◉ Technical Paper

Induced Steady Flow around Two Oscillating Circular Cylinders+

Y.K. Suh*, S.W. Oh* and K.M. Han**

振動하는 두 원통둘레의 定常流에 관한 研究

徐龍權·吳世旭·韓健模

Key Words : Oscillation(振動), Cylinder(圓筒), Streaming Motion(潮流), Eddy(소용돌이), Asymptotic Solution(漸近解), Stokes Layer(Stokes層)

초 록

두 개의 동일한 원통이 두 중심점을 잇는 선에 對하여 수직인 方向으로 振動할 때 發生되는 定常流에 對한 解를 구하였다. 관련된 諸變數들간의 어떤 假定下에서 本 問題는 Stokes問題가 될 수 있었다. 流動함수를 級數展開하였으며, 級數의 保數函數는 penta-diagonal matrix를 풀어 求할 수 있었다. 解의 結果에 의하면 원통 주위에 몇개의 소용돌이가 생겼으며 그 소용돌이의 數는 두 원통間의 거리에 따라 달라졌다. 두 원통間의 거리가 커짐에 따른 漸近的 性質을 單一 원통의 경우와 比較하여 確認하였다.

1. Introduction

Recently great efforts have been concentrated on the wave forces acting on a single object or array of objects as the model of the pile(s) of the offshore structures.[1),2)] However. most of the studies have been concerned only with the oscillating motion of the wave. Batchelor derived the steady streaming velocity on the body immersed in the standing wave or in the progressive wave.[3)] He pointed out that this streaming motion has consequences of practical importance, such as transport of sediment in the flow field. He also suggested that the motion of wave could be replaced by the oscillating motion of the body in analyzing the streaming motion.

This paper finds an analytic solution for the motion of the streaming flow induced by two equal circular cylinders oscillating in the same frequency to the direction normal to the center-line. Schlichting[4)] introduced the linearized equation of motion for the amplitude smaller than the body scale, and obtained the analytic solution valid in the thin Stokes layer near the body. The steady streaming motion around a single circular cylinder is then obtained and compared with the experimental result.

Longuet-Higgins[5)] extended the oscillating flow problem to the water waves.

Very recently, Jenkins and Inman[6)] used the matched asympotic expansion method to obtain higher-order solution for a sphere in the water waves. They found that from the boundary layer several secondary motions were excited and torques and forces were arised due to these flow motions. In

+Presented at the 1986 KCORE Autumn Conference
*Member, Mechanical Engineering Dep't, Dong-A University
**Member, Ocean Engineering Dep't, Dong-A University

this paper, bipolar coordinate is introduced to find the series solution for the steady streaming motion around the two equal circular cylinders. Assumptions are made that the amplitude and the Stokes layer thickness are much smaller than the diameter of the cylinder, and that the amplitude is much smaller than the Stokes layer thickness. The latter condition is designed in order to make the steady motion be a Stokes problem. Matched asymptotic expansion method is utilized to formulate the problem with two small parameters.

The coefficient functions of the series expansion are obtained by solving the penta-diagonal matrix using the Thomas algorithm. Numerical results for a few cases of distances between the two cylinders show that the induced motion forms eddies around the bodies. The direction of the current at a large distance from the body is reversed as ξ_0 is increased. It is also found that the solution tends to that of a single cylinder as the distance between the two obstacles is increased.

2. Formulation of the Problem

Geometry to be concerned in the present study is as shown in Fig. 1. The frame is fixed to the two circular cylinders so that oscillation of the obstacles is replaced by oscillation of the fluid. We consider only the case in which oscillation takes place in the normal direction to the line connecting two centers(center-line). The flow field is assumed to be two-dimensional, incompressible and laminar. The governing equation for this problem will be

$$\frac{\partial \nabla_*^2\psi^*}{\partial t^*}-\frac{\partial(\psi^*, \nabla_*^2\psi^*)}{\partial(x^*, y^*)}=\nu\nabla_*^4\psi^* \qquad (1)$$

where $\nabla_*^2=\frac{\partial^2}{\partial x^{*2}}+\frac{\partial^2}{\partial x^{*2}}$ is the Laplacian operator and ν is the kinematic viscosity of the fluid. As the flow field should be symmetric about the x^*-aris, only the upper half domain $y^*>0$ is of our concern. Characteristic quantities will be U_∞, d, ω upon which the dimensionless variables are defined as follows.

$$(x, y)=\frac{(x^*, y^*)}{d}, \quad \psi=\frac{\psi^*(x^*, y^*, t^*)}{U_\infty d}, \quad t=\omega t^* \qquad (2)$$

