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Generation of Large Scale Magnetic Fields by Cross Field Diffusion

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Abstract

The possibility of the cross-field diffusion of a fluid to generate magnetic fields on a large scale is examined. Application to the sunspot is discussed.

One of the major issues in plasma physics is how to explain on origin of the large scale magnetic fields in many astrophysical systems. There are many different points of views for each different system (Parker 1979, Brandt and Hodge 1964, Gubbins 1974), but the currently dominating idea, which applies to most of the systems commonly, is that the magnetic fields are generated by the motions of the materials. Dynamo theory is concerned with these fluid motions which generate magnetic fields on a large scale (Gubbins 1974, Braginskii 1964, Cowling 1934).

The difficulties of a dynamo theory can be understood at several different levels according to the complexity of the system to which the model is applied. But the fundamental difficulty lies in the requirement for any theory to deal with the fact that the fluid motions of a highly conducting material are nearly tied to the magnetic field lines, and, as a consequence, magnetic fields are not easily generated by the fluid motions. The velocity required to generate magnetic fields is not large for many systems we are interested in, but it has to be the one relative to the field lines (Cowling 1977). There are many velocities greater than the required value in the hydromagnetic motion described by the Navier-Stokes equation, and so almost all fluid motions seem to have a capacity to generate magnetic fields. On the other hand, when the material's conductivity is very high, the magnetic field lines are almost frozen to the fluid motions, so that no fluid motion seems to be able to generate magnetic fields. These paradoxical properties of magnetohydrodynamic fluid motions have been the principal source of the difficulties. It is not clear what kinds of fluid motions would generate magnetic fields, but at least it is certain that a simply connected fluid motion does not generate net magnetic fields. Because a simply connected fluid motion cannot generate magnetic field, the solution has been pursued in three dimensional geometry. But solving a complete set of the Navier-Stokes equation and Maxwell's equation in a three dimensional configuration involves prohibitive technical difficulties. Thus, many dynamo theories have obtained the status of being neither established nor discarded(Cowling 1977).

This awkward situation of the theory of dynamo appears to be inevitable, as long as the mechanism of dynamo is sought in three dimensional fluid motions. However, the fluid motion in three dimensional geometry is not the only possible solution for the mechanism of a dynamo. The possibility that other kinds of fluid motions would generate magnetic fields through a different physical process is not ruled out.

We find such an alternative approach from an observation that the Navier-Stokes equation, which is used in a self-consistent dynamo theory, does not entirely govern the fluid motions, and that there is a certain class of fluid motions, outside the Navier-Stokes equation, which actually describe the relative motions to the field lines. The cross-field diffusion, described by a transport equation, gives a finite fluid flow across the field lines. Since the velocity of this flow is a relative one to the magnetic field lines, it can generate or dissipate magnetic fields, depending on the direction of its flow. Somehow, this capability of a cross-field fluid diffusion to generate magnetic fields has been completely neglected in the theory of dynamo. Therefore, we wish to examine in this paper under what circumstances the fluid diffusion can generate magnetic fields, and when the generated magnetic fields can be sufficiently large enough to be physically interesting.

We start our investigation with a straight cylindrical model of a sunspot plasma. The radial, azimuthal and toroidal coordinates are represented by (r, υ, z). The plasma has a nonuniform magnetic field B_z in z diredtion. The plasma is confined in a cylinder with radius a, where the magnetic field lines determine the plasma boundary. There can be small amplitude fluctuating magnetic fields in the r and υ directions, but the dominant magnetic fields are assumed to be in the z direction. In the following, we shall adopt the cgs units.

The behavior of the magnetic fields is governed by Ohm's law,

$$\frac{\partial \underset{\sim}{B}}{\partial t} = \underset{\sim}{v} \times (\underset{\sim}{v} \times \underset{\sim}{B}) + \frac{\eta c^2}{4\pi} \nabla^2 B \quad \cdots\cdots \quad (1)$$

Here we assume a constant resistivity for simplicity. For the magnetohydrodynamic fluid motions described by the Navier-Stokes equation, we give the boundary conditions as $v_r = 0$ and $B_r = 0$. These conducting boundary conditions are a mathematical simplification of the line-tying condition for the cylindrical model of the sunspot plasma. The boundary condition $v_r = 0$ reflects the mass conservation. We first show that under these boundary conditions the internal fluid motions do not contribute to the net flux change.

When Eq. (1) is integrated over the cross sectional surface of the cylinder, it becomes

$$\frac{\partial}{\partial t}\int_0^a B_z 2\pi r\ dr = \oint (\underset{\sim}{v} \times \underset{\sim}{B}) \cdot d\underset{\sim}{l} + \frac{\eta c^2}{4\pi}\int (\nabla^2 \underset{\sim}{B})_z 2\pi r\ dr \quad \cdots\cdots (2)$$

$$= (- v_r B_z + v_z B_r)|_a\ 2\pi a + \frac{\eta c^2}{4\pi}\int (\nabla^2 \underset{\sim}{B})_z\ 2\pi r\ dr \quad \cdots\cdots (3)$$

where $d\ell$ denotes a vector length element in the υ direction at $r = a$. In cylindrical coordinates,

$$(\nabla^2 \underset{\sim}{B})_z = (\frac{d^2}{dr^2} + \frac{1d}{rdr})\ B_z$$

When the boundary conditions of $v_r = 0$ and $B_r = 0$ are employed, Eq. (3) becomes

