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STRESS FIELD PATTERN AT CRACK TIP IN COMPOSITE MATERIALS

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요 지

본 연구는 Orthotropic 재료에 존재하는 균열선단에서의 응력장의 양상에 대하여 지금까지의 연구들에서 상세하게 고려되지 않았던 여러 탄성계수들의 영향을 평면-변형조건 하에서 고찰하였다.

여러 하중조건과 경계조건 하에서의 일반적인 이방성 재료에 존재하는 균열선단에서의 응력장의 양상도 고찰하였으며, $a_{11}S^4-2a_{16}S^3+(2a_{12}+a_{66})S^2-2a_{26}S+a_{22}=0$인 특성방정식의 4근은 Cardano와 Ferrari의 해법을 쓴 새로운 단순기법에 의해 해석하였고 특성방정식의 4근에 미치는 일반적인 이방성 재료의 물성치 즉 $E_{xx}, E_{yy}, E_{zz}, G_{xy}, \nu_{zx}, \nu_{xy}, \nu_{yz}, \eta_{xy,x}, \eta_{xy,y}, \eta_{xy,z}$들의 영향들에 대해 상세히 연구되었다. 또한 균열선단부근에서의 응력분포 양상을 재료의 물성치와 경계조건 및 하중조건들로써 체계적으로 연구하였다.

Introduction

The elasticity problems in anisotropic bodies containing various defects have been being attracted by many researchers [1,2,3,4] as a result of the incresing demand of high performance

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composite materials and many occurrences of dangerous accidents in aircraft industries and high pressure technologies. The application of the elastic fracture mechanics theory into the anisotropic materials seems to be partially successful in orthotropic materials and unidirectional composites. Elastic fracture analyses of both generally anisotropic and isotropic materials need basic knowledges of the stress patterns at the vicinity of crack tips which are influenced by various conditions. These may contain the boundary and loading conditions, material properties and the shape of cracks. The use of the elastic fracture mechanics concepts into the generally anisotropic materials, however, has been found to be rare even though the basic stress equations for these materials are published in Ref. [5].

The general stress distributions at the vicinity of a plane-strain crack tip in an orthotropic material were calculated and the tangential stress distribution were plotted along the polar angles extending between 0° and 180° under symmetrical, skew-symmetrical and anti-plane shear loading conditions(5). Recently an iterative method based on finite element analysis was proposed to evaluate the asymptotic crack tip field of an orthotropic elastic materials(6). It is interesting to note, however, that most previous investigations of the crack tip field for the orthotropic plates have not considered the effects of some elasticity constants such as E_{zz}, ν_{yz} and ν_{zx} which appear in the coefficient of the characteristic equation. This report, thus, investigates the influence of the neglected elasticity constants on the various stress distributions around a through plane-strain crack under opening mode and in-plane shear mode loading conditions in orthotropic bodies.

For the generally anisotropic plates with a through plane strain crack under normal stresses, $-\sigma_0$, along the crack surface, the asymptotic stress expressions have been published with an implicit higher order term(5). In this report it has been attempted to formulate more accurate stress equations by addition of the explicit higher order term. These new stress equations have been used to generate a variety of stress field patterns around the crack tip. The influence of material properties and various loading and boundary conditions on the stress field patterns around a plane-strain crack tip in a generally anisotropic plate has been systematically investigated.

The classification of analytically generated various crack tip stress field patterns in generally anisotropic plates as is done in this paper can be incorporated with the isotropic birefringent coating techniques to extract stress intensity factors more effectively.

Theoretical

Following Lekhnitskii's solution(7) the stresses σ_{xx}, σ_{yy} and τ_{xy} in the vicinity of a through crack of length 2a that lies in the xz-plane in a generally anisotropic plate as shown in Figure 1 can be represented in terms of holomorphic functions $\Phi(Z_1)$ and $\Psi(Z_2)$ in the form

$$\sigma_{xx} = 2\,R_e\,[\,S_1^2\Phi'(Z_1) + S_2^2\Psi'(Z_2)\,] \quad \cdots\cdots (1)$$

$$\sigma_{yy} = 2\,R_e[\,\Phi'(Z_1) + \Psi'(Z_2)\,] \quad \cdots\cdots (2)$$

$$\tau_{xy} = -2\,R_e[\,S_1\Phi'(Z_1) + S_2\Psi'(Z_2)\,] \quad \cdots\cdots (3)$$

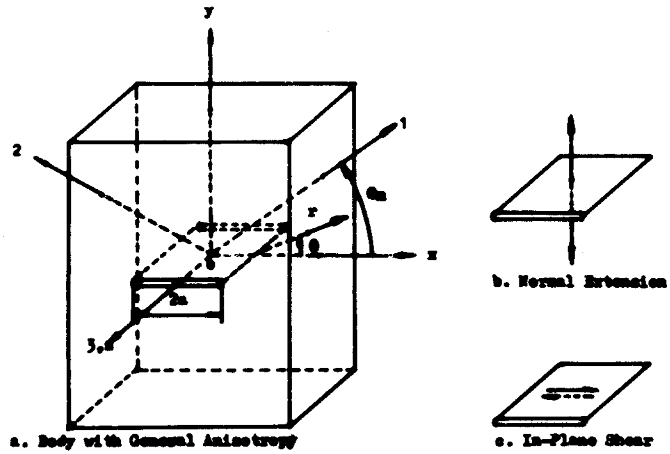

FIGURE 1 THROUGH CRACK IN AN INFINITE BODY WITH GENERAL ANISOTROPY

where the prime denotes differentiation with the respective complex variables Z_1 and Z_2 and $Z_1 = x+S_1y$ and $Z_2 = x+S_2y$. Note that Re denotes the real part of the expressions. S_1 and S_2 are roots of the characteristic equation

