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On a Subset Selection Procedure Based on Hodges-Lehmann Estimators

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ABSTRACT

In this paper, we study on a subset selection procedure based on Hodges-Lehmann estimators derived from the Wilcoxon test. To estimate the standard error of the Hodges-Lehmann estimators, the biweight A-estimator of scale is used. The Pitman efficiency of the proposed rule is compared with the Gupta's rule and the trimmed-means rule through a small-sample Monte Carlo study. The results show that the proposed rule satisfies the P^*-condition and is very efficient in various heavy-tailed distributions.

1. Introduction

Populations are often characterized by the values of a parameter. Then, it often happens that an experimenter is interested in the "best" (usually largest or smallest) parameter and he wants to select the "best" population associated with the best parameter. In this paper we are interested in selecting a non-empty subset of the given k populations which contains the best population associated with the largest location parameter.

Let $\pi_1, \cdots, \pi_k$ be $k(\geqq 2)$ independent populations with continuous and symmetric distributions $F(x-\theta_1), \cdots, F(x-\theta_k)$, respectively. Let $\theta_{[1]} \leqq \cdots \leqq \theta_{[k]}$ be the ordered values of the location parameters θ_i's. The best population is the population associated with $\theta_{[k]}$. The event of selecting a subset of populations containing the best one is called a correct selection (CS). In subset selection procedures it is usually required that any given procedure R satisfy the P^*-condition, i.e.

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$$\inf_{\theta \in \Omega} P(CS|R) \geqq P^*$$

for a preassigned P^*, where $\Omega = \{\theta_1, \cdots, \theta_k | -\infty < \theta_i < \infty,\ i=1, \cdots, k\}$.

There have been many studies on the subset selection problem. The subset selection procedure based on the sample means has been developed by Gupta (1956, 1965). But, because the sample mean is very sensitive to a few gross errors, the performance of the selection rule based on the sample means is not satisfactory for heavy-tailed distributions. We thus want to develop some robust procedures which perform reasonably well for heavy-tailed distributions and are insensitive to gross errors.

For robust subset selection procedures, we can consider some procedures based on robust location estimators, such as sample medians, trimmed means, Huber's M-estimators, Hodges-Lehmann(H-L) estimators, etc. Subset selection procedures based on sample medians are investigated by Gupta and Singh(1980) and Lorenzen and McDonald(1981). Song, Chung and Bae(1982) considered subset selection procedures based on trimmed means and H-L estimators.

Assuming that the populations have a common known variance, Gupta and Huang (1974) proposed a robust subset selection procedure based on the H-L estimators derived from the Wilcoxon signed-rank test. While, Song, Chung and Bae(1982) studied a subset selection procedure based on the H-L estimators in the case of common unknown variance. In developing the procedure they used the median absolute deviation(MAD) to estimate the asymptotic variance of the H-L estimators. But the empirical results show that the MAD usually underestimates the standard error of the H-L estimators in heavy-tailed distributions. As a result, their proposed rule based on the H-L estimators with the MAD significantly violates the P^*-condition in heavy-tailed distributions. These facts motivate us to consider some other subset selection procedures based on the H-L estimators.

In this paper, we use the A-estimator proposed by Lax(1985) to estimate the standard error of the H-L estimators and develop a subset selection rule based on the H-L estimators with the A-estimator. The Pitman efficiency of the proposed selection rule is the same as that of the Wilcoxon test in hypothesis testing. Through a small-sample Monte Carlo study the proposed rule is also compared with the other selection rules. The results of the study show that the proposed rule is successful in satisfying the P^*-condition and also efficient in heavy-tailed distributions.

2. Studentization of H-L Eatimators

2.1. A-Estimators of Scale

Lax (1985) compared the performance of several families of scale estimators by a Monte Carlo study. In his study, he introduced a family of scale estimators called A-estimators, which are finite sample approximations to the asymptotic variance of M-estimators of location.

