◉ 研究論文

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A Study on the Optimal State Estimation of a Dynamic System with an Unknown Input

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入力이 未知인 動的시스템의 最適狀態推定에 關한 研究

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要 約

未知의 操作量이나 매우 큰 外亂이 入力으로 作用하고 있는 動力시스템의 精度 높은 狀態를 推定 하려면 狀態推定에 앞서 시스템의 入力推定이 要求된다. 본 論文에서는 간략형 칼만필터 (SKF: Simplified Kalman Filter)를 利用하여 運動하고 있는 目標物의 狀態推定을 行함과 동시에 起動탐지자(Maneuvering Detector)와 入力추정자(Input Estimator)에 의해 시스템의 入力을 推定하고 이것에 의하여 SKF의 推定値를 補正해줌으로써 入力이 未知인 動的 시스템의 狀態推定에 있어서 推定精度를 改善하는 方法을 提案하며 디지탈계산기를 利用한 시뮤레이션을 통하여 본 方法의 有効性을 밝힌다.

1. Introduction

The state estimation problem for systems with unknown maneuvering inputs finds wide applications in target tracking systems and control systems for plants with large biased noises. In target tracking systems such as air or sea traffic control, weapons systems, space aircrafts, and range ships, a continuous tracking of the object may be desired. In order to provide the reliable knowlege about the targets, the input of the system should be estimated.

Many different tracking filters such as the Kalman filter, α-β filter, the Wiener filter and a simple extrapolator have been developed since Kalman first introduced the idea of the recursive filtering in the early 1960's and there have been many advances in the development of sophisticated digital filtering algorithms for tracking airborne targets.

Earlier work on the maneuvering target tracking problem includes Singer's generalized tracking model.[7] The generalized model tracks a maneuvering target fairly well provided the so-called "maneuver parameters" are appropriately chosen, but if the target is not maneuvering, the tracker degrades in performance compared to the tracker based on a constant-velocity model. Mcauray and Delinger[4] have shown that there are significant improvements in the

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tracking capability when using a maneuvering detector in two parallel models. Thorp[11] showed that a weighted combination of two Kalman filters gives good estimates in response to a detected maneuver. Moose et al.[15] combined the generalized model of Singer with the adaptive Semi-Markov maneuver model. Another technique, described by Chan et al.[3], uses a least square estimator to estimate a target's acceleration input and updates the output of the baseline tracker, i.e., the predicted estimate, by the input estimate if the detection is declared.

In this paper a maneuvering target problem is implemented by incorporating the input estimator with an update of the filtered estimate to provide some improvement in position accuracy. Measurements of target position are made in sensor coordinates and then filtering is performed in the same frame. We continue to use the constant-velocity, straight-line tracker for estimating positions. When a bias develops in the residual sequence due to the target deviation from the assumed motion, updating of the filtered estimate is performed to remove the bias. Whenever the estimate is updated, the error covariance increases. So, in order to guard against unexpected update of the estimate while the target doesn't maneuver, the likelihood ratio test is used to monitor the occurrence of maneuver at each time.

2. Modeling

A plot of the target and sensor geometry is shown in Fig. 1.

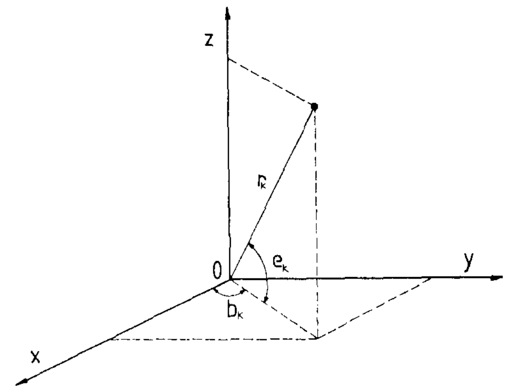

Fig. 1. Target geometry at time t_k : r_k, b_k, and e_k denote range, bearing, and elevation, respectively. The sensor is assumed at the origin.