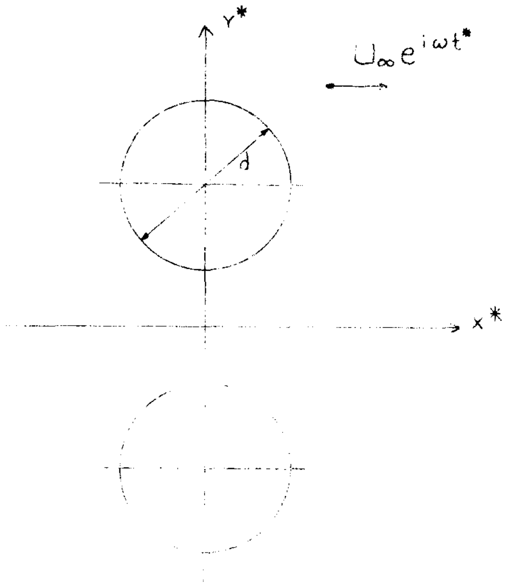

Fig. 1 Geometry of the problem

Then (1) reduces to

$$\frac{\partial \nabla^2\psi}{\partial t}-\epsilon_1\frac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)}=\epsilon_2^2\nabla^4\psi \qquad (3)$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ and

$$\epsilon_1=\frac{U_\infty}{\omega d}, \quad \epsilon_2^2=\frac{\nu}{\omega d^2} \qquad (4)$$

Boundary conditions are

$$\psi(x, 0, t)=0\ ; \ \frac{\partial\psi}{\partial s}=\frac{\partial\psi}{\partial n}=0 \text{ on } n=0\ ;$$

$$\frac{\partial\psi}{\partial y}\rightarrow \text{Real}(e^{it}) \text{ as } \sqrt{x^2+y^2}\rightarrow\infty \qquad (5)$$

where n and s are the local coordinates along the body and normal to it, respectively.

3. Matched Asymptotic Expansion Method

For the time being, ϵ_1 and ϵ_2^2 themselves only are assumed to be small. Formal expansion of ψ in the matched asymptotic expansion method is

$$\psi=\psi_0(x, y, t;\ \epsilon_2)+\epsilon_1\psi_1(x, y, t;\ \epsilon_2)+\epsilon_1^2\psi_2(x, y, t;\ \epsilon_2)+\cdots \qquad (6)$$

Substituting (6) into (3), we get

$$\left[\frac{\partial}{\partial t}\nabla^2\psi_0-\epsilon_2^2\nabla^4\psi_0\right]$$

$$+\epsilon_1\left[\frac{\partial}{\partial t}\Delta^2\psi_1-\epsilon_2^2\nabla^4\psi_1-\frac{\partial(\psi_0,\nabla^2\psi_0)}{\partial(x,y)}\right]$$
$$+\epsilon_1^2\left[\frac{\partial}{\partial t}\nabla^2\psi_2-\epsilon_2^2\nabla^4\psi_2-\frac{\partial(\psi_0,\nabla^2\psi_1)}{\partial(x,y)}-\frac{\partial(\psi_1,\nabla^2\psi_0)}{\partial(x,y)}\right]$$
$$+\epsilon_1^3\left[\frac{\partial}{\partial t}\nabla^2\psi_3-\epsilon_2^2\nabla_t\psi_3-\frac{\partial(\psi_0,\nabla^2\psi_2)}{\partial(x,y)}-\frac{\partial(\psi_2,\nabla^2\psi_0)}{\partial(x,y)}-\frac{\partial(\psi_1,\nabla^2\psi_1}{\partial(x,y)}+\cdots=0\right. \tag{7}$$

The leading order equation of (7) is

$$\frac{\partial}{\partial t}\nabla^2\psi_0-\epsilon_2^2\nabla^4\psi_0=0 \tag{8}$$

For small ϵ_2^2 we expect the thin inner layer(Stokes layer) out of which the appropriate governing equation will be

$$\frac{\partial}{\partial t}\nabla^2\psi_0=0$$

Solution of this can be represented in a separable form as follows:

$$\psi_0=\psi_p(x,y)\mathrm{Real}(e^{it})$$

where ψ_p satisfies

$$\nabla^2\psi_p=0 \tag{9}$$

$$\psi_p(x,0)=0\ ;\ \frac{\partial\psi_p}{\partial s}=0 \text{ on } n=0\ ;\ \psi_p\to y \text{ as } \sqrt{x^2+y^2}\to\infty \tag{10}$$