Let $X_1, \cdots, X_n$ be a random sample from a continuous and symmetric distribution $F(x-\theta)$ with a location parameter θ. The class of M-estimators, which was originally introduced by Huber (1964), may be defined in terms of a continuous function ψ. That is, the M-estimator of location, given a robust scale estimator S and a positive constant c, is defined to be the solution T_n of the equation

$$\sum_{i=1}^{n} \psi((X_i - T_n)/cS) = 0.$$

Under appropriate regularity conditions, the asymptotic variance of $\sqrt{n}\ T_n$ is given by

$$A_\psi(T,\ F) = \frac{\int_{-\infty}^{\infty} (cS\psi((x-T)/cS))^2 dF(x)}{\left[\int_{-\infty}^{\infty} \psi'((x-T)/cS)\ dF(x)\right]^2}$$

where $T = T(F)$ is the functional of θ. Setting T to be an estimator of the location, S_0 to be an initial estimator of the scale, and $u_i = (X_i - T)/cS_0$, Lax (1985) used the following finite sample approximation to $A_\psi(T,\ F)$:

$$S_\psi^2 = \frac{n}{n-1} n (cS_0)^2 \left(\sum_{i=1}^{n} \psi^2(u_i) / (\sum_{i=1}^{n} \psi'(u_i))^2\right). \tag{2.1}$$

He called this S_ψ an A-estimator of scale.

Note that a number of different A-estimators can be derived from different ψ-functions. But, according to the comparative study by Lax (1985), the redescending biweight and modified sine A-estimators are most successful. In this paper, we want to use the biweight A-estimator to estimate the standard error of H-L estimators. The biweight A-estimator S_b can be written as follows:

$$S_b = \frac{n\left[\sum_{|u_i|<1} (X_i - T)^2 (1-u_i^2)^4\right]^{\frac{1}{2}}}{(n-1)^{\frac{1}{2}} \left|\sum_{|u_i|<1} (1-u_i^2)(1-5u_i^2)\right|}. \tag{2.2}$$

To compute the values of $u_i=(X_i-T)/cS_0$, Lax(1985) used the sample median for T and the the MAD for S_0. The results of a Monte Carlo study performed by Lax (1985) show that the biweight A-estimators with $c=9$ and $c=10$ are noticeably robust in symmetric long-tailed distributions compared with the sample standard deviation, MAD and M-estimators of scale. Throughout this paper, we use $c=10$ for the biweight A-estimator S_b.

2.2. t-Distribution Approximation

The H-L estimator of the location parameter θ based on the Wilcoxon signed-rank test is given by

$$\hat{\theta}=\underset{i\leq j}{\text{med}}\{(X_i+X_j)/2\}.$$

Hodges and Lehmann(1963) showed that $\sqrt{n}(\hat{\theta}-\theta)$ has a limiting normal distribution with mean 0 and variance

$$\sigma_H^2=1\Big/\left[12\left(\int f^2(x)\,dx\right)^2\right].$$

Since $\sigma_H^2=\pi\sigma^2/3$ in the case of normal distribution, we may estimate the asymptotic variance of $\sqrt{n}\hat{\theta}$ by $\pi\hat{\sigma}^2/3$ with some robust scale estimator $\hat{\sigma}$. For example, Song, Chung and Bae(1982) used the MAD defined by $\text{MAD}=1.48\,\underset{i}{\text{med}}\;|X_i-\underset{j}{\text{med}}(X_j)|$ and applied this estimator to develop a robust subset selection rule based on H-L estimators.

But, the results of Monte Carlo study performed by Song, Chung and Bae(1982) indicate that the MAD estimator usually underestimates the standard error of the H-L estimators in heavy-tailed distributions. In this paper, we want to estimate the standard deviation of $\sqrt{n}\hat{\theta}$ by using the biweight A-estimator explained in Section 2.1. We thus want to approximate the distribution of the quotient

$$\sqrt{n}(\hat{\theta}-\theta)/(\sqrt{\pi/3}\;S_b)$$

by the t-distribution with $n-1$ degrees of freedom.

