7). Indeed, for K given by (11)

$$\begin{aligned} V^{-1}BKCV &= V^{-1}BW(CV)' [CV(CV)']^{-1} CV \\ &= Z' CV(CV)'[CV(CV)']^{-1} CV \\ &= Z' CV \\ &= J - V^{-1} AV. \end{aligned} \tag{12}$$

Therefore, (12) shows that

$$(A + BKC) V = VJ \tag{13}$$

as required.

(Necessity): The condition 1) follows from the property of generalized eigenvectors given in [9]. The equation (7) can be written as

$$[A-\lambda_i I_n, B] \begin{bmatrix} v_{ij} \\ KCv_{ij} \end{bmatrix} = v_{ij-1} \tag{14}$$

which shows that the second condition is also satisfied. If there exists a real matrix K satisfying (7), then there exist generalized left eigenvector chains

$$t'_{id_i} (A + BKC - \lambda_i I_n) = 0 \tag{15a}$$

$$t_{ij} \ (A + BKC - \lambda_i I_n) = t'_{i\,j+1}, \quad j=1, \ldots, d_i-1 \tag{15b}$$

such that

$$T'V = I_n. \tag{16}$$

(15) can be written equivalently as

$$[A' - \lambda_i I_n, C'] \begin{bmatrix} t_{ij} \\ z_{ij} \end{bmatrix} = t_{ij+1} \tag{17}$$

where $z_{ij} = K'B't_{ij}$ and $t_{id_i+1} = 0$. Then, (14) and (17) give rise to the following equations;

$$AV - VJ = -BW \tag{18}$$

$$T'A - JT' = -Z'C. \tag{19}$$

Multiplying T' on the left of (18) and V on the right of (19) gives

$$T'AV - T'VJ = -T'BW \tag{20}$$

$$T'AV - JT'V = -Z'CV. \tag{21}$$

Therefore, one can obtain from (16), (20) and (21) the following relation;

$$V^{-1}BW = Z'CV. \tag{22}$$

Thus, the proof has been completed.

In the case of state feedback, i.e., $C=I_n$, the condition 3) of Theorem 1 is not required and hence one can obtain the following result in [2].

Corollary 1: Let the matrix C be the identity matrix. Then, there exists a real matrix K such that for i=1, ..., s,

$$(A + BK - \lambda_i I_n) v_{i1} = 0$$

$$(A + BK - \lambda_i I_n) v_{ij} = v_{i\,j-1}, \ j = 2, \ldots, d_i$$

if and only if the following conditions are satisfied.

1) The vectors in $[\, v_{ij} \mid i=1, \ldots, s; j=1, \ldots, d_i]$ are linearly independent in C^n and $\lambda_i = \bar{\lambda}_k$ implies $v_{ij} = \bar{v}_{kj}$ $(j=1, \ldots, d_i)$.

2) There exists a set of vectors $[\, w_{ij} \mid i=1, \ldots, s; j=1, \ldots, d_i \,]$ such that

$$[A-\lambda_i I_n, B]\begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} = v_{ij-1}$$

where $v_{i0} = 0$.

Corollary 2: There exists a real matrix K such that for i=1, ..., s,

$$(A + BKC - \lambda_i I_n)\, v_{i1} = 0$$
$$(A + BKC - \lambda_i I_n)\, v_{ij} = v_{ij-1},\ j = 2, ..., d_i$$

if and only if there exist v_{ij}, t_{ij} for i=1, ..., s; j=1, ..., d_i satisfying

$$[A-\lambda_i I_n, B]\begin{bmatrix} v_{ij} \\ wij \end{bmatrix} = v_{ij-1}$$

$$[A'-\lambda_i I_n, C']\begin{bmatrix} t_{ij} \\ z_{ij} \end{bmatrix} = t_{i\,j+1}$$

such that

$$TV = I_n$$
$$t_{ij} = \bar{t}_{kj},\ \bar{v}_{ij} = \bar{v}_{kj} \text{ if } \lambda_i = \bar{\lambda}_k$$

where $v_{i0} = t_{id_i+1} = 0$.

