Density Variation within Specimen as Affected by Vibration

진동으로 인한 모래 공시체내의 밀도변화에 관한 연구

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Abstrct

To obtain meaningful data of the tests for cyclic loading triaxial test, cyclic loading simple shear test ; and shake table studies, it is necessary to test uniformly densified specimens. However, there is still some question about the assumed uniform density within a specimen when subjected to the process of densification.

A study is conducted to investigate the density variation within the specimen and analyze the effect of various parameters during the process of vibratory densification.

It is found that variation of the ideally graded sand having a homogeneous initial density results in large inhomogeneities within the specimen after vibration. The degree of density variation within the speimen becomes more pronounced by the gradation of sand, surcharge and the intensity of acceleration.

要 旨

교번 삼축 압축시험, 교번 전단응력 시험, 액상화 시험 및 진동 테이블 시험 등을 하기 위하여 제작되는 공시체로서 잘 다져진 모래를 사용하게 되며 진동에 의해 다져진 모래의 공시체가 균일한 밀도를 갖는다는 가정하에 상기한 실험을 하여 왔다. 지금까지 이러한 가정에 대한 연구가 충분히 되어 있지 않았으며 신뢰도 높은 실험을 하기 위하여는 공시체내에 모래의 밀도가 균일한 공시체를 제작하는 것이 중요하다.

이 연구에서는 진동에 의해서 모래 공시체를 제작할 경우 공시체내에 밀도의 변화가 발생하는지 여부와 각 진동인자가 밀도변화에 미치는 영향을 조사하였다. 이 연구 결과 진동 전에 균일한 밀도를 갖는 공시체가 진동 후엔 공시체의 높이에 따라 상당한 밀도의 변화가 발생하였음을 보였으며 이러한 현상은 과재 하중, 입도 및 진동가속도의 크기에 따라 상당한 영향을 받는 것으로 밝혀졌다.

1. Introduction

Most of the significant tests relating to densification of sand have assumed uniform density throughout the specimens before and after vibratory densification. To obtain meaningful data on the behavior of sand, it is necessary to test homogeneous specimens. However, there is still

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some question about the assumed uniform density within a specimen when subjected to the process of densification. Because of the insufficient amount of studies relating to the density variation within a specimen, misinterpretation of the behavior of a sand may have occurred in the past when specimens were assumed to have uniform density throughout after vibration. The studies relating to the density variation within a specimen are important to those concerned with the preparation of test specimens in such tests as cyclic loading triaxial test, cyclic loading simple shear test, and shake table studies. It is important, therefore, that the density variation within a specimen be investigated along with various parametric effects on vibratory densification of sand.

In order to ascertain whether significant non-uniformities are produced within the specimens during the process of vibratory densification, a study is conducted to investigate the density variation within the specimen as characterized by the change in packing of particles during the vibration throughout the specimens.

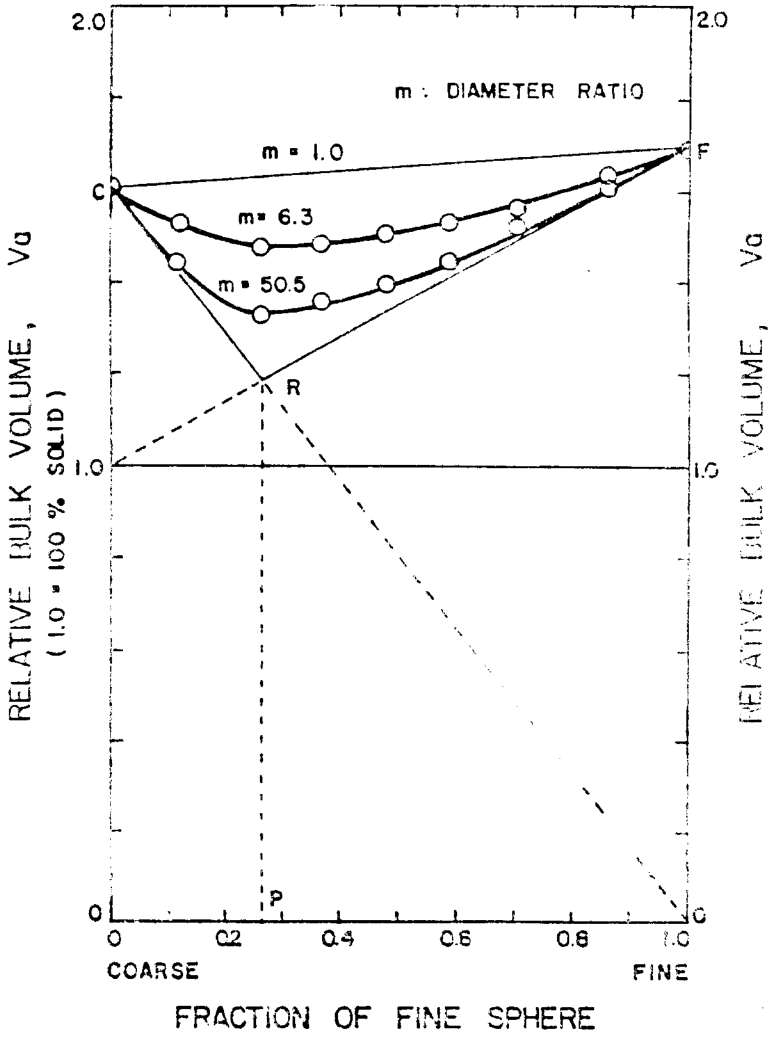


Fig. 1. Packing of particles in two-size

2. Theoretical Background

2.1 Packing of Particles

In reference to Fig. 1, it was shown that for a mixture having a diameter ratio equal to unity ($m=1$) the relative bulk volume, V_a was given by the line segment, CF, having the equation:

$$V_a = C_x + F_z \tag{1}$$

where x and z are the compositions of the coarse and fine particles, respectively. C and F represent the experimentally determined relative bulk volume of the coarse and fine particles, respectively, The values of V_a, for an infinite diameter ratio ($m=\infty$), are given by the line joining F with unity at the left side of the diagram and the line segment $\overline{CR}$ is obtained by joining C with the zero point at the right side of the diagram, the equation of these lines being:

$$V_{ax} = C_x \tag{2a}$$

$$V_{az} = x + F_z \tag{2b}$$

where V_{ax} and V_{az} are the ordinates of the points on the C and F plane.

In order to obtain a mathematical equation for the V_a curve, an empirical equation was developed by Westman (9, 10), the general form of which is as follows:

$$a^2 + 2\mathrm{mab} + b^2 = 1 \tag{3}$$

where a, b are the constants determined empirically.

If the diameter ratio m is equal to 1.0, then Eq, 3. becomes a linear equation of the form:

$$a + b = 1 \tag{4}$$

This equation represents the straight line $\overline{CF}$ in Fig. 1. For $m=\infty$, it takes the form:

$$ab = 0 \tag{5}$$

This represents the line segment $\overline{\mathrm{CRF}}$ in Fig. 3.1. By making appropriate substitutions for a and b, and using Eqs. (1) and (2), Eq. (3) can be converted into an equation which becomes Eq. (1) for $m=1$ and either one of Eq. (2a) or Eq. (2b) for $m=\infty$, In this way, the following equation can be obtained for computation of V_a:

$$\left(\frac{V_a - C_x}{F}\right)^2 + 2m\left(\frac{V_a - C_x}{F}\right)\left(\frac{V_a - x - F_z}{C-1}\right) + \left(\frac{V_a - x - F_z}{C-1}\right)^2 = 1.0 \tag{6}$$

Eq. (6) can be given in a more explicit form, but it is more convenient for computations in the form given. Westman's experimental data which are represented by the data points in Fig. 1 gave a reasonably good fit to the values computed by Eq. (6). Fig. 2 and 3 represent the relationship that exists for a mixture of three-size particles.

The theory can be verified by examining gradation curves when the change of gradation occurs.

Gradation curve A in Fig. 4 represents the grain size distribution of a mixture which is ideally mixed with 32, 39 and 29 percent for coarse, medium and fine particles, respectively. Their average diameter ratios are 14 : 5 : 1 for coarse, medium and fine particles, respectively.

Gradation curves B and C have different combination of coarse, medium and fine particles from that of gradation A. Gradation B has a combination of 32, 33 and 55 percent for coarse, medium and fine particles, respectively. The curve has a greater percentage of fine and a smaller particles compared to the gradation A. Similarly, gradation curve C has a combination of 45, 35 and 20 percent for coarse, medium and fine particles, respectively. The curve C has a greater percentage of coarse and a smaller percentage of fine particles compared to the gradation A. All curves A, B and C have a same diameter ratio.

This shows that the density of a mixture is directly related to gradation in that an ideal gradation (curve A) will produce a maximum density and gradations other than ideal, for