In order to examine the appropriateness of this approximation, we performed a small sample ($n=10$) simulation study on the standard normal, double exponential, slash, Cauchy, and contaminated normal distributions. The slash distribution is the distribution of the quotient Z/U, where Z and U are normal $N(0,1)$ and uniform $U(0,1)$ random variables, respectively, and Z and U are stochastically independent. The *cdf* of the ε-contaminated normal distribution is given by

$$F(x)=(1-\varepsilon)\Phi(x)+\varepsilon\Phi(x/\sigma),$$

where Φ is the *cdf* of standard normal.

To make a comparative study, we include the studentization of the trimmed means. For simplicity, we assume that $g=n\alpha$ is an integer throughout this paper. Then, the α-trimmed mean is defined by

$$\overline{X}_\alpha=\frac{1}{h}\sum_{i=g+1}^{n-g}X_{(i)},$$

where $h=n-2g=n-2n\alpha$ and $X_{(1)}\leqq\cdots\leqq X_{(n)}$ are the order statistics. To estimate the standard error of the α-trimmed means, Song, Chung and Bae (1982) used the Tukey-McLauglin's estimator $S_\alpha/\sqrt{h}$, where $S_\alpha^2=SS(\alpha)/(h-1)$ with $SS(\alpha)$ the Winsorized sum of squares defined by

$$SS(\alpha)=(g+1)(X_{(g+1)}-\overline{X}_\alpha)^2+(X_{(g+2)}-\overline{X}_\alpha)^2+\cdots$$
$$+(X_{(n-g-1)}-\overline{X}_\alpha)^2+(g+1)(X_{(n-g)}-\overline{X}_\alpha)^2. \qquad (2.3)$$

Huber (1970) suggested a t-distribution with $h-1$ degrees of freedom to approximate the distribution of $\sqrt{h}(\overline{X}_\alpha-\theta)/S_\alpha$.

In our simulation study we include the following three quotients:

$$\frac{\sqrt{n}(\overline{X}-\theta)}{S}\ ;\ \frac{\sqrt{h}(\overline{X}_\alpha-\theta)}{S_\alpha}\ ;\ \frac{\sqrt{n}(\hat{\theta}-\theta)}{\sqrt{\pi/3}\,S_b}, \qquad (2.4)$$

where $\overline{X}$ is the sample mean and S is the usual sample standard deviation. The distribution of the quotient $\sqrt{h}(\overline{X}_\alpha-\theta)/S_\alpha$ is to be approximated by the t-distribution with $h-1$ degrees of freedom, and the others with $n-1$ degrees of freedom.

The normal random variates with and without contamination are generated by using the subroutine GGNPM in IMSL. The uniform random variates are generated by GGUBT. The inverse integral transformation is applied to generate the double exponential and Cauchy random variates.

The simulation was repeated 1,000 times for each distribution with sample size $n=10$. The number of values exceeding $t(\nu,p)$ divided by 1,000 is the estimated probability of $P(T\geqq t(\nu,p))$, where $t(\nu,p)$ is the $100(1-p)$ percentile of the t-distribution with ν degrees of freedom and T is one of the quotients in (2.4). These estimated probabilities for $p=.40$, .25, .10, .05, .025, and .01 are summarized in Table 1.

The results in Table 1 and the empirical cdf's which are not reported in this paper show that the t-distribution approximation of the quotient of H-L estimators with biweight A-estimator is successful. The H-L estimators with $c=9$ and 11 gave almost the same results. But these are not reported in this paper. The quotient based on H-L estimators with the modified sine A-estimator was also studied. But, the results were not satisfactory comparing with the others. The quotient of H-L estimators with MAD was also inappropriate in heavy-tailed distributions.