$$\frac{\partial}{\partial t}\int_0^a B_z\ 2\pi r\ dr = \frac{\eta c^2}{4\pi}\int (\nabla^2 \underset{\sim}{B})_z\ 2\pi r\ dr \quad \cdots\cdots (4)$$

$$\sim - \eta c^2 B_z \quad \cdots\cdots (5)$$

Result (4) shows that the internal fluid motions do not particpate in the net flux change, regardless of their natures, whether they are linear, nonlinear, or turbulent, as long as the boundary conditions are given as described above. For a more general case where the dominant magnetic fields have r components as well as z components, and if the boundary conditions are still given as a line-tying condition of $\underset{\sim}{v} x \underset{\sim}{B} = 0$ at the plasma boundary, a similar conclusion can be obtained. Therefore, the ordinary hydromagnetic motions cannot supplement the flux dissipated by resistivity. This result is a consequence of the boundary conditions, and is not affected by the consideration of other nonideal effects such as viscosity, Hall term or electron inertia.

If there is a fluid motion that generates magnetic fields against the resistive dissipation, Eq. (3) requires the fluid motion to have a finite velocity at the plasma boundary. How can a plasma have a finite velocity relative to the magnetic field lines which determine the plasma boundary, without violating the conservation of mass? This condition is not easily achieved. However, if there is a plasma source outside the cylinder and a sink inside, or the other way around, then the associated transport process can give rise to a finite velocity at the boundary. This finite velocity will then induce a finite value of emf, unless $B_Z = 0$ at the boundary. Physically, a source and sink of a plasma can naturally exist if there is a temperature difference between the inside and the outside of the plasma. If there is a temperature difference, neutral atoms are ionized in the hot region, and plasma recombines in the cool region. This process produces density variations for the plasma and the neutral particles, which cause flows of plasma and neutral atoms in opposite directions. When the temperature difference is maintained, this process is repeated, forming a recycling process – ionization, a plasma flow, recombination, a neutral follow, and ionization again. In fact, the recyling process is a well known phenomenon in laboratory plasmas. We are particularly interested in the plasma flow in this recycling process since this fluid motion has the relative velocity to the field lines.

The sunspot plasma is in the equilibrium with the outside plasms, so that

$$P_s + \frac{B_s^{\,2}}{8\pi} = P_0 \qquad (6)$$

where s and o denote the spot and the outside respectively. Then, because of the large magnetic field B_S, the pressure P_S in the spot region is less than P_O in the outside region. The whole mass densities in both regions are almost equal. Then the temperature in the spot region has too be much lower than that in the outside. It is known that the spot region has a temperature of about 4000°K, while the rest of the sun has a temperature of about 6000°K. Because a temperature difference is maintained between the inside and the outside of the sunspot, a recycling process is expected. Thus there will be a plasma inflow to the spot region and a neutral outflow to the outer region. The magnetic fields in the spot region are in the same direction. Then according to Eq. (3), this plasma inflow will cause an increase of the total flux.

A possible flow associated with the recycling process is a cross-field diffusion due to the density gradient. Its motion can be described as Eq. (7).

$$v_r = -D\frac{1}{n}\frac{\partial n}{\partial r} \qquad (7)$$

where n denotes density, and D is a diffusion coefficient. The cross-field diffusion coefficient D is determined by the microscopic properties of the plasma, depending on the level of the turbulence, and is in the range of $D_{class} < D < D_{Bohm}$, where D_{class} is a classical diffusion coefficient due to the collision, with the value of $\eta \frac{nkTc^2}{B^2}$, and D_{Bohm} is the Bohm anomalous diffusion coefficient with the value of $\frac{1}{16}\frac{kTc}{eB}$. Here k is the Boltzmann constant, and e is the elementary charge. T denotes a temperaturre.

For the smallest value of D_{class}, the emf induced by the diffusion becomes, from Eq. (3),

$$-v_r B_z\ 2\pi a = D_{class} \frac{1}{n}\frac{\partial n}{\partial r} B_z\ 2\pi a \quad (8)$$

$$\sim \eta c^2 B_z \quad (9)$$

Equation (9) is exactly the same as Eq. (5). That is, the emf induced by the classical diffusion is in the same order as the flux dissipated by resistivity in a unit time. Thus we see that the cross-field diffusion near the classical value can maintain the magnetic field against the resistive dissipation.

The plasma of the sunspot is very active. A variety of solar activities are known to be connected with the sunspot. The transport phenomenon of a sunspot might be more appropriately described by a turbulent diffusion coefficient. When the Bohm diffusion coefficient is used, Eq. (3) gives, neglecting the resistive diffusion term,

$$\gamma B \sim \frac{c}{4\pi}\frac{1}{L^2}\frac{B^2}{ne} \quad (10)$$

L denotes a spatial scale, here the size of a sunspot. For the typical parameter, $B \sim 1000$ gauss, $n \sim 10^{14}/cm^3$, $L \sim 100$ km, we obtain a field amplification rate $\gamma \sim 3\times10^{-8}$/sec which is about 1/month. This value seems to be in the right range to be compared to the observational results.

Summing up, we have considered the possibility of the generation of large scale magnetic fields by cross-field diffusion. With the examples of the sunspot, we have seen that the cross-field diffusion is a plausible mechanism for the generation of large scale magnetic fields. This mechanism is, however, not limited to this example only. The generality of this mechanism together with the widespread occurrence of diffusion suggests that many other large scale magnetic fields in astrophysical and laboratory conducting media might also have a similar relation to diffusion.

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