J. Astron. Space Sci., Vol. 4, No. 2, 1987

Three Dimensional Numerical Code for the Expanding Flat Universe

Kyoung W. Min
Korea Institute of Technology
(Received December 1, 1987; Accepted December 10, 1987)

Abstract

The current distribution of galaxies may contain clues to the condition of the universe when the galaxies condensed and to the nature of the subsequent expansion of the universe. The development of this large scale structure can be studied by employing N-body computer simulations. The present paper describes the code developed for this purpose. The computer code calculates the motion of collisionless matter acting under the force of gravity in an expanding flat universe. The test run of the code shows the error less than 0.5% in 100 iterations.

I. Introduction

There have been two great discoveries in modern scientific cosmology. The first one is Hubble's discovery of the recession of galaxies (1929). From observations of the spectra of galaxies, Hubble found that the galaxies at distances r are traveling away from us with velocities V = Hr, where H is Hubble's constant. This discovery tells us that the universe is expanding and, running the expansion backwards in time, implies that the universe began with an explosion from a very dense state. The second great discovery occurred in 1965 when Penzias and Wilson detected a cosmic radiation field with a temperature of about 3K(Penzias and Wilson 1965; Dick et al. 1965). This microwave background radiation is highly isotropic. The conventional interpretation (Peebles 1971; Weinberg 1972) of this result is that it is the relic of the hot fireball stage of the universe at the Big Bang. In the standard cosmological model, the universe started out in a very hot, dense state and cooled as it expanded. When the temperature of the universe reached about 10^4 K, the matter and radiation decoupled. Since the matter in the universe is transparent to radiation after decoupling, the observations of the microwave background provide us with direct observations of the physical conditions at that period. In particular, the isotropy of the microwave background tells us that the universe was highly isotropic at decoupling. Therefore, the basis of our physical model is a homogeneous, isotropic expanding universe.

On the other hand, it is evident that the structures smaller than the scale of homogeneity form and evolve. Detailed mapping of the distribution of galaxies by measurements of their redshift velocities gives us information about the three-dimensional structure of the universe. Our current picture is one in which the galaxies and clusters of galaxies lie in even larger associations known as superclusters of galaxies(de Vaucouleurs 1981; Oort 1983). Superclusters are typically either flattened pancake-like systems, or elongated filamentary systems several tens of megaparsecs in extent. The galaxy distribution also contains large holes or voids where no galaxies are observed.

II. The Flat Universe

It is well known that the density parameter Ω_0 plays a key role in the evolution of the homogeneous background universe, where Ω_0 is given by

$$\Omega_0 = \frac{\rho_0}{\rho_{crit}}$$

Here ρ_0 is the background mass density, and

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

The value of Ω_0 can in principle be deduced from observations. Although current work places it in the range of $0.1 \leq \Omega_0 \leq 1$, we are not certain about the exact value of ρ_0. If we are interested in the structure of the scale of superclusters, the exact value of ρ_0 may not be necessary, since the universe is almost flat in this scale. That is, we can study the evolution of the superclusters within the framewark of Newtonian cosmology. The matter evolves via the Newtonian force laws in an expanding background universe. This approach is valid for velocities $v \ll C$, where C is the speed of light, and for length scales $\lambda \ll L_H$, where $L_H \sim Ct$ is the horizon size. Typical values of these quantities for galaxies in clusters and superclusters are $v \lesssim 10^3$ km/sec and $\lambda \lesssim 10^2$ Mpc; the conditions of Newtonian cosmology are thus satisfied.

There are numerous papers on the topic of galaxy clustering which employs the N-body simulation(Efstathiou and Eastwood 1981; Centrella and Melott 1983; Davis et al. 1985). All these simulations make the use of particle-mesh method (PM) in an infinite periodic universe.

Efstathiou et al. (1985) reported that PM method is adequate for studying the evolution of structure on large scale, while P^3M method employing direct interections of particles is to be preferred whenever the evolution of small-scale structure is important. In this regard, the PM code is developed to study the galaxy clustering in the homogeneous flat background universe, and will be discussed in the next section.

III. The Code

In studying the clustering of galaxies it is helpful to divide out the general expansion of the universe by introducing comoving coordinates defined by

$$\vec{r}_c(t) = \vec{r}(t)/a(t), \quad \cdots\cdots (1)$$

where a(t) is an expansion factor

$$a(t) = \frac{r(t)}{r(t_0)} \quad \cdots\cdots (2)$$

Then, the Newton equations for the particle velocity and position in the original or 'proper' coordinates are

$$\frac{d\vec{v}}{dt} = -\vec{\nabla}\phi \quad \cdots\cdots (3)$$

$$\frac{d\vec{r}}{dt} = \vec{V} \quad \cdots\cdots (4)$$

$$\nabla^2\phi = 4\pi G\rho(\vec{r}, t) \quad \cdots\cdots (5)$$

become

$$\frac{d\vec{V}_c}{dt} + 2H(t)\vec{v}_c = -\frac{1}{a^3}\vec{\nabla}_c\phi_c \quad \cdots\cdots (6)$$

$$\frac{d\vec{r}_c}{dt} = \vec{v}_c \quad \cdots\cdots (7)$$

$$\nabla_c^2 \phi_c = 4\pi G(\rho_c(\vec{r}_c, t) - \rho_b) = 4\pi G\sigma(\vec{r}_c, t) \quad \cdots\cdots (8)$$

where $H(t) = \dot{a}(t)/a(t)$, $\rho_c = a^3(t)\rho$, and ρ_b is the constant average mass density. In the flat universe a(t) and H(t) have simple expressions