Table 1. Estimated Probability of $P(T\geq t(\nu, p))$ Based on 1,000 Replications

Distribution	T \ p	.40	.25	.10	.05	.025	.01
Normal	T_1	.382	.245	.089	.044	.030	.011
	T_2	.388	.254	.111	.055	.022	.008
	T_3	.403	.253	.094	.056	.035	.015
Double Exponential	T_1	.399	.254	.111	.053	.025	.009
	T_2	.399	.267	.102	.056	.022	.008
	T_3	.404	.247	.090	.051	.018	.005
Contaminated Normal ($\varepsilon=.1$, $\sigma=5$)	T_1	.441	.282	.098	.039	.010	.003
	T_2	.394	.263	.110	.060	.029	.008
	T_3	.399	.248	.103	.058	.034	.012
Slash	T_1	.419	.303	.087	.036	.007	.002
	T_2	.439	.292	.096	.036	.013	.003
	T_3	.406	.265	.106	.058	.030	.013
Cauchy	T_1	.411	.320	.084	.021	.009	.001
	T_2	.432	.284	.097	.028	.009	.004
	T_3	.401	.273	.125	.063	.031	.012

T_1, T_2, T_3 denote the quotients $\sqrt{n}(\bar{X}-\theta)/S$, $\sqrt{h}(\bar{X}_a-\theta)/S_a$, $\sqrt{n}(\hat{\theta}-\theta)/\sqrt{\pi/3}S_b$, respectively.

3. A Subset Selection Procedure Based on H-L Estimators

3.1. Selection Procedures

Consider again the k independent populations $\pi_1, \cdots, \pi_k$ with cdf's $F(x-\theta_1), \cdots, F(x-\theta_k)$ respectively. We assume that they have a common unknown variance σ^2. Let $X_{i1}, \cdots, X_{in}$ be an independent sample of size n from π_i, $i=1, \cdots, k$. Here we are interested in selecting a subset which contains the "best" population associated with the largest parameter $\theta_{[k]}$.

Gupta (1956, 1965) has suggested the following subset selection rule R based on the sample means:

$$R: \text{Select } \pi_i \text{ if and only if } \bar{X}_i \geqq \bar{X}_{[k]} - dS_\nu/\sqrt{n}, \qquad (3.1)$$

where $\bar{X}_i$ is the sample mean from the ith population, $\bar{X}_{[k]}$ is the largest sample mean, $d=d(k,n,P^*)$ is to be determined to satisfy the P^*-condition and S_ν^2 is the usual pooled sample unbiased estimator for the common variance σ^2 with $\nu=k(n-1)$ degrees of freedom. Assuming normality, the constant d is a solution of

$$\int_0^\infty\int_{-\infty}^\infty \Phi^{k-1}(u+dy)\phi(u)q_\nu(y)\,du\,dy = P^* \qquad (3.2)$$

where ϕ is the pdf of standard normal and $q_\nu(y)$ is the density of $\chi_\nu/\sqrt{\nu}$. Gupta and Sobel (1957) have tabulated the values of d for various combinations of k, ν and P^*.

As a robust procedure, Song, Chung and Bae (1982) have proposed a subset selection rule R_α based on the trimmed means. The selection rule R_α is defined by

$$R_\alpha: \text{Select } \pi_i \text{ if and only if } \bar{X}_{i\alpha} \geqq \bar{X}_{[k]\alpha} - d_\alpha S_{\nu\alpha}/\sqrt{h}, \qquad (3.3)$$

where $\bar{X}_{i\alpha}$ is the α-trimmed mean associated with the population π_i, $\bar{X}_{[k]\alpha}$ is the largest α-trimmed mean, $h=n-2n\alpha$, and $S_{\nu\alpha}/\sqrt{h}$ is the pooled sample estimated standard error of the α-trimmed mean defined by

$$S_{\nu\alpha} = \left\{\sum_{i=1}^k SS_i(\alpha)/(k(h-1))\right\}^{\frac{1}{2}}$$

with $SS_i(\alpha)$ the Winsorized sum of squares defined in (2.3) for the ith sample. The constant d_α is also to be determined to satisfy the P^*-condition. But, since the distributions of $\bar{X}_\alpha$ and $S_{\nu\alpha}$ are too complicated to determine d_α, they used d in (3.2) with $\nu=k(h-1)$ for d_α in (3.3). The results of a simulation study presented in Section 2.2 indicate that the distribution of $\sqrt{h}(\bar{X}_\alpha-\theta)/S_\alpha$ is successfully approximated by a t-distribution with $h-1$ degrees of freedom. This is a heuristic reason why d_α can be approximated by d with $\nu=k(h-1)$.