Problem of (9) with (10) is just the potential flow past the two circular cylinders ; the uniform flow direction is clearly along the x-axis. Series solution of (9) with (10) is[7]

$$\psi_p=a\left[1+\sum_{k=1}^{\infty}\{2(-1)^k e^{-k\xi}+C_k\sinh k\xi\}\right]\cos k\eta\ ;$$

$$C_k=\frac{-2(-1)^k}{1e^{k\xi_0}\sinh k\xi} \tag{11}$$

Where ξ,η are the bipolar coordinates defined as

$$x=\frac{a\sin\eta}{\cosh\xi+\cos\eta},\quad y=\frac{a\sinh\xi}{\cosh\xi+\cos\eta},$$

$$a=\frac{1}{2}\sinh\xi_0$$

and ξ_0 is ξ value of $n=0$.[7] The relationship between (x,y) and (ξ,η) is shown in Fig.2. Note that series (11) is convergent except for $\xi=0$ near which the following expression can be used.

$$\psi_p=a\left[\frac{\sinh\xi}{\cosh\xi+\cos\eta}+\sum_{k=1}^{\infty}C_k\sinh k\xi\cos k\eta\right]$$

With the inner variables

$$\psi_0=\sqrt{2}\,\epsilon_2\,\varphi_0(s,Y),\quad n=\sqrt{2}\,\epsilon_2 Y$$

the leading order equation for the Stokes layer

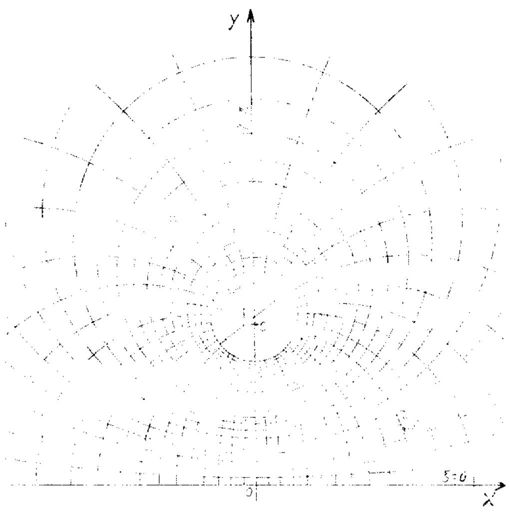

Fig. 2 The bipolar coordinate system

becomes

$$\frac{\partial}{\partial t}\frac{\partial^2}{\partial Y^2}\varphi_0=\frac{1}{2}\frac{\partial^4}{\partial Y^4}\varphi_0 \tag{12}$$

Boundary conditions are

$$\varphi_0(s,0)=\frac{\partial\varphi_0}{\partial Y}(s,0)=0\ ;\ \frac{\partial\varphi_0}{\partial Y}\to V_0(s)\mathrm{Real}(e^{it}) \text{ as } Y\to\infty,$$

$$V_0(s)=\frac{\partial\psi_p}{h\partial\xi}=\frac{D_1}{2}+\sum_{k=1}^{\infty}E_k\cos k\eta,$$

$$D_k=\frac{2k(-1)^{k+1}(\cosh k\xi_0+\sinh k\xi_0)}{e^{k\xi_0}\sinh k\xi_0}$$

$$E_k=D_k\cosh\xi_0+\frac{1}{2}(D_{k-1}+D_{k+1}),\quad (D_0=0)$$

h in the above expression is the scale factor in the (ξ,η) plane.

Solution of (12) can also be expressed in a separable form as follows :

$$\varphi_0=V_0(s)\ \mathrm{Real}\,[W_0(Y)e^{it}]$$
$$=Y+\frac{(1-i)}{2}[\exp\{-(1+i)Y\}-1] \tag{13}$$

The second order equation for ψ_1 in (7) is

$$\frac{\partial}{\partial t}\nabla^2\psi_1-\epsilon_2^2\nabla^4\psi_1=\frac{\partial(\psi_0,\nabla^2\psi_0)}{\partial(x,y)} \tag{14}$$

where the right-hand side contains the steady term as well as the unsteady one since $\{\mathrm{Real}(e^{it})\}^2=\frac{1}{2}\cos 2t+\frac{1}{2}$. Consequently we split ψ_1 as follows :

$$\psi_1=\widehat{\psi_1}+\widetilde{\psi_1} \tag{15}$$

where $\frown$ stands for "unsteady" and $\sim$ for "steady". The unsteady part of (14) is then

$$\frac{\partial}{\partial t}\nabla^2\hat{\psi}_1 - \epsilon_2^2\nabla^4\hat{\psi}_1 = \left[\frac{\partial(\psi_0, \nabla^2\psi_0)}{\partial(x, y)}\right]^{\wedge} \tag{16}$$