$$a_{11}S^4 - 2a_{16}S^3 + (2a_{12} + a_{66})S^2 - 2a_{26}S + a_{22} = 0 \qquad (4)$$

where $a_{ij}(i,j=1,2,6)$ represents the elements of compliance matrix of the generally anisotropic material and are given by(7):

$$a_{11} = (1/E_{xx}) - (\nu_{zx}{}^2/E_{zz}) \qquad (5a)$$

$$a_{22} = (1/E_{yy}) - (E_{zz}\nu_{yz}\,2/E_{yy}{}^2) \qquad (5b)$$

$$a_{66} = (1/G_{xy}) - (\eta_{xy,z}\,2/E_{zz}) \qquad (5c)$$

$$a_{12} = -(\nu_{xy}/E_{xx}) - (\nu_{zx}\,\nu_{yz}/E_{yy}) \qquad (5d)$$

$$a_{16} = (\eta_{xy,x}/E_{xx}) + (\nu_{zx}\,\eta_{xy,z}/E_{zz}) \qquad (5e)$$

$$a_{26} = (\eta_{xy,y}/E_{yy}) + (\nu_{yz}\,\eta_{xy,z}/E_{yy}) \qquad (5f)$$

Note that $a_{16} = a_{26} = 0$ and $a_{11} = a_{22} = 2a_{12} + a_{66}$ for isotropic materials and $a_{16} = a_{26} = 0$ for orthotropic materials.

For an orthotropic material, the characteristic equation (4) simplifies to the form

$$a_{11}S^4 + (2a_{12} + a_{66})S^2 + a_{22} = 0 \qquad (6)$$

The effects of the second term of the right hand side of eq.(5) on the roots of the characteristic equation have been usually neglected in the orthotropic materials. In this paper, the second portion of the right hand side of eq.(5), i.e., $-(\nu_{zx}{}^2/E_{zz})$, $-(E_{zz}\ \nu_{yz}{}^2/E_{yy}{}^2)$, $-(\eta_{xy,z}{}^2/E_{zz})$ and $-(\nu_{zx}\nu_{yz}/E_{yy})$ for the coefficients of a_{11}, a_{22}, a_{66} and a_{12}, respectively, is included. The influence of these terms on the tangential, radial and shear stresses is investigated as a function of the polar angle(θ) around the crack tip under opening and in-plane shear mode conditions.

For the generally anisotropic material, the influence of material properties such as E_{xx}, E_{yy}, E_{zz}, G_{xy}, ν_{zx}, ν_{xy}, ν_{yz}, $\eta_{xy,x}$, $\eta_{xy,y}$ and $\eta_{xy,z}$ and various loading and boundary conditions on the stress field pattern at the vicinity of a crack tip has been studied systematically. Where E_s are the elastic moduli in the principal directions x,y and z. ν_s are Poisson's ratios, G is the shear modulus and η_s are known as the mutual influence coefficiants.

Omitting few analytical approximation steps, the following asymptotic stress expressions may be yielded under opening mode loading conditions with normal stresses, $-\sigma_o$, along the plane-strain crack surfaces.

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \mathrm{Re}\left[\frac{S_1 S_2}{S_1 - S_2}\left[\frac{S_2}{\sqrt{\cos\theta + S_2 \sin\theta}} - \frac{S_1}{\sqrt{\cos\theta + S_1 \sin\theta}}\right]\right] + \mathrm{Re}\left[\frac{K_I}{\sqrt{\pi a}} S_1 S_2\right] \qquad \text{(7a)}$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \mathrm{Re}\left[\frac{1}{S_1 - S_2}\left[\frac{S_1}{\sqrt{\cos\theta + S_2 \sin\theta}} - \frac{S_2}{\sqrt{\cos\theta + S_1 \sin\theta}}\right]\right] - \mathrm{Re}\left[\frac{K_I}{\sqrt{\pi a}}\right] \qquad \text{(7b)}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \mathrm{Re}\left[\frac{S_1 S_2}{S_1 - S_2}\left[\frac{1}{\sqrt{\cos\theta + S_2 \sin\theta}} - \frac{1}{\sqrt{\cos\theta + S_1 \sin\theta}}\right]\right] \qquad \text{(7c)}$$

where K_I is the stress intensity factor under the opening mode conditions and a is the half crack length. r and are polar coordinates originated at the crack tip as shown in Figure 1.

The term $K_I/\sqrt{\pi a}$ appeared in eq.(7) may be affected by boundary conditions since it includes the term of half crack length, a. $K_I/\sqrt{\pi a}$ may, thus, be represented in terms of a series like $A_i r^n$ as is usually modelled for the isotropic cases[8] where A_i (i=1,2,....) are constants and n (0,0.5,1,1.5,....) are real numbers.

In this report, a constant takes the place of the term, $K_I/\sqrt{\pi a}$ appeared in eq.(7) as is commonly done for the isotropic cases[9]. It is noted that the term $K_I/\sqrt{\pi a}$ appeared in the σ_{xx} is coupled with S_1 and S_2 being contrary to the case of the σ_{yy}. Therefore, $K_I/\sqrt{\pi a}$ is substituted by a constant term, denoted by σ_{oy}, in this paper.