We now propose a subset seletion rule R_b based on the H-L estimators with the biweight A-estimators of scale, which is defined as follows:

$$R_b: \text{Select } \pi_i \text{ if and only if } \hat{\theta}_i \geqq \hat{\theta}_{[k]} - d_b\sqrt{\pi/3}\,S_{\nu b}/\sqrt{n}, \quad (3.4),$$

where $\hat{\theta}_i$ is the H-L estimator of θ_i, $\hat{\theta}_{[k]}$ is the largest of $\hat{\theta}_i$'s, $\sqrt{\pi/3}\,S_{\nu b}/\sqrt{n}$ is the pooled sample estimated standard error of the H-L estimator, defined by

$$S_{\nu b} = \left\{\sum_{i=1}^k S_{ib}^2/k\right\}^{\frac{1}{2}}$$

with S_{ib} the biweight A-estimator defined in (2.2) for the ith sample. Since the exact values of d_b satisfying the P^*-condition are not available, we want to approximate d_b in (3.4) by the values of d in (3.2) with $\nu=k(n-1)$. The studentization considered in Section 2.2 justifies this approximation.

We now consider the asymptotic relative efficiency of the proposed selection rule R_b relative to the Gupta's rule R in (3.1). Following the definition of Hsu (1980), the Pitman efficiency of the subset selection procedures is closely related to that of tests. According to Theorem 4.3 of Hsu (1980), the Pitman efficiency of the procedure R_b based on H-L estimators relative to the Gupta's procedure R is the same as that of the Wilcoxon test relative to the t-test.

3.2. Small-Sample Monte Carlo Study

In this section we compare the subset selection rules discussed in the previous section by a small-sample Monte Carlo study. The rules included in our simulation study are the Gupta's procedure R, the rule R_α based on the trimmed means with $\alpha=.1$, and the proposed rule R_b based on the H-L estimators and the biweight A-estimator of scale with $c=10$.

We compare these rules in various underlying distributions including the standard normal, double exponential, Cauchy, slash, and contaminated normal distributions. The parameters we considered are the equally spaced parameters, i.e.

$$\theta_i=\theta_0+(i-1)\delta\sigma, \quad i=1,\cdots,k$$

where $\delta>0$ is a given constant and σ is a standard deviation of each underlying distribution. For Cauchy distribution, we used the value of $F^{-1}(0.84)-F^{-1}(0.5)=1.8326$ for σ, which corresponds to 1 in standard normal case. Since the slash distribution is similar to the Cauchy distribution we also used 1.8326 for σ. The constants used in our simulation study are $k=5$, $n=10$ and $\delta\sqrt{n}=0, 2, 4$. For the contaminated normal distribution $\varepsilon=.1$ and $\sigma=5$ are used. 1,000 replications were performed for each distribution and for each value of δ.

When $\delta\sqrt{n}=0$, the average number of selected populations divided by 1,000 can be interpreted as the empirical P^*. These values are presented in Table 2. The empirical results show that the proposed rule R_b successully satisfies the P^*-condition for various distributions. The parametric selection rule R significantly violates the P^*-condition for

$P^*=.75$ in extremely heavy-tailed distributions such as slash or Cauchy. The trimmed-means selection rule R_a slightly violates the P^*-condition in normal and double exponential distributions. The maximum shortage of the empirical values of $P(CS)$ from the required values P^* is .011 which occurs at $P^*=.750$ in the normal distribution. The proposed rule R_b also slightly violates the P^*-condition, but the violation is not significant. The maximum shortage of the empirical values of $P(CS)$ is .002 which occurs at $P^*=.975$ in the contaminated normal distribution. The performances of the rules based on the H-L estimators with MAD and with the modified sine A-estimator were not satisfactory in terms of the P^*-condition. Thus, these results are not reported in this paper.