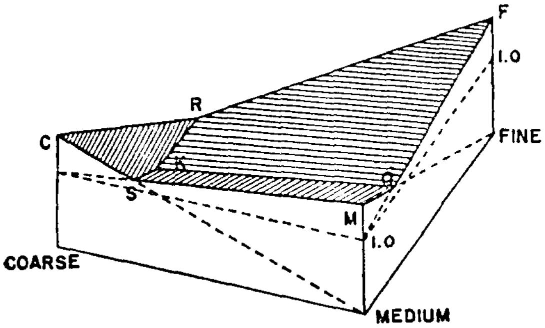


Fig. 2. Packing of particles in three-size system

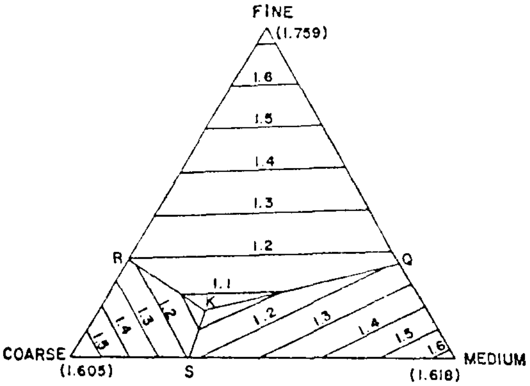


Fig. 3. Packing of particles in three-size system $(m=\infty)$

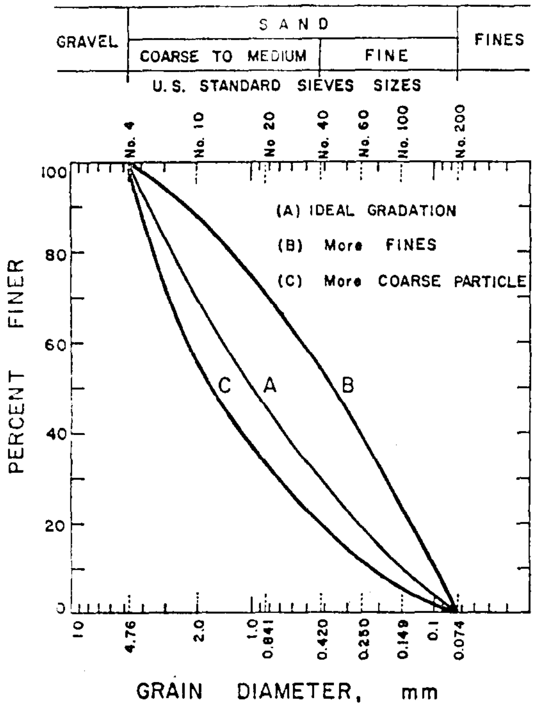


Fig. 4. Gradation curves for three-size system

example, curve B or C in Fig. 4 will produce lower densities. Consequently, if vibration causes a change in gradation of a mixture from one type to the other, there will be a change in density resulting from vibrational process,

3. Test Procedures

A series of laboratory tests were conducted on a uniform sand and an ideally graded sand to investigate the relative effects of various parameters associated with vibratory densification.

3.1 Material

A brief description of the engineering properties is given here. The specific gravity of the sand was 2.65. The ideally graded sand was artificially mixed with a definite amount for each size based on Weymouth's theory of particles interference (11). The uniform sand used in the tests passd sieve No. 40 and was retained all on the sieve No. 60. The Uniformity Coefficients were 8.9 for the ideally graded sand 1.4 for the uniform sand. The maximum density for the uniform and the ideally graded sands, obtained by a series of tests, were 12.90 and 15.47KN/m^3, respectively (here, 1.0 ton/m^3=9.807 KN/m^3, 1.0 kg/cm^2=98.07 Kpa). Fig. 5 shows the grain size distribution of the sand tested.

3.2 Experimental Setup and Procedure

An overall experimental setup is shown in Fig. 6. It consists of a lucite mold, the vibration table, the oscilloscope and control console, the pressure supply and the hydraulic power supply

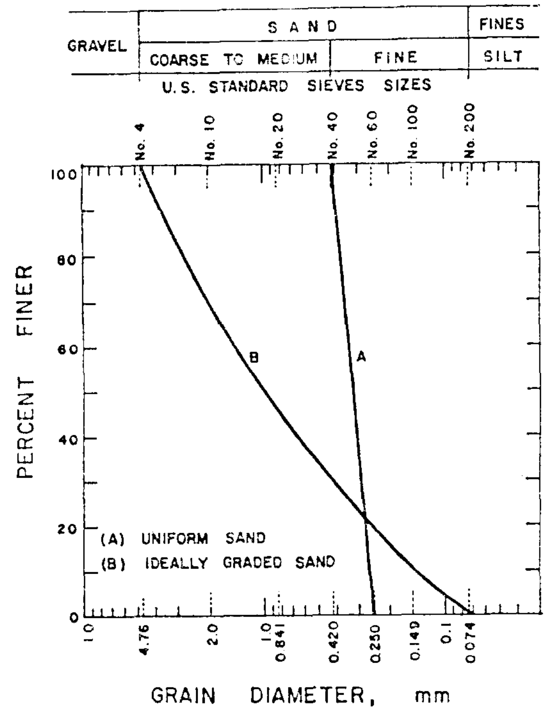


Fig. 5. Gradation curves of material tested

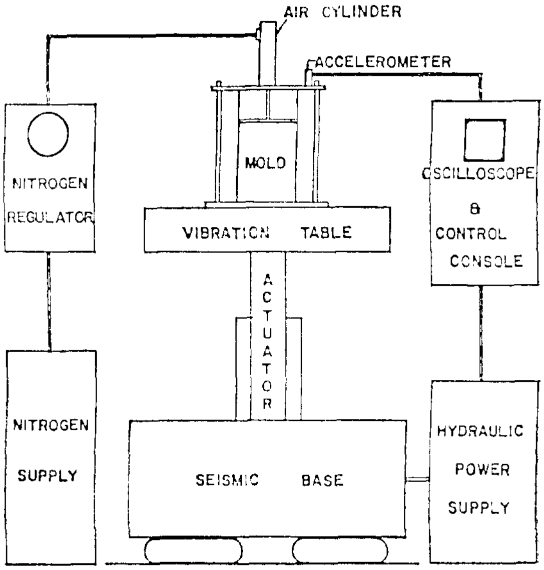


Fig. 6. Schematic diagram of experimental set-up

which serves to drive the vibration table. The lucite mold was 19.05cm in height and 12.7cm in diameter. As shown in Fig. 7, the mold was made up of five tiered sections of equal height and held together by vertical screws and fastened to the table of a Model 840 Servohydraulic Vibration Test System made by MTS System Corporation. A Model 308 A Quartz Accelerometer was mounted upright on the upper plate of the mold for the response measurement. Nitrogen cylinder was used to apply confining pressure through a rod connected to the piston. This rod extended through an opening in the upper plate and acted on an aluminium plate which distributed the load to sand specimens.

After a specimen was prepared in a loose initial condition for each test, the mold was transfered to the vibration table and clamped in place with care in order to minimize any disturbance. Vibration of the system was initiated for a given period of time and the mold was dismantled tier by tier so that the measurement of density at each section could be made.

