Design of an Improved Weakly-Coupled Power Divider and a Generalized n-Way Power Divider for CATV and/or MATV Systems

(CATV 및 MATV 시스템용 개량된 약결합 전력분배기와 일반화된 n-분기 전력분배기의 설계에 관한 연구)

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要 約

CATV 및 MATV 시스템용 개량된 약결합 전력분배기(tap unit 또는 power divider)를 제안하여 설계의 자유도를 높힘으로써, 상당히 조밀한 간격으로의 회로설계를 가능하게 하게 하고 있다. 실현 가능한 권선비는 이산적인 값(권수가 0.5 또는 1.0의 배수) 뿐이므로, 종래형식의 전력분배기는 결합간격에 상당히 큰 제한을 받고 있으나, 본 논문에서 이러한 문제점을 실용상 지장이 없을 정도로 해결하고 있다.

한편, 다선조 이상변성기 및 저항만으로 이루어지는, 임의의 분배율의 일반화된 n-분기 전력분배기의 설계이론이 설명되어 있다. 이 회로는 주파수 의존성을 가지지 않는 소자만으로 구성되므로 이론적으로 대단히 넓은 주파수 대역폭을 가진다. 또한, 실험에 의하여 본 논문에 제한된 설계이론의 타당성이 입증되어 있다.

Abstract

In this paper, we proposed an improved weakly-coupled power divider (TAP UNIT) for CATV and/or MATV systems, by which the degree of freedom in design and density of coupling interval are signifcantly increased compared with the intrinsic one even though the turn ratios are still of discrete values.

On the other hand, the new design theory of a generalized n-way power divider with arbitrary dividing ratios, for CATV and/or MATV systems, which consists of ideal multiwinding transformers and resistors only was presented. Since the circuit elements is not frequency dependent, the proposed power divider is to be of considerably broad bandwidth.

Furthermore, the experimental verification has been achieved, and, hence, the validity of the design theory proposed here is confirmed.

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I. Introduction

Since the turn number of coils composing the transformers can be realized only in discrete numbers in the weakly-coupled power divider[1], free choice of coupling cannot be obtained but only the restricted coupling can be obtained. How can the degree of freedom in design and density of coupling interval be increased? This problem is solved in II.

According to network theory, the circuit elements without frequency dependence are those which do not store any energy in themselves; e.g., ideal transformer and resistor, the former does not dissipate any power and the latter does. It was recognized that the weakly-coupled power divider[1] is of extremely broad bandwidth. From [1] it was also confirmed that the characteristics of the circuit could be obtained fairly accurately even though we assumed that the circuit is composed of only ideal transformers and resistors, and that the design process is of very good outlook and prospects well. Furthermore, the weakly-coupled power divider has a limit of coupling to TAP, the reason of which was not known physically but mathematically. Here, the following questions arise;

(1) Does there exist any essential limit of coupling to TAP of the power divider constructed by transformers and resistors?

(2) Is it needed really to change the circuit form according to the desired degree of coupling?

(3) What is the essential capability of the circuit which is constructed only by transformers and resistors? These problems are solved in III.

II. Improved Weakly-Coupled Power Divider

In [1], we saw that the coupling less than about 8dB to TAP can only be obtained even if arbitrary turn ratios could be realized. Since, however, the turn numbers of coils composing the transformers can be realized only in discrete numbers of integers or multiples of 0.5, free choice of coupling cannot be obtained but only the restrictedly discrete coupling can obtained.

To resolve this problem, we propose a modified weakly-coupled power divider as shown in Fig. 1, where two other transformers were added. Let r_1 and r_2 be the ratios of turn numbers, n_1/n_2 and n_3/n_4, respectively. When we apply the simplified design method described in [1] to the modified weakly-coupled power divider, it can be concluded that the characteristics of the modified one are just same as those of the intrinsic one except that the turn ratio of the modified one takes $r_1/1-r_2$ instead of r. Therefore, the optimum normalized isolation resistance $\widetilde{R}_L$, and the reflection and transmission coefficients are given as follows;