Table 2. Empirical P^* Based on 1000 Replications

Distribution	Rule	P^* .75	.90	.95	.975	.99
Normal	R	.753	.900	.954	.978	.992
	R_a	.739	.896	.946	.970	.988
	R_b	.749	.902	.949	.974	.991
Double Exponential	R	.752	.901	.950	.975	.991
	R_a	.743	.895	.945	.975	.992
	R_b	.783	.910	.954	.981	.992
Contaminated Normal ($\varepsilon=.1, \sigma=5$).	R	.737	.891	.944	.974	.991
	R_a	.763	.908	.956	.980	.991
	R_b	.763	.904	.953	.973	.989
Slash	R	.685	.912	.965	.984	.996
	R_a	.775	.923	.964	.984	.996
	R_b	.777	.906	.954	.975	.989
Cauchy	R	.687	.918	.966	.986	.996
	R_a	.769	.921	.964	.985	.993
	R_b	.770	.906	.952	.974	.989

Next, we compare the small-sample efficiencies of the selection rules by the concept of relative efficiency suggested by Song and Oh(1981). The relative efficiency of the procedure R' relative to the procedure R is defined by

$$e(R', R)=\frac{E(S|R)}{E(S|R')} * \frac{P(CS|R')}{P(CS|R)},$$

where $E(S|R)$ is the expected number of populations to be selected with a given rule R. To estimate the relative efficiencies of the considered rules relative to the Gupta's

Table 3. Empirical Relative Efficiencies Based on 1000 Replications

Efficiency	$\delta\sqrt{n}$	P^* .75	.90	.95	.975	.99
				Normal		
$e(R_a, R)$	2	.997	.980	.999	.993	.990
	4	.985	.982	.988	.980	.993
$e(R_b, R)$	2	.973	.958	.980	.973	.968
	4	.993	.994	.992	.977	.980
				Double Exponential		
$e(R_a, R)$	2	1.035	1.036	1.042	1.038	1.053
	4	1.037	1.041	1.051	1.054	1.041
$e(R_b, R)$	2	1.058	1.036	1.029	1.033	1.043
	4	1.038	1.047	1.058	1.060	1.039
				Contaminated Normal		
$e(R_a, R)$	2	1.173	1.204	1.210	1.226	1.221
	4	1.062	1.130	1.171	1.199	1.234
$e(R_b, R)$	2	1.203	1.229	1.243	1.257	1.262
	4	1.072	1.145	1.189	1.208	1.244
				Slash		
$e(R_a, R)$	2	1.396	1.330	1.273	1.221	1.165
	4	1.708	1.686	1.647	1.598	1.508
$e(R_b, R)$	2	1.619	1.519	1.450	1.381	1.284
	4	2.034	2.070	2.005	1.945	1.861
				Cauchy		
$e(R_a, R)$	2	1.559	1.480	1.429	1.359	1.286
	4	1.758	1.777	1.783	1.750	1.691
$e(R_b, R)$	2	1.887	1.822	1.743	1.664	1.542
	4	2.198	2.298	2.294	2.248	2.164

rule R, we counted the number of times that each population is selected in 1,000 replications. These relative efficiencies are summarized in Table 3.

The results in Table 3 show that the performances of the robust selection procedures R_a and R_b are satisfactory. Only in normal distribution, they are slightly worse than the rule based on sample means. The proposed rule has almost the same efficiency as that of the rule based on trimmed means in the normal and double exponential distributions. But, in heavy-tailed distributions such as contaminated normal, slash and Cauchy, the proposed rule is consistently better than the rule based on trimmed means.

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