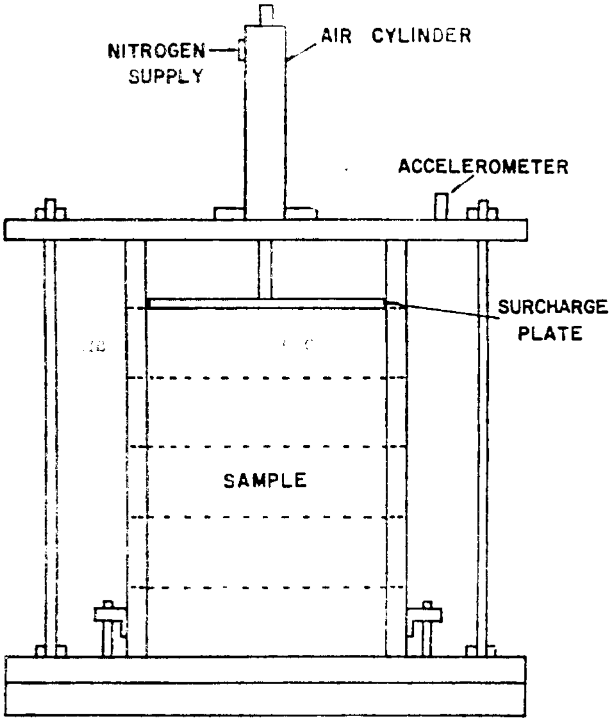


Fig. 7. Close-up schematic diagram of mold

3.3 Test Schedule

The phase of experimental tests performed were for finding the density variation along the height of the specimen. The tests were performed at a constant frequency and surcharge for different levels of acceleration, and at a constant level of acceleration and surcharge for different frequencies. With a constant level of acceleration and frequency, three different surcharge were applied to the specimen.

The tests were run to investigate the effect of water contents at varying accelerations and surcharges while other factors were remaining constant. More tests were performed to investigate the effect of initial density and the density variation after preparation of the sample.

4. Presentation and Discussion of Test Results

4.1 Ideally Graded Sand

4.1.1 Density Variation during Sample Preparation

In Fig. 8 is shown the test value of dry density of each tier and the range of variation along height of the specimen before vibration. It is noted that the density increases slightly with depth from top to bottom despite every effort made to place the soil homogeneously in the mold. This is, of course, because of an unavoidable overburden effect during the preparation and placement of the sample. The density variation, however, is less than 4 percent, and thus the samples prepared may be considered homogeneous throughout.

It is noted that before or after vibration an average density of the whole mold appears to agree well with a density averaged from each of the five tiers.

4.1.2 Effect of Acceleration and Frequency

Fig. 9 shows the variation of density as a function of distance from bottom of the mold at a frequency of 20Hz and a surcharge of 6.89KPa and at various acceleration levels. It can be seen that the density varies significantly throughout the specimen for the acceleration levels of 1 to 5 g. It is of interest to note that the degree of density variation becomes more pronounced as acceleration increases up to the 3 g level, beyond which the effect seems to start diminishing as

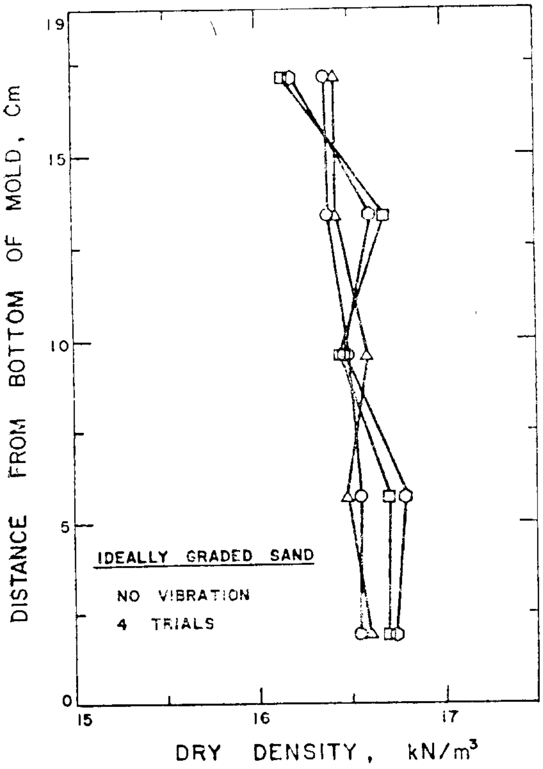


Fig. 8. Initial density variation (ideally graded sand)