The stress field around the crack tip in the generally anisotropic body under the in-plane shearing mode condition can be developed through the same manner as is done for the opening mode condition. The final results show that τ_{xy} contains an isolated constant term while σ_{xx} does a constant coupled with S_1 and S_2. The stress patterns around the crack tip in the generally anisotropic bodies under the mixed mode conditions can also be generated by combining the stress fields under the opening mode with those of the in-plane shearing mode.

Generation of Isochromatics

The distributions of a maximum in-plane shear stress, τ_m, and a principal strain differences, ε_p, at the vicinity of the crack tip may be represented by

$$\tau_m = [\{(\sigma_{yy} - \sigma_{xx})/2\}^2 + \tau_{xy}^2]^{1/2} \quad \cdots\cdots (8)$$

$$\varepsilon_p = \varepsilon_1 - \varepsilon_2 = [(\varepsilon_{yy} - \varepsilon_{xx})^2 + 4\varepsilon_{xy}^2]^{1/2} \quad \cdots\cdots (9)$$

regardless of the classification of the materials. Notice, however, is needed because the isochromatic patterns in composite materials differ from those in photoelastic coatings notwithstanding the coincidence of the principal strain differences, ε_p.

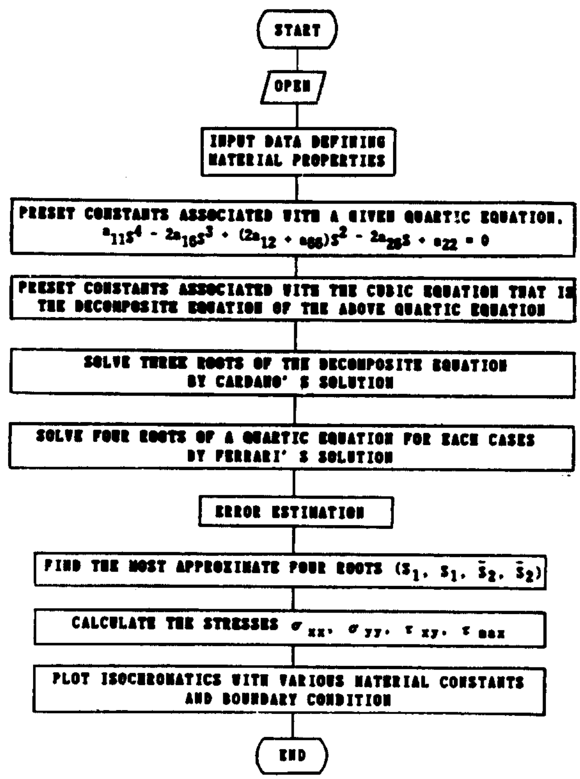

FIGURE 2 FLOW CHART TO SOLVE THE CHARACTERISTIC EQUATION

The characteristic equation (4) should be solved numerically to obtain four distinguished roots, S_1, S_2, $\overline{S}_1$ and $\overline{S}_2$ where the overbar designates complex conjugates. A new-simple numerical algorithm developed with the help of Cardano and Ferrari's solutions(12) and utilized to determine four roots of the characteristic equation is shown in Figure 2. These four roots are mainly related to the material properties such as E_{xx}, E_{yy}, E_{zz}, G_{xy}, ν_{zx}, ν_{xy}, ν_{yz}, $\eta_{xy,x}$, $\eta_{xy,y}$ and $\eta_{xy,z}$ of a given generally anisotropic plate. Using four roots of the characteristic equation with combinations of the material properties, the stress field around the crack tip incorporating various loading conditions such as a symmetric or a skew-symmetric or a mixed-mode can be generated as described in eqs.(7) and (8). The isochromatics which may be visualized with the help of photoelastic coating techniques can be generated as have been done in Refs.[1,9,10].

Results and Discussion

The variation of tangential stresses along the polar angles, , under opening mode are plotted in Figure 3. The results for the case of $\alpha_o = \sqrt{E_{xx}/E_{yy}} = 5$ (E_{xx} = 180000(Ksi), $E_{yy} = E_{zz}$ = 7200(Ksi), G_{xy} = 7200(Ksi), ν_{xy} = 0.25) with $q = \nu_{zx} / \nu_{yz} = 0$ agree remarkedly well with those of Refs.[5] and [6]. The influence of q on the distribution of tangential stresses are clearly seen in Figure 3. The peak values of the tangential stresses are increased with the increase of q. The peak value of tangential stresses with q = 0 occurs at = 60°. Figure 3 also indicates how the locations of the peak values move from 60° with q = 0 to 70° with q = 1.6.

The variation of radial and shear stresses along the polar angles, θ, is also shown in Figures 4 and 5, respectively. The consequences for the case of α_o= 5 with q = 0 agree very well with