$$\widetilde{R}_L = \frac{2(1-r_1^2) - r_1^2}{2(1-r_2^2) - 3r_1} \qquad (1)$$

and

$$S_{11} \cong \frac{-r_1^2}{2[(1-r_2)^2 - r_1^2]}$$

$$S_{22} \cong \frac{r_1^2}{2[(1-r_2)^2 - r_1^2]}$$

$$S_{33} \cong 0$$

$$S_{12} \cong \frac{2(1-r_2)^2 - 3r_1^2}{2[(1-r_2)^2 - r_1^2]} = 1 + S_{11}$$

$$S_{13} \cong \frac{r_1}{1-r_2}$$

$$S_{23} \cong 0 \qquad (2)$$

Here, the additional transformers with turn ratio of r_2 are the same as those used in the tightly-coupled power divider[1].

If we want to get the coupling of 12 dB to Tap, r=0.25 (1:4) is the only one solution for the turn ratio in the intrinsic weakly-coupled power divider. By the improved one, however, we can design in several combinations of turn ratios, e.g., r_1=0.2 (1:5) and r_2=0.222 (1:4.5), r_1=0.667 (1:6) and r_2=0.333(1:3), and so on.

Thus, one can change the turn numbers of r_1 and r_2 simultaneously so that the degree of freedom in design and density of coupling interval are increased extremely even though the turn ratios are still of discrete values. So, the power divider shown in Fig. 1 is referred to as the improved weakly-coupled power divider.

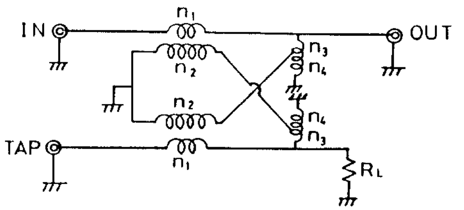

Fig. 1. Improved weakly-coupled power divider.

III. Design Theory of Generalized n-Way Power Divider

A. Two-Way Power Divider Based on Scattering Matrix

The scattering matrix [S] for an ideal 2-way power divider is given by

$$[S] = \begin{bmatrix} 0 & t_1 & t_2 \\ t_1 & 0 & 0 \\ t_2 & 0 & 0 \end{bmatrix} \tag{3}$$

with reference to Fig. 2, where t_1 and t_2 are real numbers. The eq.(3) means that the circuit is of perfect match and isolation.

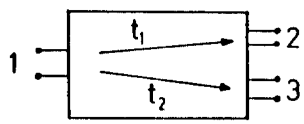

Fig. 2. Schematic diagram of two-way power divider.

Since the circuit is passive, we get

$$t_1^2 + t_2^2 \leq 1 \tag{4}$$

Considering the efficiency of transmission, let us impose that

$$t_1^2 + t_2^2 = 1 \tag{5}$$

Now we calculate the dissipation matrix [Q] and its rank[2]. The dissipation matrix [Q] or Q-matrix is defined by

$$[Q] = [U] - [S]^{\dagger}[S] \tag{6}$$

where [U] is the 3x3 identity matrix and (†) indicates a conjugate transpose or adjoint. Substituting eq.(3) into eq.(6),

$$[Q] = \begin{bmatrix} 1-t_1^2-t_2^2 & 0 & 0 \\ 0 & 1-t_1^2 & -t_1 t_2 \\ 0 & -t_1 t_2 & 1-t_2^2 \end{bmatrix} \tag{7}$$

and

$$\det [Q] = (1-t_1^2 - t_2^2) = 0^2. \tag{8}$$

Thus, the dissipation matrix [Q] is of rank 1. In the case forming a network, the rank of Q-matrix means the number of resistors needed[2,3]. If t+t 1, the dissipation matrix [Q] is of rank 3.

Although the scattering matrix [S] given in eq. (3) is not unitary, it is possible to make it a unitary matrix by expanding the dimension (increasing the rows and columns) of the given matrix[S]. A unitary scattering matrix stands for a dissipationless network. Since, moreover, all the elements of the expanded untiary scattering matrix [Σ] are real, the network with the expanded scattering matrix can be realized only with ideal transformers. Since, in this case, the dissipation matrix [Q] is of rank 1, the dimension to be expanded is also 1.