We first of all consider the outer region. In this region (16) becomes

$$\frac{\partial}{\partial t}\nabla^2\psi_1 = 0 \tag{17}$$

The solution of the homogeneous equation (17) is $\hat{\psi}_1 = 0$ since the boundary conditions are also homogeneous. For the inner region, by $\hat{\psi}_1 = \sqrt{2}\epsilon_2\hat{\Psi}$, (16) becomes

$$\frac{\partial}{\partial t}\frac{\partial^2}{\partial Y^2}\hat{\Psi}_1 - \frac{1}{2}\frac{\partial^4}{\partial Y^4}\psi_1 = \frac{1}{2}V_0\frac{dV_0}{ds}\text{Real}\,[(W_0W_0''' - W_0'W_0'')e^{2it}]$$

$$\hat{\Psi}_1(s, 0) = \frac{\partial\hat{\Psi}_1}{\partial Y}(s, 0) = 0$$

$$\frac{\partial\hat{\Psi}_1}{\partial Y} \to 0 \text{ as } Y \to \infty \tag{18}$$

Solution of (18) is

$$\hat{\Psi}_1 = V_0\frac{dV_0}{ds}\text{Real}\,\{\hat{W}_1(Y)e^{2it}\}$$

$$\hat{W}_1(Y) = \frac{1+i}{4\sqrt{2}}[e^{-\sqrt{2}(1+i)Y} - 1] + \frac{i}{2}Ye^{-(1+i)Y}$$

Now we consider the lowest order equation of the steady part in the outer region. The leading order of the steady part in (7) is either $O(\epsilon_1\epsilon_2^2)$ (the second term of the second line) or $O(\epsilon_1^3)$(the fifth term of the fourth line) depending on the order of $\frac{\epsilon_2}{\epsilon_1}$. The full equation for ψ_1 is then

$$-\frac{\partial(\tilde{\psi}_1, \nabla^2\tilde{\psi}_1)}{\partial(x, y)} = \frac{1}{\tilde{R}_e}\nabla^4\tilde{\psi}_1 \tag{19}$$

This is the steady Navier-Stokes equation for $\tilde{\psi}_1$ with

$$\tilde{R}_e = \left(\frac{\epsilon_1}{\epsilon_2}\right)^2 \tag{20}$$

The inhomogeneous boundary condition may be set by matching with the solution in the Stokes layer. The inner expansion for $\tilde{\psi}_1$ is $\tilde{\psi}_1 = \sqrt{2}\,\epsilon_2\,\tilde{\Psi}_1$ upon which the steady part of the second line in (7) reduces to

$$\frac{\partial^4}{\partial Y^4}\tilde{\Psi}_1 = \frac{1}{2}V_0\frac{dV_0}{ds}\text{ Real}\,[W_0\bar{W}_0''' - W_0'\bar{W}_0'']$$

$$\tilde{\Psi}_1(s, 0) = \frac{\partial\tilde{\Psi}_1}{\partial Y}(s, 0) = 0$$

$$\frac{\partial\tilde{\Psi}_1}{\partial Y} \to \text{finite as } Y \to \infty \tag{21}$$

where the symbol bar denotes the conjugate of a complex variable. Note that the boundary condition for $Y \to \infty$ is inevitable due to the intrinsic nature of the solution. Solution of (21) is

$$\tilde{\Psi}_1 = V_0\frac{dV_0}{ds}\tilde{W}_1(Y)$$

$$\tilde{W}_1(Y) = \frac{13}{8} - \frac{3}{4}Y - \frac{1}{8}e^{-2Y} - e^{-Y}\left(\frac{3}{2}\cos Y + \sin Y + \frac{1}{2}Y\sin Y\right) \tag{22}$$

Matching condition requires that

$$\left[\frac{\partial\tilde{\psi}_1}{h\partial\xi}\right]_{\xi_0} = \left[\frac{\partial\tilde{\Psi}_1}{\partial Y}\right]_{Y\to\infty} = -\frac{3}{4}V_0\frac{dV_0}{ds} \tag{23}$$

where h is the scale factor in the (ξ, η) plane. Now the streaming motion of the fluid in the outer region is governed by (19) with one of the boundary conditions (23) at the wall. In the present analysis we consider only the case for small $\tilde{\text{Re}}$ i.e, for

$$\epsilon_1 \ll \epsilon_2 \tag{24}$$

by which (19) is approximated to the leading order as follows :

$$\nabla^4\hat{\psi}_1 = 0 \tag{25}$$

Before attempting to solve the Stokes problem (25), we contemplate about the three restrictions for ϵ_1, ϵ_2 in connection with the physical quantities.