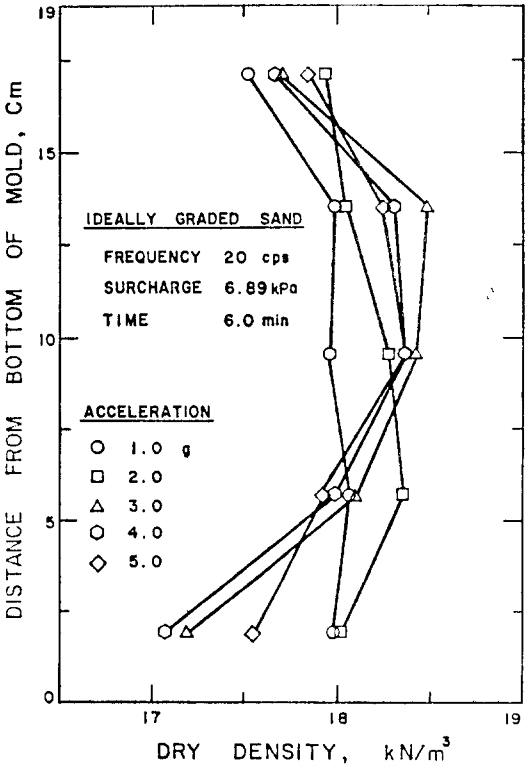
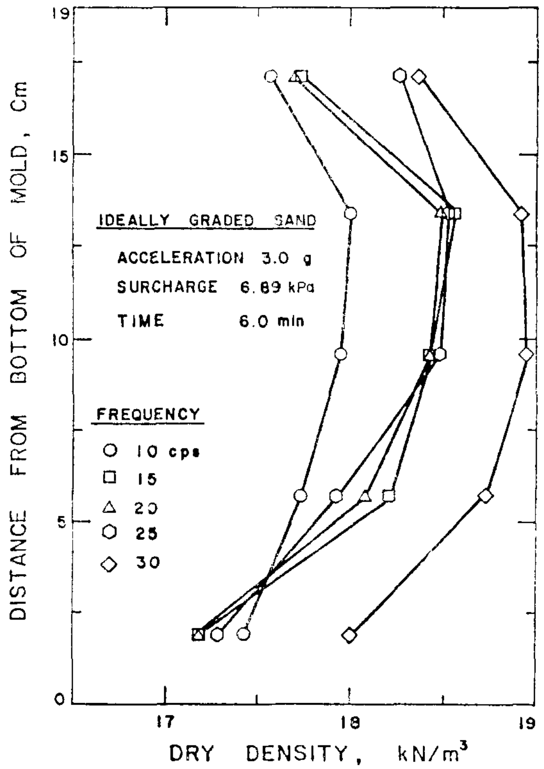


Fig. 9. Density variation along height of specimen as affected by acceleration



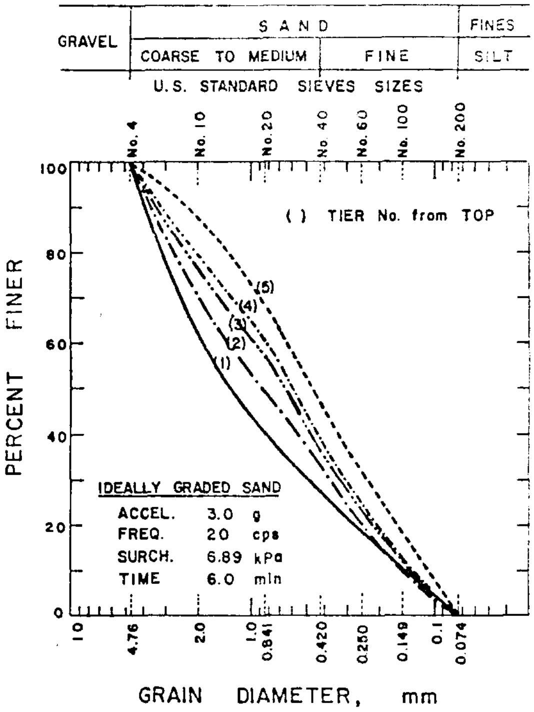


Fig. 10. Gradation curve for each tier after vibration

Fig. 11. Density variation along height of specimen as affected by frequency

acceleration is further increased. It is observed that the densest condition at 1 and 2 g acceleration levels occurs at a height of approximately 5cm (second tier from bottom) from bottom of mold while the densest condition is obtained between 10 to 15 cm (second tier from top) from bottom at 3, 4 and 5 g acceleration levels. The density near the surface is not high for two reasons: one, due to the surcharge, the particles near the surface are not allowed to relocate to their favorable position, and two, grain size distribution changes from an ideal (which produces a high density) to relatively uniform (which produces a low density) as fine particles fall downward into the voids.

Another observation to be made in Fig. 9 is that the lowest density occurs at bottom of the specimen at all acceleration levels. First of all, this behavior may be explained by the fact that particles in the lowest tier, are subjected to the greatest amount of confining pressure. During the vibration, the tendency of a material to densify is affected by confining pressure increase with depth under the same surcharge.

Secondly, this behavior can be explained in terms of the limit of packing of particles that was described previously.

As can be seen in Fig. 10 there is shift in grain size distribution after vibration. The grain size distribution curve for the top tier (tier 1) of the mold now includes a greater percentage of coarse particles and lesser fine particles compared to the original grain size distribution curve of the ideally graded sand (loss of fines). A reverse change in grain size distribution occurred at bottom tier of the mold: it now includes a smaller percentage of coarse particles and a greater percentage of fine particles compared to the original grain size distribution (gain of fines). In either case, there has been a shift in grain size distribution from the ideal to a more uniform distribution. However, the grain size distribution curves from the midsections of the mold show only small changes in grain size distribution compared to the original gradation of the ideally graded sand. Based on the theory of packing of particles developed by Westman and Hugill (9) and McGeary (8), one can expect an ideally graded sand to be compacted to a maximum terminal density, Therefore, the maximum terminal density is expected to occur at midsection of the mold because there is no significant change in grain size distribution here after the vibration. The terminal density of the top and bottom sections would be lower than that of the midsection because of the shift in gradation from that of ideal to relatively uniform. The terminal density is the lowest at bottom of the mold because of the additional aleffect of confining pressure.