The generalized method to form an expanded scattering matrix has been established in [3]. Since the matrix in eq. (1) is very simple, the expanded scattering matrix [Σ] can be found easily as eq. (9) by inspection.

$$[\Sigma] = \left[\begin{array}{ccc:c} 0 & t_1 & t_2 & 0 \\ t_1 & 0 & 0 & \pm t_2 \\ t_2 & 0 & 0 & \mp t_1 \\ \hdashline 0 & \pm t_2 & \mp t_1 & 0 \end{array}\right] = \left[\begin{array}{c:c} [S] & \begin{matrix} 0 \\ \pm t_2 \\ \mp t_1 \end{matrix} \\ \hdashline \begin{matrix} 0 & \pm t_2 & \mp t_1 \end{matrix} & 0 \end{array}\right] \tag{9}$$

The theory of synthesis of a network with an orthogonal scattering matrix as in eq. (9) was also established in [4]. But, in this case, the synthesis can be readily performed as follows by inspection because the eq. (9) is of very simple form. If the port numbers of 1, 2, 3, and 4 is renumbered into 1, 4, 2 and 3, the expanded scattering matrix becomes as follows;

$$[\Sigma] = \begin{bmatrix} 0 & 0 & t_1 & t_2 \\ 0 & 0 & \pm t_2 & \mp t_1 \\ t_1 & \pm t_2 & 0 & 0 \\ t_2 & \mp t_1 & 0 & 0 \end{bmatrix}$$

Then, the network with the scattering matrix of eq. (10) is an all-pass network and is formed by multi-winding ideal transformers (See Appendix I). As a result the network shown in Fig. 2 is that with the scattering matrix $[\Sigma]$ in eq.(10). Thus, the network with the scattering matrix of eq. (3) can be easily realized by terminating the port-4 with a unit resistor.

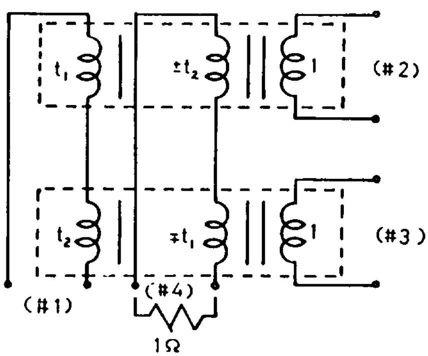

Fig. 3. New (generalizgd) type two-way power divider.

Therefore, the network shown in Fig. 3 operates as an ideal 2-way power divider. If transformers with arbitrary turn ratios could be realized, the 2-way power dividers with arbitrary dividing ratio can always be realized as the form of the network shown in Fig. 3. So it is not needed to change the form of network according to dividing ratio of incident power.

B. n-Way Power Divider

The method of synthesis of a two-way power divider based on scattering matrix is readily expanded to n-way power divider. The process of synthesis of n-way power divider is just the same as the above. The scattering matrix [S] for an ideal n-way power divider is expressed by

$$[S] = \begin{bmatrix} 0 & t_1 & t_2 & \cdot & \cdot & \cdot & t_n \\ 0 & 0 & 0 & 0 & . & . & . \\ t_2 & 0 & 0 & . & . & . & 0 \\ \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & & & & \\ \cdot & \cdot & \cdot & & \mathbf{0} & & \\ t_n & 0 & 0 & & & & \end{bmatrix}$$

where $t_1^2 + t_2^2 + \ldots + t_n^2 = 1$. In this case, the dissipation matrix [Q] is of rank n-1, and, thus, the number of the required resistors is also n-1.[5]

In general, there exist n-1 linearly independent vectors orthogonal to an n-dimentional vector $\mathbf{t} = (t_1, t_2, \ldots, t_n)^t$, where $(\ )^t$ indicates a transpose of ( ). By using the orthonormalization method of Schmidt, we can obtain n-1 normalized and linearly independent vectors orthogonal to the vector. Thus, the n x n matrix [T] formed by the vector $\mathbf{t}$ and the n-1 orthonormalized vectors $\mathbf{X}_i$ (i=1, 2, . . ., n-1) is expressed as

$$[T] = (\mathbf{t}, \mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_{n-1}) \qquad (12)$$

the elements of which indicate the turn ratios of the ideal multi-winding transformers the network with which is a 2n-port one (See Appendix II). Therefore, an n-way power divider with any degree of coupling can be realized theoretically by forming the network shown in Fig. 4, if arbitrary turn ratios of coils could be realized.