Corollary 2 follows from the proof of Theorem 1. A special case of Corollary 2 is proved by Kimura [7] under the superfluous additional hypothesis that eigenvalues of the closed-loop system matrix (A + BKC) are distrinct and do not include any eigenvalues of the open-loop system A.

In the following, matrices V_o, W_o, T_o and Z_o are defined as

$$V_o=[V_1, V_2, ..., V_p]$$
$$W_o=[W_1, W_2, ..., W_p]$$
$$T_o=[T_{p+1}, T_{p+2}, ..., T_s]$$
$$Z_o=[Z_{p+1}, Z_{p+2}, ..., Z_s]$$

where V_i is an n x d_i submatrix of the form

$$V_i = [v_{i1}, v_{i2}, ..., v_{idi}]$$

and similarly matrices T_i, W_i, Z_i have same forms.

Theorem 2: Let $\Lambda = \Lambda_1 U \Lambda_2$ such that $\Lambda_1 = [\lambda_1, ..., \lambda_p]$ and $\Lambda_2 = [\lambda_{p+1}, ..., \lambda_s]$ are symmetric with $\sum_{i=1}^{p} d_i = r$ and $\sum_{i=p+1}^{s} d_i = n - r$. If $\Lambda = \Lambda_1 U \Lambda_2$ exists such that there exist vectors v_{ij}, for i = 1, ..., p; j=1, ..., d_i and t_{ij}, for i=p+1, ..., s; j = 1, ..., d_i satisfying

1) $T_o' V_o = 0$
2) CV_o is of full rank and $\lambda_i = \lambda_k$ implies $v_{ij} = \bar{v}_{kj}$

where

$$[A-\lambda_i I_n, B]\begin{bmatrix} v_{ij} \\ w_{ij} \end{bmatrix} = v_{ij-1} \tag{23}$$

$$[A'- \lambda_i I_n, C']\begin{bmatrix} t_{ij} \\ z_{ij} \end{bmatrix} = t_{i\,j+1} \tag{24}$$

with $v_{i0}=t_{id_i+1} = 0$, then there exists a real matrix K such that a set of eigenvalues of the closed-loop system (A + BKC) is Λ, and v_{ij} (i=1, ..., p; j=1, ..., d_i) and $t_{i\,j}$ (i=p+1, .., s; j=1, ..., d_i) constitute corresponding generalized right eigenvector and left eigenvector sets respectively.

Proof of Theorem 2: From (23) and (24), we can obtain the following equations;

$$AV_o - V_o J_r = - BW_o \tag{25}$$
$$T_o'A - J_{n-r} T_o' = - Z_o'C. \tag{26}$$

Then, an output feedback gain

$$K = W_o (CV_o)^{-1} \tag{27}$$

satisfies the following equation;

$$(A + BKC)V_o = V_o J_r. \tag{28}$$

Multiplying T_o' on the left of (25) and V_o on the right of (26) gives

$$T_o'AV_o - T_o'V_o J_r = - T_o'BW_o, \tag{29}$$

$$T_o'AV_o - J_{n-r}T_o'V_o = - Z_o'CV_o. \tag{30}$$

Using the condition 1), one can obtain the following equation;

$$T_o' B W_o = Z_o' C V_o. \tag{31}$$

From (27) and (31), one can show that

$$\begin{aligned} T_o (A+BKC) &= T_o' A + T_o' B W_o (CV_o)^{-1} C \\ &= T_o' A + Z_o' C \\ &= J_{n-r} T_o'. \end{aligned} \tag{32}$$

Equations (28) and (32) show that the closed-loop system matrix (A+BKC) has Λ as a set of eigenvalues and columns of V_o and T_o constitute corresponding generalized right eigenvector and left eigenvector sets.

A design procedure based on Theorem 2 for finding a desired feedback matrix K is given in the following.