Fig. 11 is a plot showing the variation of density as a function of depth for five different frequencies at a 3 g acceleration and with a 6.89 KPa surcharge. It is noted that similarity in shape of the dry density curves for the specimen exists at all frequencies. Lower density is obtained near bottom and top of the specimen due to again the effect of confining pressure and the shift in grain size distribution from the original ideal packing configuration.

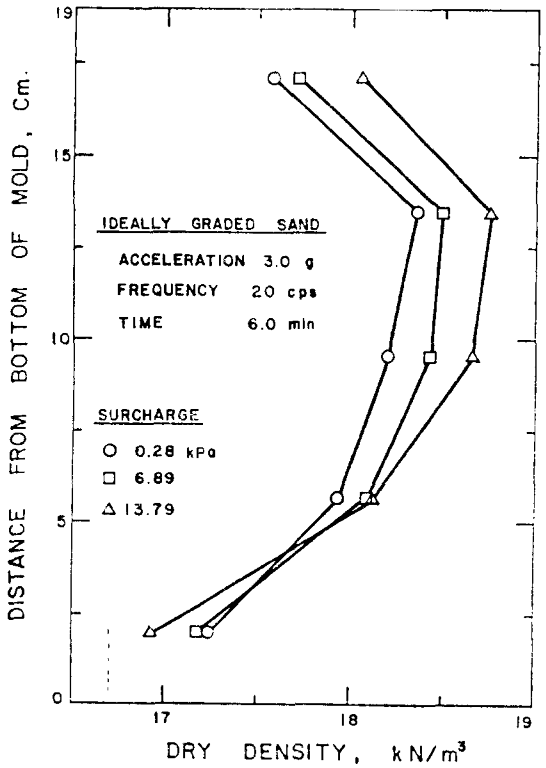


Fig. 12. Density variation along height of specimen as affected by surcharge

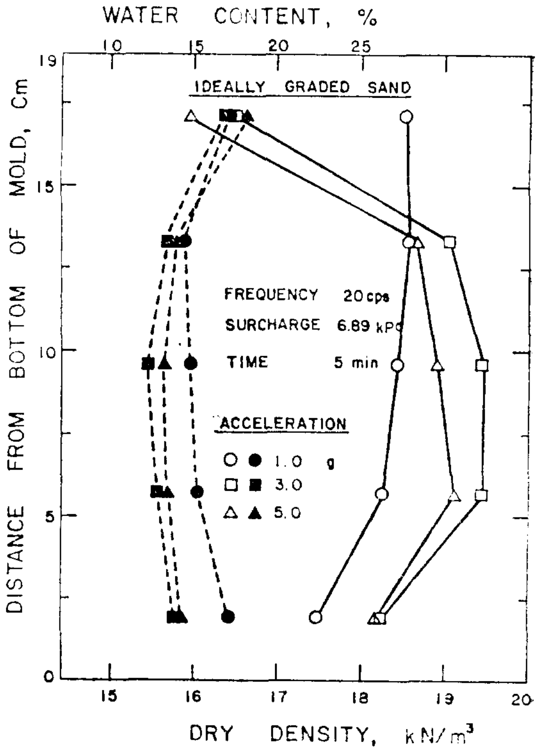


Fig. 13. Density and water content variation along heiget of specimen as affected b acceleration

4.1.3 Effect of Surcharge

Fig. 12 illustrates the variation of dry density as a function of distance for different surcharges and at a 3 g acceleration level and a frequency of 20 Hz. The results show resemblance to the previous discussed curves under the effect of acceleration and frequency.

Once again near the bottom of the specimen the change in relative density is quite low due to the effects of the change in grain size distribution. The variation of density at bottom of the mold is more pronounced by the effect of confining pressure.

Although the change in relative density varies throughout the specimen for given conditions, the variation as affected by surcharge seems not to be very significant (less than 10 percent).

4.1.4 Effect of Saturation

Saturated sands were tested to determine the behavior during vibration. Fig. 13 shows the variation of density as a function of distance from bottom of the specimen for three acceleration levels at a frequency of 20Hz and a surcharge of 6.89 KPa. The dotted lines in Fig. 13 illustrate the variation of water content within the specimen under the same vibratory conditions. The variation of density at the 1 g acceleration level is not as prominent as that at the 3 and 5 g acceleration levels. It should be noted that the density at top of the specimen is very low at 3 and 5 g acceleration levels. This may be explained by the fact that fine particles have been washed out through the holes in the surcharge plate during the vibration leading to an unavoidable experimental error. It is of interest to note that the maximum density is obtained in the fourth tier (around 5 cm from bottom) from top of the mold for the saturated sand as propposed to dry