IV. Experimental Results

To examine whether or nor the design theory of the generalized n-way power divider with frerrite toroids for CATV and/or MATV system is valid, we performed an experiment for generalized 2-way power divider with the

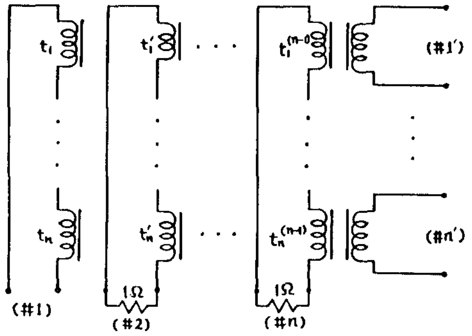

Fig. 4. Generalized n-way power divider.

coupling degree of 14 dB.

Since the degree of coupling to TAP is 14 dB, the matrix [T], the elements of which are formed by turn ratios of multiwinding transformers, is given by

$$[T] = \begin{bmatrix} 0.9799 & \pm 0.1995 \\ 0.1995 & \mp 0.9799 \end{bmatrix} \qquad (13)$$

Using eq. (13), we can construct a 2-way power divider of generalized form readily. Figure 5 shows the circuit constructed and tested, where the ferrite cores used are DL-2 opw 7-3.5-3-1.2H2 manufactured by Nippon Ferrite Co., Ltd in Japan. Figure 6 shows the experimental frequency characteristics for the generalized 2-way power divider with coupling degree of 14 dB constructed. We can see from Fig. 6 that a power divider with generalized circuit form has very good characteristics in the frequency range from 5 to about 500 MHz. When, however, the operating frequency becomes higher than 500 MHz, the responses deteriorate rapidly. It is regarded as due to a resonance occuring at about 1,000 MHz. To remove the resonance and broaden the bandwidth is a remainder of work to be solved. Furthermore, the experimental investigation for the generalized power dividers more than 2-way divisions is also a remainder of topic to be performed.

V. Conclusion

An improved weakly-coupled power divider for CATV and/or MATV systems was proposed.

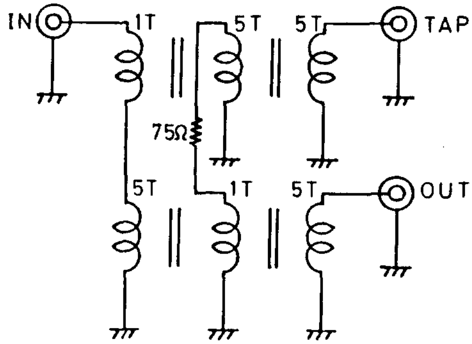

Fig. 5. Generalized (New) type 2-way power divider with coupling degree of 14 dB contstructed and tested. (The coils in 5T and 1T are of Polyurethane wires of o. 0.2mϕ and 0.29mmϕ, respectively.)

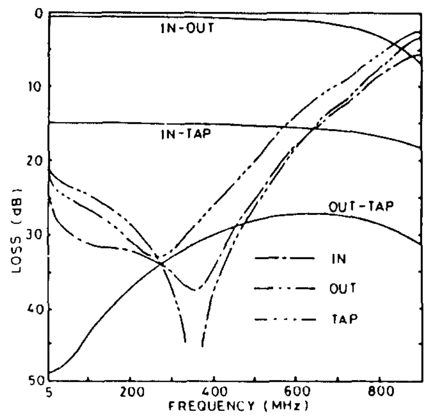

Fig. 6. Experimental frequency characteristics for the generalized two-way power divider shown in Fig. 5.

By which the degree of freedom in design and density of coupling interval can be significantly increased.

Furthermore, the design theory of the generalized n-way power divider was proposed. The generalized n-way power divider with the coupling degree of 14 dB to TAP was fabricated and tested, The frequency characteristics of which are very good in the range of 5 to 500 MHz in spite of errors of turn numbers. Thus, the validity of the proposed design theory of the generalized n-way power divider was confirmed.