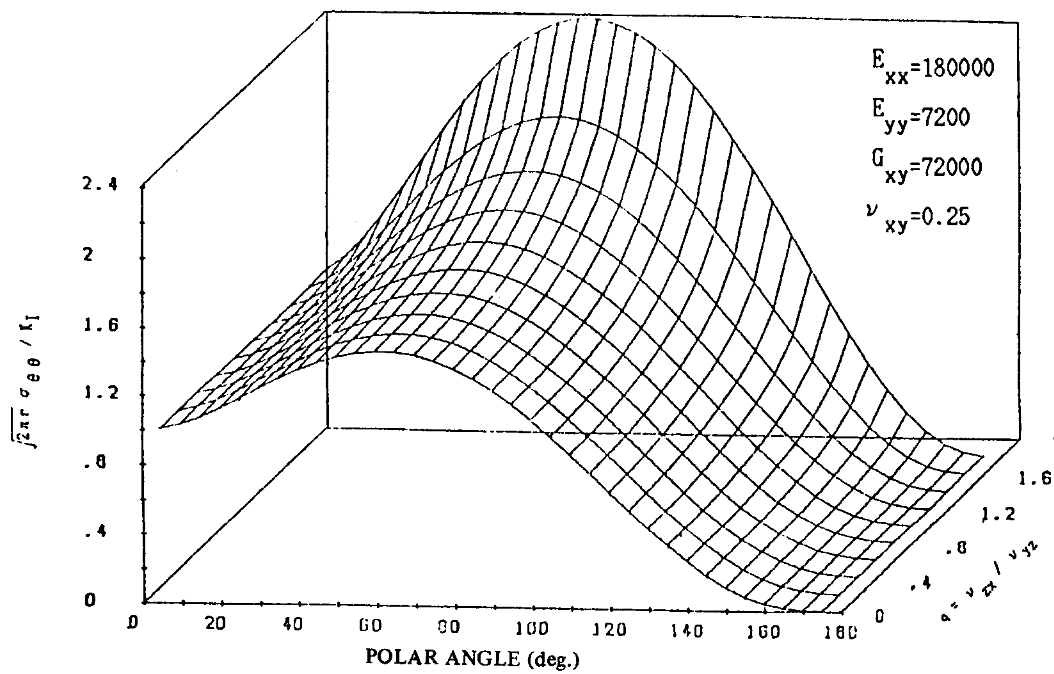

FIGURE 3 THE INFLUENCE OF $q = \nu_{zx}/\nu_{yz}$ ON THE TANGENTIAL STRESSES ($\sigma_{\theta\theta}$) UNDER THE OPENING MODE (Dimension in Ksi)

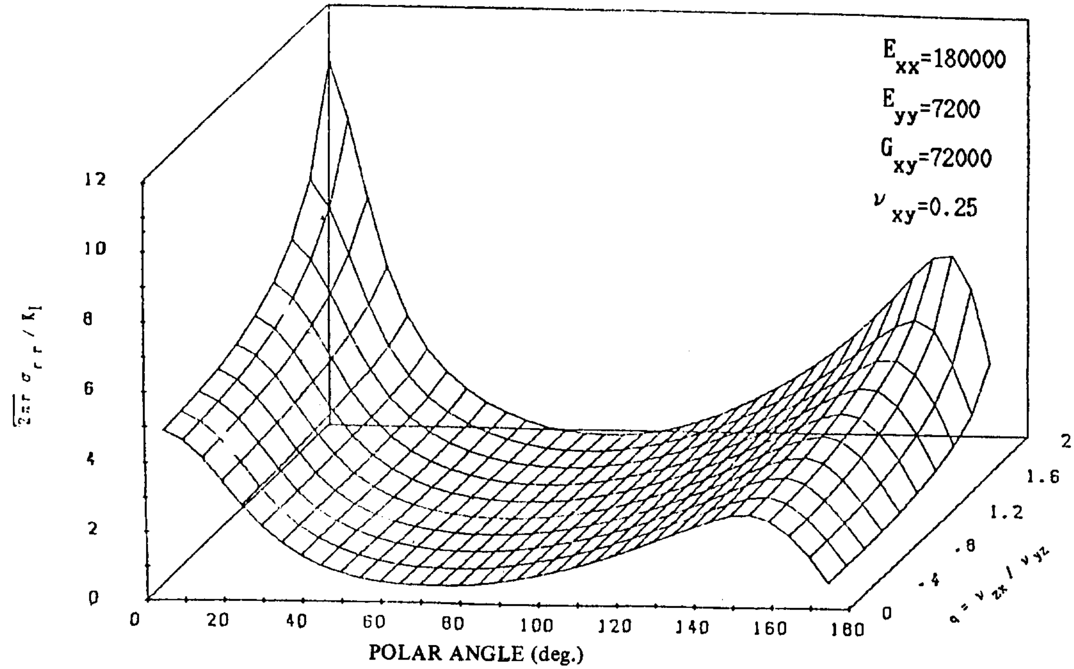

FIGURE 4 THE INFLUENCE OF $q = \nu_{zx} / \nu_{yz}$ ON THE RADIAL STRESSES (σ_{rr}) UNDER THE OPENING MODE (Dimension in Ksi)