By $U_\infty = \omega\delta_1$ where δ_1 is the amplitude, (4) becomes

$$\epsilon_1 = \frac{\delta_1}{a}, \quad \epsilon_2 = \frac{\sqrt{\nu/\omega}}{d}$$

which means that ϵ_1 is the dimensionless amplitude. ϵ_2 turned out to be a dimensionless thickness of the Stokes layer. Hence in terms of those quantities the three assumptions made in the present study are that the amplitude δ_1 and the Stokes layer thickness $\sqrt{\nu/\omega}$ are much smaller than the body size and that $\delta_1 \ll \sqrt{\nu/\omega}$. For 20°C water, the above three restrictions correspond roughly to $\frac{1}{d^2} \ll \omega \ll \frac{1}{\delta_1^2}$ [1/*sec*] where d and δ_1 are in *mm*.

3. Steady Streaming Motion Based on the Stokes Model

In the (ξ, η) plane (25) becomes

$$\nabla_1^2\{(\cosh\xi + \cos\eta)^2\nabla_1^2\tilde{\psi}_1\} = 0 \tag{26}$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}$$

The boundary conditions are

$$\tilde{\psi}_1(0, \eta) = \frac{\partial^2}{\partial\xi^2}\tilde{\psi}_1(0, \eta) = 0 \tag{27a}$$

$$\widetilde{\psi}_1(\xi_0, \eta)=0 \tag{27b}$$

$$\frac{\partial\widetilde{\psi}_1}{h\partial\xi}(\xi_0, \eta)=-\frac{3}{4}V_0\frac{dV_0}{ds} \tag{27c}$$

$$\widetilde{\psi}_1(\xi, \eta)=\frac{\partial^2\widetilde{\psi}_1}{\partial\eta^2}(\xi, \eta)=0 \text{ at } \eta=0, \ \pi \tag{27d}$$

The boundary condition (27c) can be expressed in a series form as follows:

$$\left[\frac{\partial\widetilde{\psi}_1}{\partial\xi}\right]_{\xi_0}=\frac{3}{8}\sum_{k=1}^{\infty}H_k\sin k\eta \tag{28}$$

where

$$H_k=D_1kE_k+F_k+G_k$$

$$F_k=\sum_{i=1}^{k-1}iE_iE_{k-i} \ (F_1=0), \quad G_k=k\sum_{i=1}^{\infty}E_iE_{i+k}$$

Considering (28), we may try to find the solution of the Stokes problem in the form

$$\widetilde{\psi}_1=\sum_{k=1}^{\infty}A_k(\xi)\sin k\eta$$

Then (26) reduces to

$$A_k''-k^2A_k=B_k \tag{29a}$$

$$f_k''-k^2f_k=0 \tag{29b}$$

$$f_k=\frac{1}{4}B_{k-2}+\cosh\xi\ B_{k-1}+\left(\frac{1}{2}\cosh 2\xi+1\right)B_k + \cosh\xi\ B_{k+1}+\frac{1}{4}B_{k+2}, \ (B_k=0 \text{ for } k<1) \tag{29c}$$

The boundary conditions for A_k are

$$A_k(0)=A_k''(0)=0, \quad A_k(\xi_0)=0, \quad A_k'(\xi_0)=\frac{3}{8}H_k \tag{30}$$

Since from the first of (30) $B_k(0)=0$ for all k, the general solution of (29b) must be the basis function $\frac{\sinh k\xi}{\sinh k\xi_0}$ multiplied by an arbitrary constant.

Basis for each B_k can be decided upon giving condition at $\xi=0$ or that at $\xi=\xi_0$.