Step 1: Find maximal rank matrices

$$N_i = \begin{bmatrix} N_{1i} \\ N_{2i} \end{bmatrix}, \; S_i = \begin{bmatrix} S_{1i} \\ S_{2i} \end{bmatrix},$$

$$\bar{N}_k = \begin{bmatrix} \bar{N}_{1k} \\ \bar{N}_{2k} \end{bmatrix}, \; \bar{S}_k = \begin{bmatrix} \bar{S}_{1k} \\ \bar{S}_{2k} \end{bmatrix}$$

for i=1, ..., p; k=p+1, ..., s satisfying the following relation;

$$[A - \lambda_i I_n, B] \; [S_i, N_i] = [I_n, 0] \tag{33 a}$$

$$[A' - \lambda_k I_n, C'] \; [\bar{S}_k, \bar{N}_k] = [I_n, 0] \tag{33 b}$$

where $N_i \in C^{(n+m)\times m}$, $S_i \in C^{(n+m)\times n}$, $\bar{N}_k \in C^{(n+r)\times r}$ and $\bar{S}_k \in C^{(n+r)\times n}$.

Step 2: Form the generalized right eigenvectors and left eigenvectors for i=1, ..., p; k=p+1, ..., s as follows;

$$v_{ij} = S_{1i} v_{ij-1} + N_{1i} p_{ij}, \; j = 1, ..., d_i \tag{34 a}$$

$$t_{kj} = \bar{S}_{1k} t_{kj+1} + \bar{N}_{1k} p_{kj}, \; j=1, ..., d_k \tag{34b}$$

where $v_{i0} = t_{kd_k+1} = 0$ and vectors p_{ij} (i=1, ..., s; j=1, ..., d_i) are selected to satisfy condition 1) and 2) of Theorem 2.

Step 3: Calculate vector chains as follows;

$$w_{ij} = S_{2i} v_{ij-1} + N_{2i} p_{ij}, \quad i=1, ..., p; j=1, ..., d_i \tag{35}$$

Step 4: Calculate the output feedback gain

$$K = W_o (CV_o)^{-1}. \tag{36}$$

Remark 1: If the matrix C is the identity matrix (in case of state feedback), condition 1) of Theorem 2 is not required. The eigenvalue-assignability follows readily for this case. Assume that Λ does not include any open-loop eigenvalues. Then, the following matrices

$$\begin{aligned} N_{1i} &= (\lambda_i I_n - A)^{-1} B \\ N_{2i} &= I_m \\ S_{1i} &= -(\lambda_i I_n - A)^{-1} \\ S_{2i} &= 0 \end{aligned}$$

satisfy (33). (34a) and (35) can be written as

$$\begin{aligned} v_{i1} &= N_{1i} p_{i1} \\ &= (\lambda_i I_n - A)^{-1} B p_{i1} \\ v_{ij} &= S_{1i}^{j-1} N_{1i} p_{i1} + \dots + N_{1i} p_{ij} \\ &= (-1)^{j-1} (\lambda_i I_n - A)^{-j} B p_{i1} + \dots + \\ &\quad (\lambda_i I_n - A)^{-1} B p_{ij}, \; j=2, ..., d_i \\ w_{ij} &= p_{ij}, \quad j = 1, ..., d_i. \end{aligned}$$

Noting that [6]

$$\frac{d^k}{d\lambda^k} S(\lambda_i) = (-1)^k (k!) (\lambda_i I_n - A)^{-(k+1)} B, \quad k = 1,2, ...$$

where $S(\lambda) = (\lambda I_n - A)^{-1} B$ and using (36), the following relationship is obtained

$$KS(\lambda_i) p_{i1} = p_{i1}$$

$$K[\frac{1}{(j-1)!} \frac{d^{j-1}}{d\lambda^{j-1}} S(\lambda_i) p_{i1} + \dots + S(\lambda_i) p_{ij}] = p_{ij}$$

$$\text{for } i = 1, ..., s; j=1, ..., d_i.$$

This is corresponding to the previous result [4] obtained using Brogan's approach [10-11].

An algorithm for the computation of the maximal rank matrices satisfying $(A - \lambda_i I_n, B)$

$N_i=0$ and $(A-\lambda_i I_n, B) S_i = I_n$ is given by the following theorem, where the synthesis of output feedback regulators by entire eigenstructure assignment is greatly facilitated.