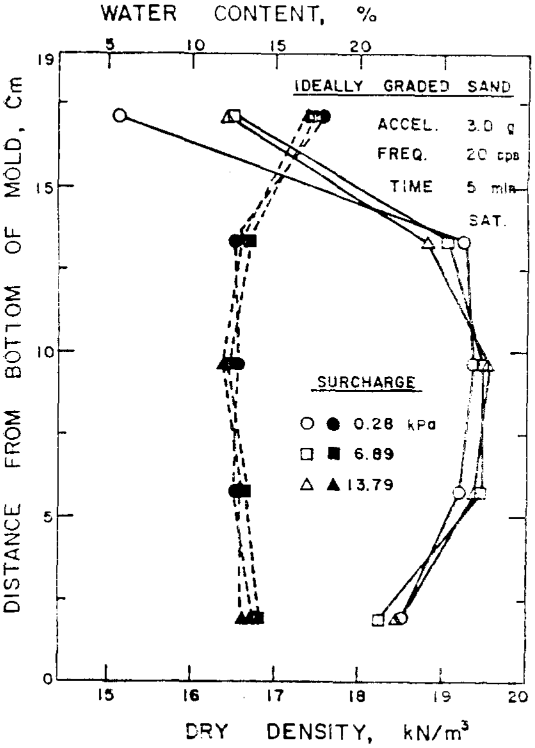


Fig. 14. Gradation curve for each tier after vibration (saturated)

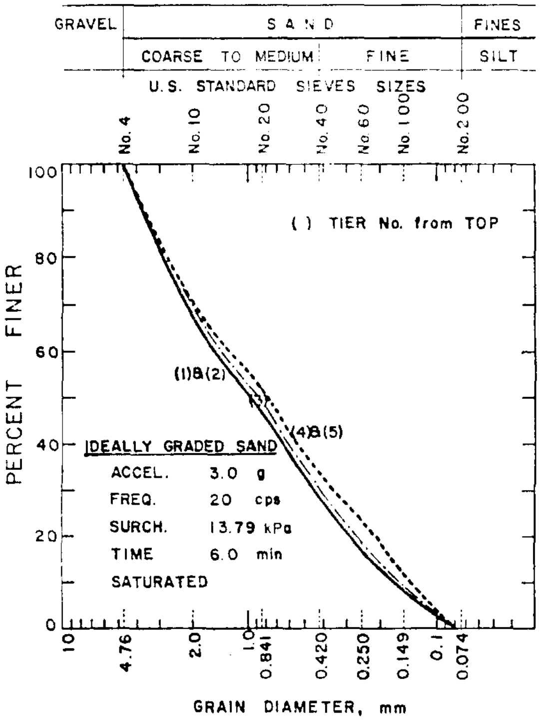


Fig. 15. Density and water content variation along height of specimen as affected by surcharge

sand for which the maximum density was obtained in the second or third tier (see Fig. 9). This may be explained by reasoning that fine particles, under saturation, are prevented from settling further down through the pores because of high pore water prressure developed during the vibration. This fact can be verified with an examination of grain size distribution curves shown in Fig. 14. As seen in the figure, there is no significant change in grain size distribution throughout the specimen. Thus, the effect of change in grain size distribution can be neglected in the case of saturated sand, and densification is solely controlled by surcharge and intensity of vibration (acceleration).

The variations of water content along height of the samples at various accelerations are shown in Fig. 13. It can readily be seen that the water content distribution is not uniform throughout the height of the specimen, and the specimen tends to lose more water toward the center. Again, the top tier has a high water content because of loosening or the loss of fine particles leading to more voids.

A maximum densification occurred around the midsection of specmen so that the void to be filled with water is small. The water content at bottom of the specimen is higher than that at the midsection because of hindrance of particle settling due to high pore water pressure.

The role of the surcharge during the vibration of the saturated sand is depicted in Fig. 15. The dotted lines in Fig. 15 show the variation of water within the specimen. It is seen again that neither the density nor the water content distribution is uniform throughout height of the specimen. It should be pointed out that the variation of density of water content due to an increase in surcharge is not significant.

4.2 **Uniform Sand**

4.2.1 Effect of Acceleration and Frequency

Fig. 16 shows the variation of initial dry density within the specimen after it had been prepared into the mold and placed for testing. It is seen that there is no apparent inhomogenity of the initial dry density within the specimen as the dry density is uniform throughout height of specimen.

Fig. 17 presents the results obtained by vibrating the uniform sand with a surcharge of 6.80 KPa at a frequency of 20Hz. The figure shows the variation of the density within the specimens as a function of the distance at five different acceleration levels. The position for the maximum density (the densest packing) is, in general, lowered with an increase of acceleration. For example, the densest packing is found in the second tier from top of the mold at the acceleration level of 1 g, in the third tier at 2g, and in the fourth tier at 3, 4 and 5 g acceleration levels. This phenomenon, again, can be explained by existance of confining pressure. A tendency of a material to densify is affected by confining pressure increase with depth under the same surcharge. For the uniform sand, there is no effect of shift in grain size distribution curve because the uniform sand has a single size of particles. Thus the lower terminal density obtained at bottom of the specimen can only be explained in terms of the effect of confining pressure.

It is noted that the relationship between the variation of density and distance from bottom of the mold is similar in pattern for all frequencies, and that the effect of frequency of the variation of density along height of the specimen is relatively small.