In the present study, we choose $b_k(\xi_0)=1$ where b_k represents the basis of B_k so that with an arbitrary constant Γ_{1k}

$$B_k(\xi)=\Gamma_{1k}\,b_k(\xi) \tag{31}$$

Then from (29c) we obtain

$$\frac{1}{4}b_{k-2}+\beta b_{k-1}+\alpha b_k+\beta b_{k+1}+\frac{1}{4}b_{k+2}=r_k \tag{32}$$

where

$$b_k=0 \text{ for } k<1 \text{ and } k>K$$

$$\alpha=\frac{1}{2}\cosh 2\xi+1, \quad \beta=\cosh\xi$$

$$r_k=\begin{cases}(\alpha+\beta+1/4)\sinh k\xi/\sinh k\xi_0 \text{ for } k=1,\ K\\ (\alpha+2\beta+1/4)\sinh k\xi/\sinh k\xi_0 \text{ for } k=2, K-1\\ (\alpha+2\beta+1/2)\sinh k\xi/\sinh k\xi_0 \text{ for } 3\leq k\leq K-2\end{cases}$$

and K is the number of equations of (32) after truncated. It is assumed here that the series (28) be well convergent for all $\xi\geq 0$ so that the truncated amount could not be that significant. Thomas algorithm can be applied to (31) to yield

$$b_k=c_kb_{k+1}+d_kb_{k+2}+e_k \tag{33}$$

where

$$c_k=\{d_k(c_{k-2}/4+\beta)+\beta\}/(-V), \quad e_k=[r_k-\{e_{k-2}/4+e_{k-1}(c_{k-2}/4+\beta)\}]/V,$$

$$d_k=1/(-4V), \quad V=c_{k-1}(c_{k-2}/4+\beta)+d_{k-2}/4+\alpha;$$

$$c_k=d_k=e_k=0 \text{ if } k<1.$$

First, c_k, d_k and e_k are calculated from $k=1$ up to $k=K$. Then (33) is used to evaluate b_k from $k=K$ to $k=1$. Solution of (29a) can be obtained by use of the undetermined coefficient method as follows:

$$A_k=\Gamma_{0k}\sinh k\xi+\Gamma_{1k}g(\xi) \tag{34}$$

$$g(\xi)=\frac{1}{2k}\left[e^{k\xi}\int_0^{\xi}b_k(z)e^{-kz}dz-e^{-k\xi}\int_{\xi_0}^{\xi}b_k(z)e^{kz}dz\right.$$

where Γ_{0k} and Γ_{1k} are determined from the boundary conditions (30) as

$$\Gamma_{0k}=-T_{23}T_{12}/(T_{11}T_{22}-T_{12}T_{21})$$

$$\Gamma_{1k}=T_{11}T_{23}/(T_{11}T_{22}-T_{12}T_{21})$$

$$T_{11}=\sinh k\xi_0, \quad T_{21}=\cosh k\xi_0, \quad T_{12}=\frac{1}{2k}g(\xi_0)$$

$$T_{22}=\frac{1}{2k}\left[e^{k\xi_0}\int_0^{\xi_0}b_k(z)e^{-kz}dz+e^{-k\xi_0}\int_0^{\xi_0}b_k(z)e^{kz}dz\right]$$

$$T_{23}=\frac{3}{8k}H_k$$

4. Asymptotic Solution near the Body for a Large ξ_0

Near the body for a large ξ_0, the solution of the present problem should approach that of a single cylinder. In this case the boundary condition (28) is approximated as

$$\left[\frac{\partial\psi_1}{\partial\xi}\right]_{\xi=\xi_0}=\frac{3}{2}\sin 2\eta \tag{35}$$

which is of the same form as that of a single cylinder of the radius 0.5.

Further, (26) reduces to the leading order to

$$\nabla_2^2\left(\frac{\partial^2}{\partial r_1^2}+\frac{\partial}{r_1\partial r_1}+\frac{\partial^2}{r_1^2\partial\eta^2}\right)\widetilde{\psi}_1=0 \tag{36}$$

where $\nabla_2^2=r_1\frac{\partial}{\partial r_1}\left(r_1\frac{\partial}{\partial r_1}\right)+\frac{\partial^2}{\partial\eta^2}$, and $r_1=e^{\xi_0-\xi_1}$ is twice the distance of a point in the flow field from the center of the upper cylinder. It is clear that since the asymptotic solution is only valid for large

ξ the boundary conditions on $\xi=0$ can not be imposed. Instead.

$$\widetilde{\psi}_1 \rightarrow \text{finite as } \xi_1 \rightarrow \infty \tag{37}$$

shall be used. Solution for (36) with (27b), (27d), (35) and (37) is well known ;

$$\widetilde{\psi}_1 = -\frac{3}{4}(1-e^{-2\xi_1}) \sin 2\eta \tag{38}$$

Note that $\widetilde{\psi}_1 \rightarrow -\frac{3}{4} \sin 2\eta$ for $\xi_1 \rightarrow \infty$.