Theorem 3: If the pair (A, B) is controllable and the matrix B is full rank, the maximal rank matrices N_i and S_i satisfying $(A-\lambda_i I_n, B)N_i = 0$ and $(A-\lambda_i I_n, B) S_i = I_n$ can be determined through column operations as follows:

$$\sim \begin{bmatrix} A-\lambda_i I_n, B \\ I_{n+m} \end{bmatrix} = \begin{bmatrix} I_n, 0 \\ U_1, U_2 \end{bmatrix}$$

where $\sim[\ ]$ represents column operations of a matrix $[\ ]$. Then $N_i = U_2$ and $S_i = U_1$ respectively.

Proof of Theorem 3: The rank of the matrix $(A-\lambda_i I, B)$ is n if and only if the pair (A, B) is controllable. Therefore we can always obtain the following matrix form through some column operations:

$$\sim \begin{bmatrix} A-\lambda_i I_n, B \\ I_{n+m} \end{bmatrix} = \begin{bmatrix} I_n, & 0 \\ x & x \end{bmatrix}$$

The column operations imply

$$\sim \begin{bmatrix} A-\lambda_i I_n, B \\ I_{n+m} \end{bmatrix} = \begin{bmatrix} A-\lambda_i I_n, B \\ I_{n+m} \end{bmatrix} [U_1\ U_2]$$

$$= \begin{bmatrix} (A-\lambda_i I_n, B)\, U_1, & (A-\lambda_i I_n, B)\, U_2 \\ U_1, & U_2 \end{bmatrix},$$

where $U_1 \in C^{(n+m)\times n}$ and $U_2 \in C^{(n+m)\times m}$. Thus $S_i=U_1$ and $N_i=U_2$.

III. A Numerical Example

Let us now consider an example not covered by the theory developed in the previous work.

Consider a system given by

$$A = \begin{bmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 1&0&1&0 \end{bmatrix}, \quad B = \begin{bmatrix} 0&0 \\ 1&0 \\ 0&0 \\ 0&1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \end{bmatrix}$$

Let a set of the desired eigenvalues be $\Lambda = [-1, -2]$ such that $d_1 = 2$ and $d_2 = 2$. Then, elements of the maximal rank matrices can be found using Theorem 3 as follows;

$$N_{11} = \begin{bmatrix} 0&-1 \\ 0&1 \\ -1&-9 \\ 1&9 \end{bmatrix}, \quad S_{11} = \begin{bmatrix} 0&0&0&0 \\ 1&0&0&0 \\ 0&0&1&0 \\ 0&0&0&0 \end{bmatrix},$$

$$N_{21} = \begin{bmatrix} 1&8 \\ 0&1 \end{bmatrix}, \quad S_{21} = \begin{bmatrix} -1&1&-1&0 \\ 0&0&-1&1 \end{bmatrix},$$

$$\bar{N}_{12} = \begin{bmatrix} 17&-30 \\ 3&-3 \\ -2&2 \\ 1&-1 \end{bmatrix}, \quad \bar{S}_{12} = \begin{bmatrix} 0&0&0&0 \\ 0&0&1&-2 \\ 0&0&0&1 \\ 0&0&0&0 \end{bmatrix},$$

$$\bar{N}_{22} = \begin{bmatrix} -38&61 \\ 5&8 \end{bmatrix}, \quad \bar{S}_{22} = \begin{bmatrix} 1&0&0&0 \\ 0&1&-2&4 \end{bmatrix},$$

The generalized eigenvectors can be represented as

$$\begin{aligned} v_{11} &= N_{11} p_{11} \\ v_{12} &= S_{11} N_{11} p_{11} + N_{11} p_{12} \\ t_{22} &= \bar{N}_{12} p_{22} \\ t_{21} &= \bar{S}_{12} \bar{N}_{12} p_{22} + \bar{N}_{12} p_{21} \end{aligned}$$

To select eigenvectors satisfying the conditions 1) and 2) of Theorem 2, we first check the condition 1) in the following. From the above equation, one can obtain

$$\begin{aligned} t'_{22} v_{11} &= p'_{22} \bar{N}'_{12} N_{11} p_{11} \\ &= p'_{22} \begin{bmatrix} 3 & 13 \\ -3 & 0 \end{bmatrix} p_{11}. \end{aligned}$$