Appendix I. Multi-Winding Ideal Transformer

The input and output characteristics of a conventional 2-port ideal transformer are expressed in terms of voltage V and V′, current I and I′, and turn ratio n by eq. (A1.1) with reference to Fig. A1.1.

$$V = nV'$$
$$nI = I' \qquad \text{(A1. 1)}$$

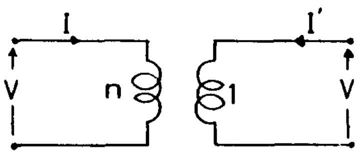

Fig. A1. 1

Generalizing to 2n-port multiwinding ideal transformer as shown in Fig. A1. 2, we get the following relations;

$$[V] = \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ \cdot \\ V_n \end{bmatrix} = \begin{bmatrix} C_{11} & \cdots & C_{1n} & V_1' \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ C_{n1} & & I_1 & V_n' \end{bmatrix} = [C]\,[V']$$

$$[C]^t\,[I] = \begin{bmatrix} C_n & \cdots & C_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot\ \cdot \\ \cdot & & \cdot \\ C_{1n} & \cdots & C_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ \cdot \\ \cdot \\ \cdot \\ I_n \end{bmatrix} \begin{bmatrix} I_1' \\ \cdot \\ \cdot \\ \cdot \\ I_n' \end{bmatrix} = [I'] \qquad \text{(A1. 2)}$$

The scattering matrix for the network shown in Fig. A1. 2 is readily obtained from the following relations,

$$[a]+[b]=[V], \qquad [a']+[b']=[V']$$
$$[a]+[b]=[I], \qquad [a']-[b']=[I'] \qquad \text{(A1. 3)}$$

where [a] and [a′] indicate the incident waves, and [b] and [b′] indicate the reflected waves, respectively. Substituting eq. (A1.2) into eq. (A1. 3) and arranging in terms of incident and reflected waves, we get

$$\begin{bmatrix} b \\ b' \end{bmatrix} = \begin{bmatrix} -(U+CC^t)^{-1}(U-CC^t) & 2(U+CC^t)^{-1}C \\ 2C^t(U+CC^t)^{-1} & (U+C^tC)^{-1}(U+C^tC) \end{bmatrix} \begin{bmatrix} a \\ a' \end{bmatrix} \qquad \text{(A1. 4)}$$

where a, b, a′, b′, C, and **U** mean [a], [b], (a′), (b′), (C) and identity matrix, respectively. Since $CC^t = C^tC = U$ when, especially, C is an orthogonal matrix, we get

$$\begin{bmatrix} b \\ b' \end{bmatrix} = \begin{bmatrix} 0 & C \\ C^t & 0 \end{bmatrix} \begin{bmatrix} a \\ a' \end{bmatrix} \qquad \text{(A1. 5a)}$$

or

$$[S] = \begin{bmatrix} 0 & C \\ C^t & 0 \end{bmatrix} \qquad \text{(A1. 5b)}$$

Therefore the scattering matrix [S] indicates that of an all-pass network.

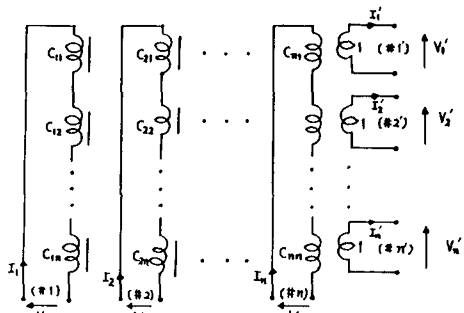

Fig. A1. 2

Appendix II. Rank of Dissipation Matrix [Q] and Derivation of Expanded Unitary Scattering Matrix

The dissipation matrix [Q] for the scattering matrix [S] given by eq. (11) is expressed as follows;

$$[Q]=[U]-[S]^*[S]=\begin{bmatrix} 1-\|tt\|^2 & 0 \\ 0 & 1-t\cdot t^t \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 1-P \end{bmatrix} \qquad \text{(A2. 1)}$$

where, $\mathbf{t}=(t_1, t_2, \ldots, t_n)$ and

$$P=[P]=\mathbf{t}\cdot\mathbf{t}^t=\begin{bmatrix} t_1 t_2 & \cdots & t_1 t_n \\ \vdots & & \vdots \\ t_n t_1 & \cdots & t_n t_n \end{bmatrix}$$