5. Numerical Results and Discussions

Numerical calculation is simple and straightforward. The discretization scheme is applied only to the integration. Streamline patterns ($\psi_1=$const.) are given in Fig.3 for $\xi_0=1.0$, 2.0. In the quarter

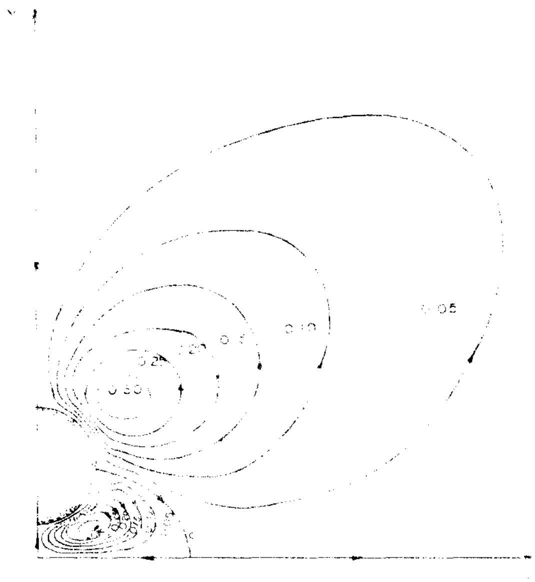

Fig.3(a) Streamline pattern for $\xi_0=1.0$

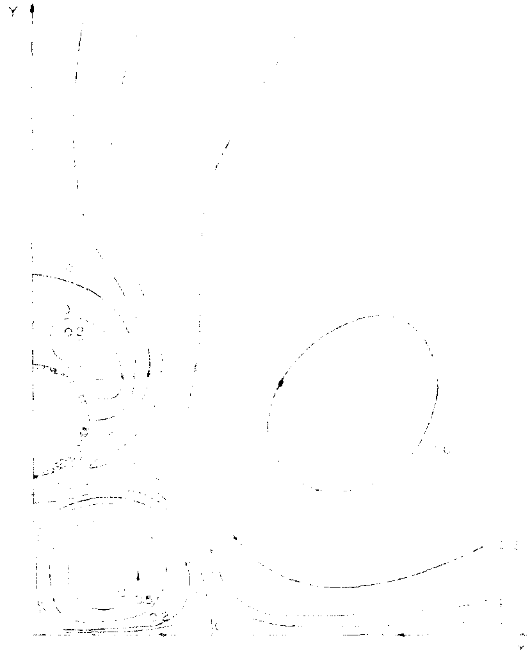

Fig.3(b) Streamline pattern for $\xi_0=2.0$

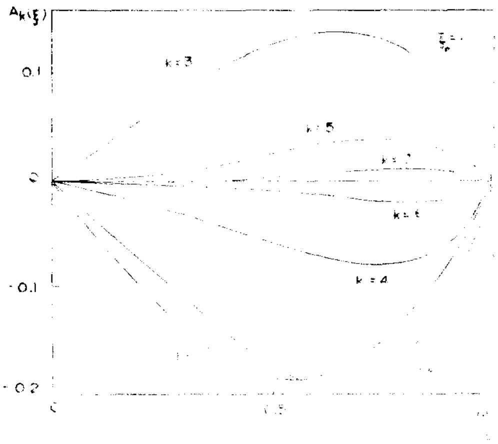

Fig.4(a) Coefficient functions for $\xi_0=1.0$

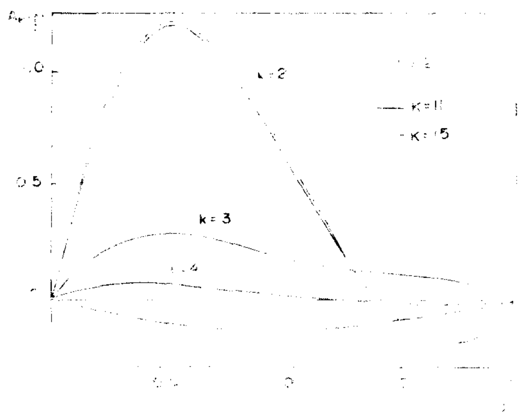

Fig.4(b) Coefficient functions for $\xi_0=2.0$

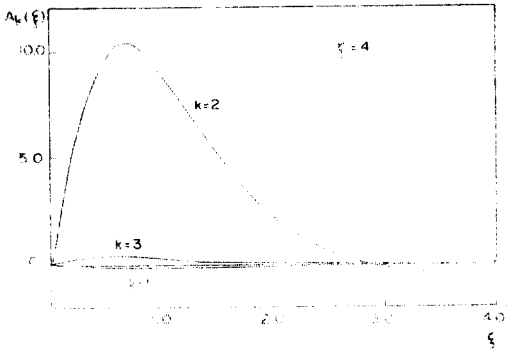

Fig. 4(c) Coefficient functions for $\xi_0 = 4.0$

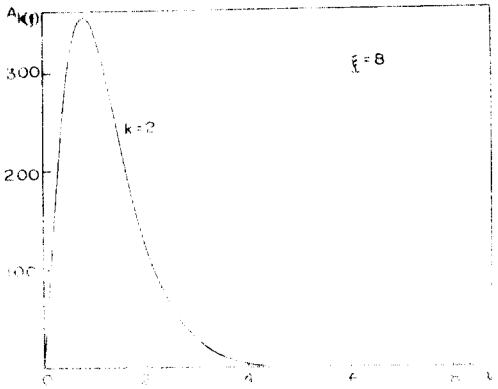

Fig. 4(d) Coefficient functions for $\xi_0 = 8.0$

plane ($x \geq 0$, $y \geq 0$), there exist two bubbles for $\xi_0 = 1.0$. For $\xi_0 = 2.0$, there occurs one additional bubble so that the direction of the flow at a large distance is reversed. Coefficient functions are plotted in Fig. 4. We note that $A_2(\xi)$ becomes more dominent as ξ_0 is increased ; this is consistent with the asymptotic nature of the solution as analyzed in section 4. The effect of truncation on the accuracy of the numerics is confirmed for $\xi_0 = 2.0$ as shown in Fig. 4(b) and Table 1. It is seen that this effect is negligible for the change of the number of equations from 15 to 11. The asymptotic nature of the present numerical solution is shown in Fig. 5 ; it is noted that the range of validity of the asymptotic solution (38) is widened and the numerical solution tends to it as ξ_0 is increased. Finally the velocity at $\xi = 0$, $\eta = \frac{\pi}{4}$ are calculated and shown in Table 2. It is to be noted that the order of the velocity on the line of symmetry $y = 0$ is not so much decreased as the distance between the two cylinders increases.