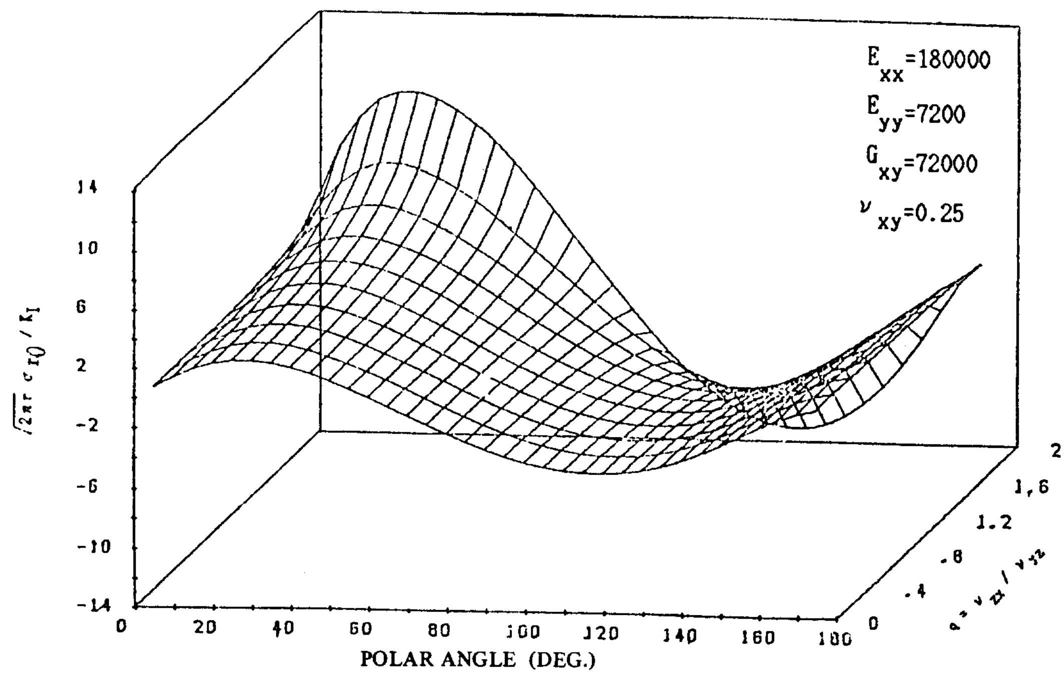

FIGURE 5 THE INFLUENCE OF $q = \nu_{zx}/\nu_{yz}$ ON THE SHEAR STRESSES ($\sigma_{r\theta}$) UNDER THE OPENING MODE (Dimension in Ksi)

those of Ref.[6]. From these theoretical calculations of stress distribution, it may be interesting to note that the influence of the second term of the right hand side of eq.(5) could not be neglected if the magnitude of q is large. However, for the in-plane shear mode, the influence of the q seems to be negligible as shown in Figure 6 and the stress distribution for the case of $\alpha_o = 5$ with $q = 0$ agrees very well with those of Ref.[5].

Figure 7 shows some generated isochromatics which may be developed around the crack tip in generally anisotropic plates whose material properties and the values of S_1 and S_2 are listed on the figure. The influence of material constants and the boundary effect with the magnitude of stress intensity factors on the configuration of isochromatic loops is clearly seen in Figure 7.

Some interesting features on the stress field pattern have been noticed as shown in Figure 8 in terms of the tilt angles, θ_{m1} and θ_{m2}, of the isochromatic loops which are shown in Figure 7. Various combinations of the material properties have been used to generate analytical stress loops. Figure 8 shows the effects of the ratios of E_{zz}/E_{yy} and σ_{oy}/K_I when $\eta = \eta_{xy,x} = \eta_{xy,y} = \eta_{xy,z}$ on the tilt angles, θ_{m1} and θ_{m2}, of the isochromatics loops.
The tilt angles, θ_{m1} and θ_{m2}, are found to be decreased with increment of the ratio of E_{xx}/E_{yy}. The decreement of η makes the tilt angles, θ_{m1} and θ_{m2}, decrease. The influence of σ_{oy}/K_I on the tilt angle becomes less pronounced than does in isotropic materials within the range of $-0.2 \succ \sigma_{oy}/K_I$ in these limited cases as shown on Figure 8. It is also interesting to note that the isochromatic loops are not symmetric even under a symmetric loading condition as in Figure 7.

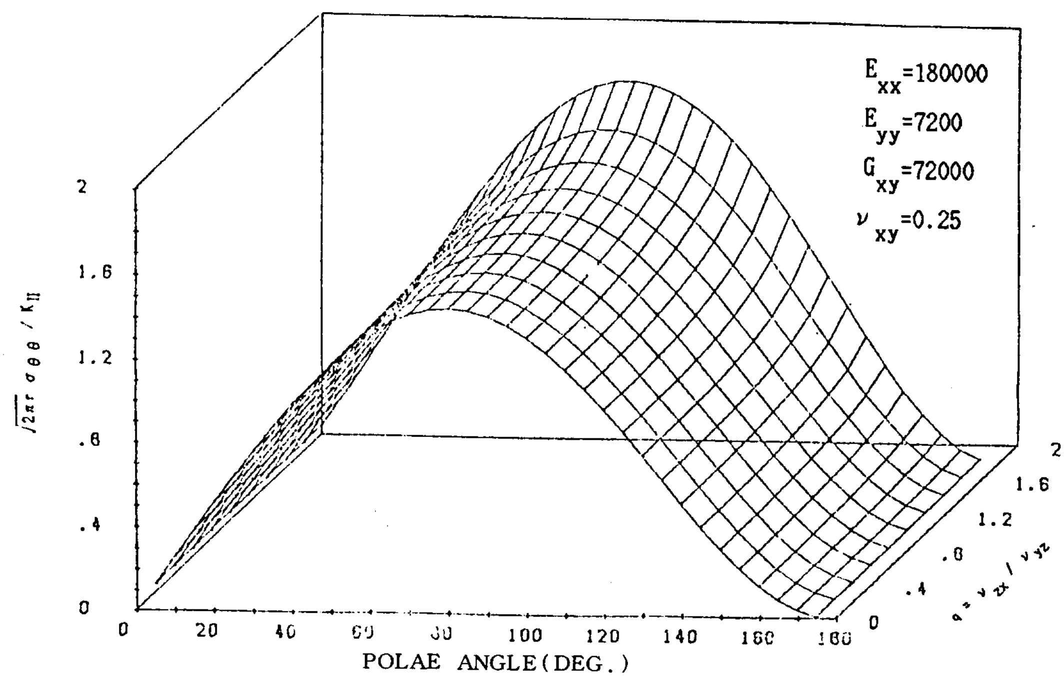

FIGURE 6 THE INFLUENCE OF $q = \nu_{zx}/\nu_{yz}$ ON THE TANGENTIAL STRESSES ($\sigma_{\theta\theta}$) UNDER THE INPLANE SHEAR MODE (Dimension in Ksi)