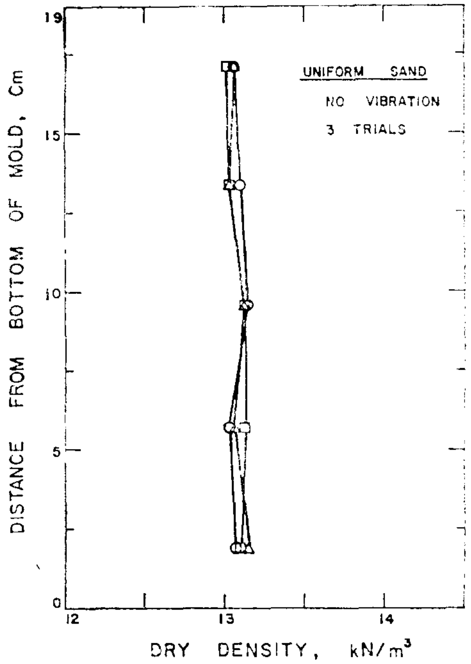


Fig. 16. Initial density variation(uniform sand)

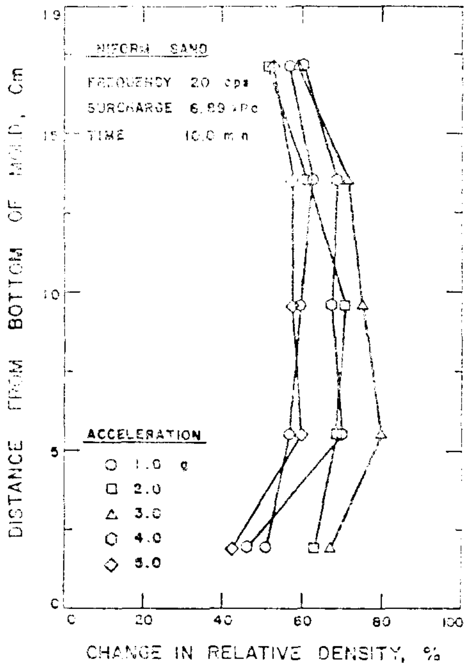


Fig. 17. Change in relative density variation along height of specimen as affected by acceleration (uniform)

4.2.2 Effect of Saturation

Fig. 18 presents the results for the saturated uniform sand subjected to three different levels

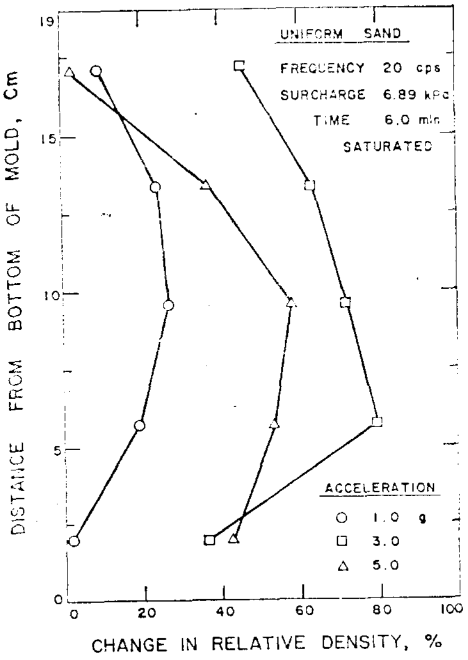


Fig. 18. Density and water content variation along height of specimen as affected by acceleration (uniform sand)

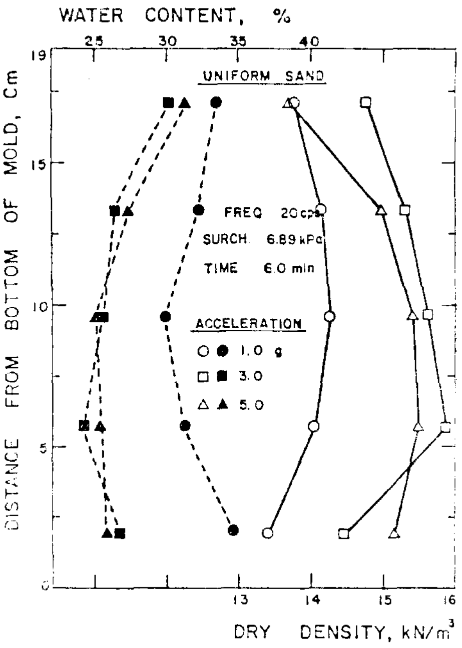


Fig. 19. Change in relative density variation along height of specimen as affected by acceleration (uniform & sat.)

of accelerations at a given frequency of 20 Hz and a surcharge of 6.89 KPa. The variation of density along specimen height is similar to that for the saturated ideally graded sand shown in Fig. 13. The variation of water content is shown by dotted lines in Fig. 18. The water content varies with depth of the specimen but in a way opposite to the variation of density within the specimen.

It is known that the change in relative density is quite uniform throughout the specimen at the high surcharge of 13.79 KPa. However, with low surcharge values the variation becomes quite significant. This is in contradiction to the behavior observed with the ideally graded saturated sand for which the surcharge had no effect at all on the vibration (1).

5. Conclusion

1. Vibration of the ideally graded sand having a homogeneous initial density throughout the specimen results in large inhomogeneities within the specimen due to shift in gradation from that of ideal to relatively uniform and the effect of confining pressure.

2. Density varies greatly throughout the specimen with the intensity of vibration. The degree of density variation within the specimen becomes more pronounced as acceleration increases up to optimum, beyond which the effect seems to start diminishing as acceleration is further increased.

3. Variation of relative density througthout the specimen for the ideally graded sand is not significantly affected by change in surcharge (less than 10 percent). The uniform sand is, however, gently affected by change in surcharge.

4. Density within the specimen for the uniform sand varies with the intensity of vibration (acceleration) but the resulting degree of inhomogenity is relatively small because the mechanism of density variation is only governed by confining pressure, and there is no shift on gradation of the material.

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