The numerical solution presented in this study is valid near the body within the distance of the body scale. As is common to the low Reynolds number flow problem, the better solution will be obtained by using the Oseen approximation method.

Table 1 Maximum absolute values of coefficient functions for $\xi_0 = 2.0$ for $K = 11, 15$

k	$K = 11$	$K = 15$
[illegible]	[illegible]	[illegible]
2	0.12[illegible] E + 01	[illegible]
3	[illegible]	[illegible]
4	[illegible] E − 01	0.7614 E [illegible] 01
5	0.128[illegible] E [illegible]	0.3424 E [illegible]
6	[illegible] E − [illegible]	0.2444 E − 02
7	0.2881 E − 03	0.3845 E [illegible]
8	0.3631 E − 04	0.5671 E − 04
9	0.[illegible]978 E − [illegible]	0.78[illegible]2 E − 05
10	[illegible] E − [illegible]	[illegible]
11	0.42[illegible] E − 07	[illegible]
12	—	[illegible] E [illegible]
13	—	[illegible]
14	—	[illegible] E − 0[illegible]
15	—	0.1967 E − 10

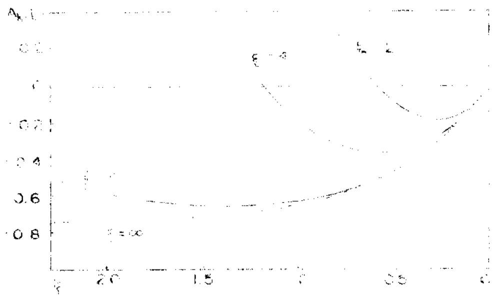

Fig. 5 $A_2(\xi_1)$ for different ξ_0 values

Table 2 Velocities at $\xi = 0$, $\eta = \frac{\pi}{3}$ for $\xi_0 = 1.0, 2.0, 4.0, 8.0$

[illegible]	[illegible]
1.0	− 1.677
2.0	4.286
4.0	3.583
8.0	1.757

Another important point of view is that the Stokes approximation treated in this study may be far from being able to describe the flow field in the severe ocean. For the actual application, therefore, equation (19) must be solved numerically for large $\widetilde{Re}$. One method is to use the discretization scheme for the full Navier-Stokes equation to the finite region around the bodies. The other method is if possible to use the matched asymptotic expansion method for the three locally characterized region; the thin boundary layer near the bodies where the classical boundary layer equation is valid, the jet flow region where the asymptotic solution for the boundary layer equation may be attained, and the inviscid region where the Euler equation is valid. Further investigation concerning this problem will be of great interest.

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