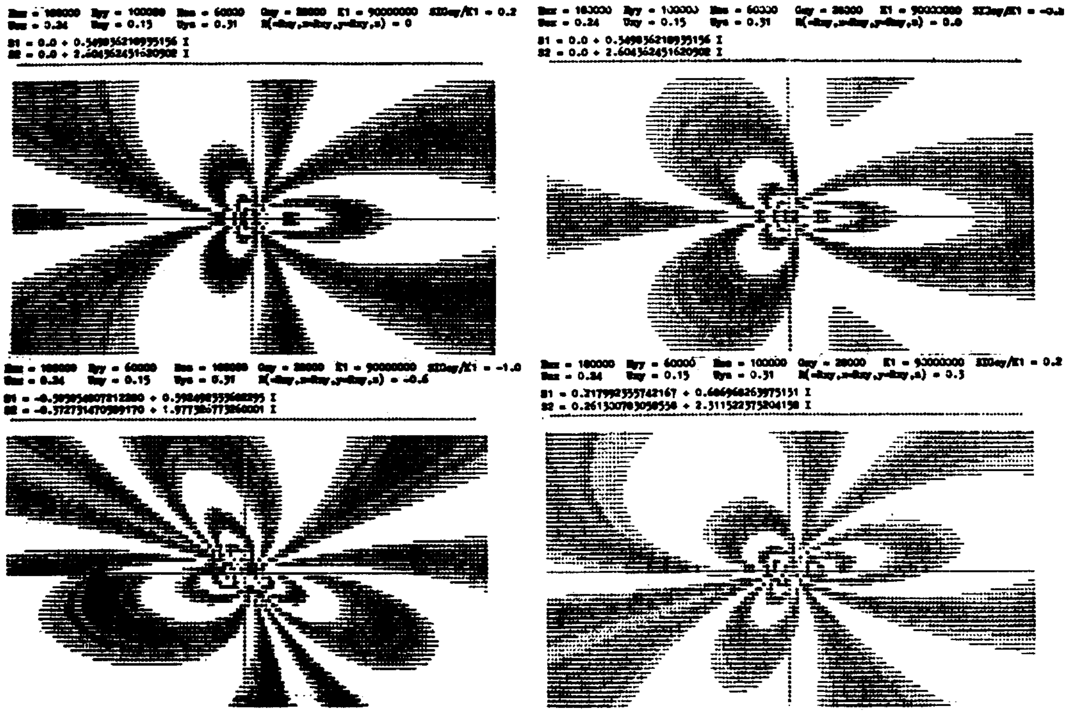

FIGURE 7 SOME ANALYTICALLY GENERATED ISOCHROMATICS AROUND THE CRACK TIP WITH VARIOUS MATERIAL PROPERTIES AND BOUNDARY CONDITIONS IN A GENERALLY ANISOTROPIC PLANT

An interesting phenomenon is also noted in Figure 9. The symmetrical and non-symmetrical isochromatic loops with respect to x-axis are found to be corresponded to the case of $|\eta| \neq 0.6$ and $|\eta| = 0.6$, respectively, with the same combination of material properties. The effects of the value of η on the four roots, S_1, S_2, $\overline{S}_1$ and $\overline{S}_2$, of the characteristic equation (4) have been noted for some limited cases as the following.

when

η = negative : α_1 , α_2 = negative

$\eta = 0$: $\alpha_1 = \alpha_2 = 0$

η = positive : α_1 , α_2 = positive

for $|\eta| = |-\eta|$: $\beta_1 = \beta_2$, $|\alpha_1| = |-\alpha_2|$

where

$$S_1 = \alpha_1 + i\beta_1 , \quad S_1 = \alpha_1 - i\beta_1$$
$$S_2 = \alpha_2 + i\beta_2 , \quad S_2 = \alpha_2 - i\beta_2$$

Wu found that the elastic constants of balsawood and glass-fiber-reinforced plastic (parallel fila-