$$a(t) = (6\pi G\rho_0 t^2)^{\frac{1}{3}} \quad \cdots\cdots (9)$$

$$H(t) = \frac{2}{3t} \quad \cdots\cdots (10)$$

since $\rho_0 = \rho_{crit}$ in the flat universe. The accuracy of the code is tested by the criterion of energy conservation. The energy equation is given by Layzer(1963);

$$\frac{d}{dt}(a^4 T) + a\frac{dw}{dt} = 0 \quad \cdots\cdots (11)$$

where the kinetic energy

$$T = \sum_i \frac{1}{2} v_c^2$$

and the potential energy

$$W = -\frac{1}{2} G \int\int \frac{\sigma(\vec{r}_c, t)\, \sigma(\vec{r}'_c, t)}{|\vec{r}_c - \vec{r}'_c|} d\tau_c d\tau'_c$$

The program used for the development of the present code was that written for the simulation of the cold collapse(Kim et al. 1986; Park and Kim 1987). The old code employs the superparticle with the Gaussian shape of mass distribution(Park 1988), which removes inter-particle collisions effectively to yield the collisionless motion of particles in the mean field. This superparticle technique is also used in the new code. However, since the space itself expands, the size of the superparticle also increases accordingly. The computations done in the code can be described as follows.

(i) Input; z (initial redshift parameter), $\vec{r}_c$, and $\vec{v}_c$.

(ii) Calculate a, t, and H from z using the equations (9) and (10).

(iii) Calculate ρ_c on the grid points. Since we are using the comoving coordinates, the expansion factor a in $\rho_c = a^3(t)\rho$ is automatically taken care of.

(iv) Subtract ρ_b from ρ_c, and calculate ϕ_c using fast Fouier transform technique. It should be noted that ρ_b is always equal to 1, since the same number of particles is used as the number of grid points.

(v) Particle velocities and positions are advanced using equations (6) and (7). These equations are represented by central differences,

$$\vec{v}_c^{\,n+\frac{1}{2}} = \vec{v}_c^{\,n-\frac{1}{2}} \frac{(1 - H(t)dt)}{(1 + H(t)dt)} + \frac{\vec{F}_c^n dt}{(1 + H(t)dt)} \quad \cdots\cdots (12)$$

$$\vec{v}_c^{\,n+1} = \vec{r}_c^{\,n} + \vec{v}_c^{\,n+\frac{1}{2}} dt \quad \cdots\cdots (13)$$

where $\vec{F}_c^n = -[\vec{\nabla}_c \phi_c / a^3]^n$, and dt is the time step.

(vi) Calculate new a from t^{n+1}, where $t^{n+1} = t^n + dt$. Proceed again from step (iii).

The energy equation (11) is written in the code as

$$(a^4T)^n + (aW)^n - \sum_{m=1}^{n} dt W^m \dot{a}^m = C \quad \cdots\cdots (14)$$

where $C = (a^4T)^0 + (aW)^0$. Thus, the left hand side of (14) is calculated at every tenth step, and compared to C of the initial value.

The code was tested with initial z = 10, and dt = 0.001. It was found that the error is less than 0.5% of the initial value C in 100 iterations. The error can be mode smaller by choosing smaller value of the dt.

IV. Discussion

The N-body computer code suitable for simulating the large scale structure of the expanding universe was developed and the algorithm of the code was described in this paper. The test run showed that the energy is conserved well and the code is very accurate. The code will be used to test several cosmological models. Different initial conditions yield different clusterings, which can be compared to the observations by computing correlation functions, and so on. For example, the cosmological model containing both hot matter and cold matter can be studied by employing two types of superparticles with different sizes. Hot material may be represented by the particles of large effective volume, while the cold material may be represented by the particles of small effective volume.

Acknowledgements

I acknowledge financial support by the Korea Science and Engineering Foundation and the Korea Institute of Technology.

Reference

Centrella, J. and Melott, A. L. 1983, *Nature,* **305**, 196.

Davis, M., Efstathiou, G., Frenk, C. S., and White, S. D. M. 1985, *Astrophys. J.* **292**, 371.

de Vaucouleurs, G. 1981, *Bull. Astr. Soc. Ind.*, **9**, 1.

Dicke, R., Peebles, P., Roll, P., and Wilkinson, D. 1964, *Astrophys. J.*, **142**, 414.

Efstathiou, G. and Eastwood, J. W. 1981, *M.N.R.A.S.*, **194**, 503.

Efstathiou, G., Davis, M., Frenk, C. S., and White, S.D.M. 1985, *Astrophys. J. Suppl.*, **57**, 241.

Hubble, E. 1929, *Proc. Nat. Acad. Sci.*, **15**, 169.

Kim, S. Y., Lee H. M., and Min, K. W. 1986, *The Use of Supercomputers in Stellar Dynamics*, ed. P. Hut and S. McMillan (New York: Springer-Verlag).

Oort, H. J. 1983, *Ann. Rev. Astr. Ap.*, **21**, 373.

Park, J. H. and Kim, S. Y. 1987, *J. Astron. Space Sci.*, **4**, 1.

Park, J. H. 1988, MS Thesis, Yonsei University.

Peebles, P.J.E. 1971, *Physical Cosmology* (Princeton: Princeton University Press).

Penzias, A. and Wilson, R. 1965, *Astrophys. J.*, **142**, 419.

Weinberg, S. 1972, *Gravitation and Cosmology* (New York: Wiley).