If p_{11} and p_{22} are selected as $p_{11} = [0\ 1]'$ and $p_{22} = [0\ 1]'$, the above equation becomes zero. Then,

$$\begin{aligned} t'_{22} v_{12} &= p'_{22} \bar{N}'_{12} S_{11} N_{11} p_{11} \\ &\quad + p'_{22} \bar{N}'_{12} N_{11} p_{12} \\ &= p'_{22} \begin{bmatrix} 2 & 15 \\ -2 & -15 \end{bmatrix} p_{11} + p'_{22} \begin{bmatrix} 3 & 13 \\ -3 & 0 \end{bmatrix} p_{12} \\ &= -15 + [-3\ \ 0]\, p_{12}. \end{aligned}$$

Therefore, p_{12} is selected as $p_{12}=[-5\ \ 0]'$ such that the above equation becomes zero. Since

$$t_{21}'v_{11} = p_{22}'\bar{N}_{12}'\bar{S}_{12}'N_{11}p_{11} + p_{21}'\bar{N}_{12}'N_{11}p_{11}$$
$$= p_{22}'\begin{bmatrix} -1 & -13 \\ 1 & 13 \end{bmatrix} p_{11} + p_{21}'\begin{bmatrix} 3 & 13 \\ -3 & 0 \end{bmatrix} p_{11}$$
$$= 13 + p_{21}'[13\ \ 0]',$$

similarly $p_{21} = [-1\ \ 0]'$. Then, one can show that

$$t_{21}'v_{12} = p_{22}'\bar{N}_{12}'\bar{S}_{12}'S_{11}N_{11}p_{11} + p_{21}'\bar{N}_{12}'S_{11}N_{11}p_{11} + p_{22}'\bar{N}_{12}'\bar{S}_{12}'N_{11}p_{12} + P_{21}'\bar{N}_{12}'N_{11}P_{12}$$
$$= p_{22}'\begin{bmatrix} -1 & 5 \\ 1 & 5 \end{bmatrix} p_{11} + p_{21}'\begin{bmatrix} -2 & -15 \\ 2 & 15 \end{bmatrix} p_{11}$$
$$p_{22}'\begin{bmatrix} 1 & 13 \\ -1 & 13 \end{bmatrix} p_{12} + p_{21}'\begin{bmatrix} 3 & 13 \\ -3 & 0 \end{bmatrix} p_{12}$$
$$= 0.$$

Since $T_o' V_o = 0$, the condition 1) of Theorem 2 is satisfied. Thus, the generalized right eigenvectors and left eigenvectors selected as above

$$v_{11} = [\ -1 \quad 1 \quad -9 \quad 9]'$$
$$v_{12} = [\ \ 0 \quad -1 \quad -4 \quad -5]'$$
$$t_{21} = [\ -17 \quad 1 \quad 1 \quad -1]'$$
$$t_{22} = [\ -30 \quad -3 \quad 2 \quad -1]'$$

also satisfy the condition 2) of Theorem 2. Then,

$$w_{11} = N_{21}p_{11} = [8\ \ 1]'$$

$$w_{12} = S_{21}v_{11} + N_{21}p_{12} = [6\ \ 18]'$$

$$W_o = \begin{bmatrix} 8 & 6 \\ 1 & 18 \end{bmatrix}$$
$$CV_o = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

so that the output feedback matrix determined by (36) is

$$K = \begin{bmatrix} -14 & -6 \\ -19 & -18 \end{bmatrix}$$

and therefore

$$(A+BKC) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -14 & -6 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -18 & -18 & 1 & 0 \end{bmatrix}$$

which has the Jordan canonical form

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

together with the generalized left eigenvectors v_{11}, v_{12} for $\lambda_1 = -1$, and the generalized right eigenvectors t_{21}, t_{22} for $\lambda_2=-2$, as required.

IV. Conclusion

In this paper, a generalization of entire eigenstructure assignment by output feedback for linear time-invariant multivariable systems has been presented without using assumptions that eigenvalues of the closed-loop system are distinct or different from any eigenvalues of the open-loop system. Necessary and sufficient conditions show that the closed-loop eigenstructure assignment by output feedback is constrained by the requirement that the generalized right eigenvectors and left eigenvectors lie in certain subspaces. The presented method has been illustrated by designing an output feedback regulator for a fourth-order two-input two-output continuous system.

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