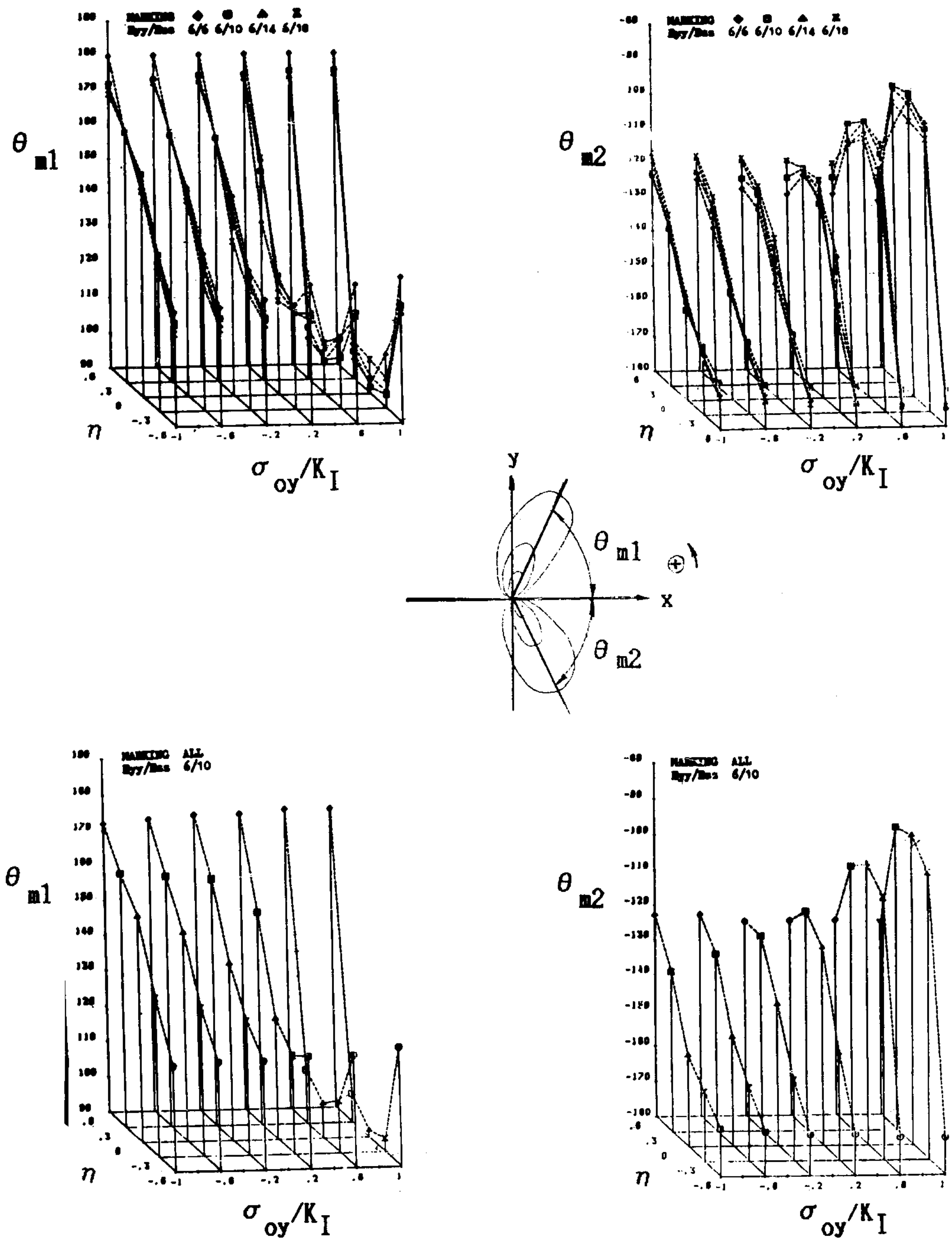

FIGURE 8 INFLUENCE OF E_{yy}/E_{zz}, σ_{oy}/K_I, η ON THE θ_{m1} AND θ_{m2}

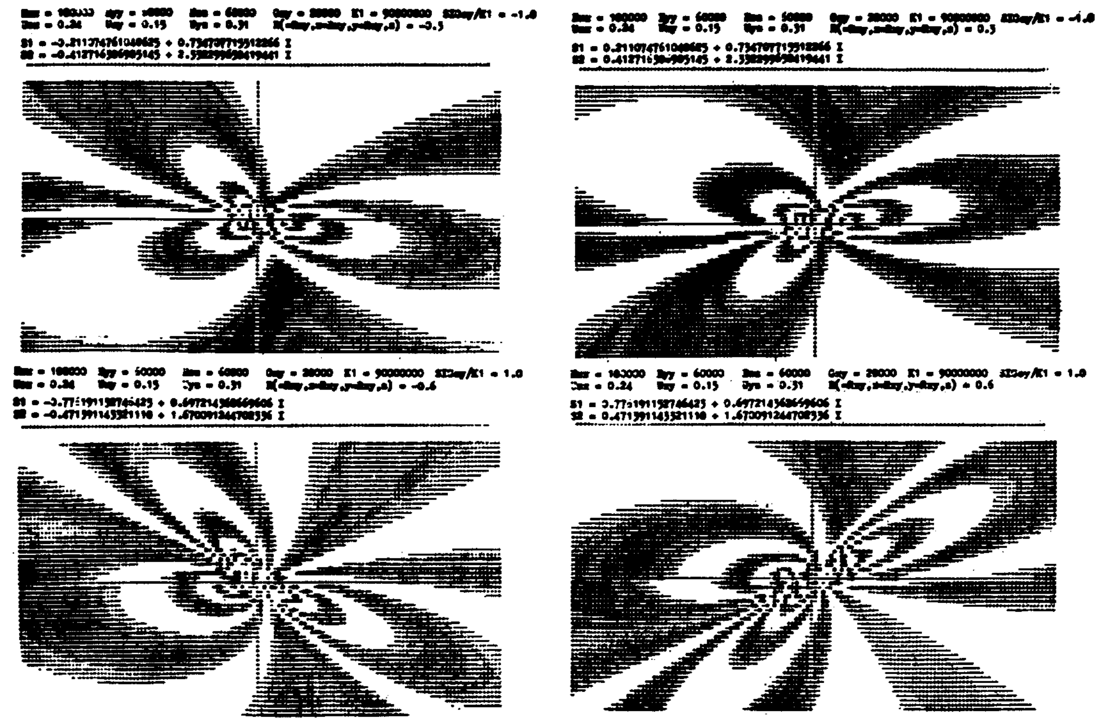

FIGURE 9 SYMMETRY AND NON-SYMMETRY WITH RESPECT TO X-AXIS OF ISOCHROMATIC LOOPS WITH VARIOUS VALUES OF $\eta = \eta_{xy,x} = \eta_{xy,y} = \eta_{xy,z}$