The relation between the matrix [P] and its eigenvalues λ can be obtained from the eigenvalue equation as

$$[P]\,\mathbf{X}=\lambda\mathbf{X} \qquad \text{(A2.2)}$$

Since $[P]\mathbf{t}=(\mathbf{t}\cdot\mathbf{t}^t)\mathbf{t}=|\mathbf{t}|^2\mathbf{t}$, an eigenvalue of [P] is $|\mathbf{t}|^2$ the eigenvector for which is $\mathbf{t}$. Since, now, there exist n-1 linearly independent normalized vectors X_i (i=1, 2, . . . , n-1) orthogonal to $\mathbf{t}$ in n-dimensional space and

$$[P]X=\mathbf{t}\cdot\mathbf{t}^t X=O\cdot X_i \qquad \text{(A2.3)}$$

the other eigenvalues of the matrix [P] are 0 of (n-1)-fold degenerate the eigenvectors for which are X_i(i=1, 2, . . . , n-1) orthogonal to $\mathbf{t}$. Furthermore, the matrix [P] can be diaagonalized by the orthogonal matrix $[T]=(\mathbf{t}, X_1, \ldots, X_{n-1})$ as follows since it is a real symmetric square matrix.

$$[P]=[T]\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}[T]^t \qquad \text{(A2.4)}$$

Thus,

$$[U]-[P]=[T]\,[T]^t-[P]$$

$$=[T]\begin{bmatrix} 0 & & & & & \\ & 1 & & & 0 & \\ & & 0 & & & \\ & & & \ddots & & \\ & 0 & & & & \\ & & & & & 1 \end{bmatrix}[T]^t. \qquad \text{(A2.5)}$$

The diagnoal matrix eq. (A2.5) is obviously of rank n-1. Since the matrix [T] is a non-singular matrix, the matrix [P] is also of rank n-1 and the dissipation matrix is of rank n-1 as the same as [P].

Furthermore, the expanded unitary scattering matrix [Σ] expressed by eq. (A2. 6) using the above normalized eigenvectors.

$$[\Sigma]=\begin{bmatrix} 0 & \mathbf{t}^t & 0 & \cdots & 0 \\ \mathbf{t} & 0 & X_1 & \cdots & X_{n-1} \\ 0 & X_1^t & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & X_{n-1}^t & 0 & \cdots & 0 \end{bmatrix} \qquad \text{(A2. 6)}$$

Now, if the port-number (1, 2, 3, . . . , n+1, n+2, . . ., 2n) are changed for the new port-number (1, n+1, n+2, . . ., 2n, 2, 3, . . ., n), the new expanded unitary scattering matrix [Σ] is given by

$$[\Sigma]=\begin{bmatrix} 0 & T \\ T & 0 \end{bmatrix} \qquad \text{(A2. 7)}$$

the network with which can be formed as was shown in Fig. 4.

References

[1] Dong Il Kim, et al., "Optimum design of the power dividers with ferrite toroids for CATV and/or MATV systems," *IEEE Trans. on CE*, vol. CE-29, no. 1, pp.27-38, Feb. 1983.

[2] H. J. Carlin et al., *Network Theory: An Introduction to Reciprocal and Non-reciprocal Circuits*, Prentice-Hall, Englewood Cliffs, N.J., ch. 4, 1964.

[3] Y. Oono and K. Yasuura, "Synthesis of finite passive 2n-terminal networks with prescribed scattering matrices," *Annales des Telecommunication,* vol. 9, pp.73-80, 1954.

[4] D.C. Youla, "Single frequency synthesis from a prescribed scattering matrix," *IRE Trans. on Circuit Theory,* vol. CT-4, pp.340-344, Dec. 1959.

[5] A.A.M. Saleh, "Theorems on match and isolation in multiport networks," *IEEE Trans. on MTT,* vol.28, no. 4, pp.428-429, Apr. 1980.

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