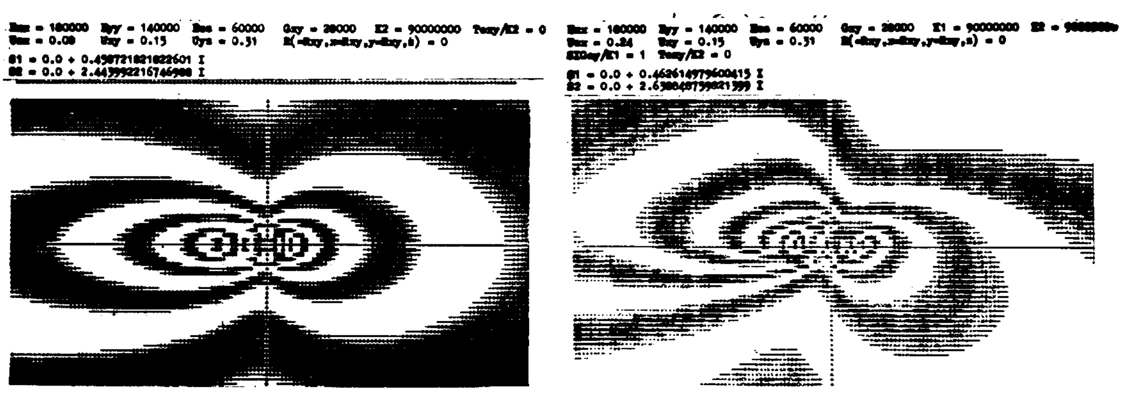

FIGURE 10 SOME ANALYTICALLY GENERATED ISOCHROMATICS AROUND THE CRACK TIP UNDER IN-PLANE SHEAR MODE LOADING CONDITIONS

ment) were such that α_s are very small and, in fact, appears to be zero(11). It is found that the above result in this paper agree well with that of Ref.[11].

Figure 10 shows some analytically generated isochromatic loops under the in-plane shear mode loading conditions. Isochromatic loops in the orthotropic plate are seen to be recovered by substitution of $\tau_{ox}/K_{II} = 0$ and with $a_{16} = a_{26} = 0$ as shown in Figure 10.

An overdeterministic least square (OLSQ) algorithm has been used to extract the stress intensity factors in isotropic polymeric materials. The final results obtained by OLSQ method have been found to be affected by the initial guessed values in the above algorithm. For these cases, the classification of the stress loops as is done in this report may help us to guess appropriate initial values.

Concluding Remarks

The influence of elasticity constants, which have been usually neglected in previous studies, on the stress distribution at the vicinity of the plane-strain crack tip in the orthotropic materials are investigated. It is noted that the ratio of $q = \nu_{zx} / \nu_{yz}$ affects the tangential, radial and shear stresses in term of its peak values and locations of peak values under the opening mode loading condition. However, the stress distribution under the in-plane shear mode loading condition is found to be little influenced by the ratio of $q = \nu_{zx} / \nu_{yz}$.

The stress field patterns at the vicinity of the crack tip in the generally anisotropic body under various loading and boundary conditions are studied systematically by the observation of the theoretically generated isochromatic loop around the crack tip. The boundary effects on the isochromatics are incorporated by extending the existing nearfield solution of the generally anisotropic materials.

The overdeterministic least square (OLSQ) algorithm used in the case for the isotropic photoelasticity to extract accurate stress intensity factors can also be utilized for the anisotropic material with the aid of the pre-determinated shape of the isochromatic loops as is done in this report.

Acknowledgement

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References

1. Rossmanith, H.P., "Analysis of Mixed-Mode Orthotropic Crack Tip Stress Patterns-I : Pattern Evaluation", Engineering Fracture Mechanics, 22, (1985), 547.
2. Jacob, K.A., Dayal, V. and Ranganayakamma, B., "On Stress Analysis of Anisotropic Composite Through Transmission Optical Patterns: Isochromatics and Isopachics", Experimental Mechanics, 23, (1983), 49.
3. Khalil, S.A., Sun, C.T. and Hwang, W.C., "Application of a Hybrid Finite Element Method to determine Stress Intensity Factors in Unidirectional Composites", International Journal of Fracture, 31, (1986), 37.
4. Satapath, P.K. and Parhi, H., "Stresses in an Orthotropic Strip containning a Griffith Crack", International Journal of Engineering Science, 16, (1978), 147.
5. Sih, G.C., "Mechanics of Fracture 6 (Cracks in Composite Materials)", Noordhoff International Publishing Leyden, (1981).
6. Barsoum, R.S., "Crack in Anisotropic Materials - an Iterative Solution of the Eigenvalue Problem", International Journal of Fracture, 32, (1986), 59.

7. Lekhnitskii, S.G., "Theory of Elasticity of an Anisotropic Elastic Body", translated by P. Fern, Holden-Day Inc., San Francisco, USA (1963).

8. Rossmanith, H.P. and Chona, R., "A Survey of Recent Developments in the Evaluation of Stress Intensity Factors from Isochromatic Crack-Tip Fringe Patterns", ICF 5, (5), (1981), 2507.

9. Ramulu, M., Kobayashi, A.S. and Barker, D.B., "Analysis of Dynamic Mixed-Mode Isochromatics", Experimental Mechanics, 25, (1985), 344.

10. Lee, O.S. and Han, M.K., "Stress Field at Crack Tip in Compostes", Proceeding of 1986 Spring Conference of KSME, (1986), 72.

11. Wu, E.M. and Reuter, R.C., "Crack Extention in Fiberglass Reinforced Plastics", TAM Rept. No. 275., University of Illinois, Urbana, Illionois.

12. Keller, M.W., "College Algebra", Purdue Univ., Houghton Mifflin Co., 331.