The selection of spherical coordinates(r, b, e) rather than Cartesian coordinates(x, y, z) for our target and sensor modeling is due to the fact that the measurement error covariance becomes diagonal. The true target motion is a nonlinear coupled differenital equation in the range, bearing, and elevation variables but the approximation of target motion by a linear system can be found. An approximate spherical dynamic[5] of the target can be represented in the matrix form as

$$X_{\kappa+1} = A_\kappa X_\kappa + B_\kappa U_\kappa + G_\kappa W_\kappa, \qquad (1)$$

where

$$X_\kappa = \begin{pmatrix} r_\kappa \\ \dot{r}_\kappa \\ b_\kappa \\ \dot{b}_\kappa \\ e_\kappa \\ \dot{e}_\kappa \end{pmatrix} \quad A_\kappa = \begin{pmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B_\kappa = \begin{pmatrix} T^2/2 & 0 & 0 \\ T & 0 & 0 \\ 0 & T^2/2d_1 & 0 \\ 0 & T/d_1 & 0 \\ 0 & 0 & T^2/2d_2 \\ 0 & 0 & T/d_2 \end{pmatrix} \quad G_\kappa = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

and

X_κ = state vector at time t_κ

A_κ = state transition matrix

$U_\kappa = [u_r(k),\ u_b(k),\ u_e(k)]^T$, deterministic input vector

$W_\kappa = [w_r(k), w_b(k), w_e(k)]^T$, noise on the state

T = sampling period

$d_1 = \hat{r}_{\kappa/\kappa}\cos(\hat{e}_{\kappa/\kappa})$

$d_2 = \hat{r}_{\kappa/\kappa}$ with $\hat{r}_{\kappa/\kappa}$ and $\hat{e}_{\kappa/\kappa}$ provided by the filtered estimate $\hat{X}_{\kappa}/_{\kappa}$.

The observation equation can be written as

$Z_{\kappa+1} = H_{\kappa+1} X_{\kappa+1} + V_{\kappa+1}$,

where

$$H_\kappa = \begin{pmatrix} 1&0&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&0&1&0 \end{pmatrix} \tag{2}$$

$Z_\kappa = [z_r(k),\ z_b(k),\ z_e(k)]^T$, measurement at time t_κ

$V_\kappa = [v_r(k),\ v_b(k),\ v_e(k)]^T$, measurement noise.

$\{W_\kappa\}$ and $\{V_\kappa\}$ are assumed as independent zero mean, white noises with

$$E\{W_\kappa W_n^T\} = Q_\kappa \delta_{\kappa n} \tag{3}$$

$$E\{V_\kappa V_n^T\} = R_\kappa \delta_{\kappa n} \tag{4}$$

$$E\{W_\kappa V_n^T\} = \mathbf{0}$$

for all $k, n = 0, 1, \cdots$, where $\delta_{\kappa n}$ is the Kronecker delta. It is further assumed that the initial state is independent of the two noises.

3. Kalman Filter Equations

By dropping the time indices of the constant matrices A_κ, G_κ, and H_κ for convenience, the Kalman filter equations are given by

$$\hat{X}_{\kappa+1/\kappa+1} = \hat{X}_{\kappa+1/\kappa} + K_{\kappa+1}[Z_{\kappa+1} - H\hat{X}_{\kappa+1/\kappa}] \tag{5}$$

$$\hat{X}_{\kappa+1/\kappa} = A\hat{X}_{\kappa/\kappa} + B_\kappa U_\kappa \tag{6}$$

$$K_{\kappa+1} = P_{\kappa+1/\kappa} H^T [H P_{\kappa+1/\kappa} H^T + R_{\kappa+1}]^{-1}$$

$$P_{\kappa+1/\kappa} = A P_{\kappa/\kappa} A^T + G Q_\kappa G^T$$

$$P_{\kappa+1/\kappa+1} = [I - K_{\kappa+1} H] P_{\kappa+1/\kappa}. \tag{7}$$

In order to start the recursive filtering operation, the Kalman filter equations should be initialized. Assuming that after the first two measurements, i.e., Z_1 and Z_2, are received, the optimum state vector $\hat{X}_{2/2}$ can be initialized as in (8)

$$\hat{X}_{2/2} = \begin{pmatrix} \hat{r}_{2/2} \\ \hat{\dot{r}}_{2/2} \\ \hat{b}_{2/2} \\ \hat{\dot{b}}_{2/2} \\ \hat{e}_{2/2} \\ \hat{\dot{e}}_{2/2} \end{pmatrix} = \begin{pmatrix} z_r(2) \\ [z_r(2) - z_r(1)]/T \\ z_b(2) \\ [z_b(2) - z_b(1)]/T \\ z_e(2) \\ [z_e(2) - z_e(1)]/T \end{pmatrix} \tag{8}$$

The corresponding covariance of the errors in the optimum estimate, as shown in Appendix A in detail, is

$P_{2/2} = S_1 R_1 S_1^T + S_2 R_2 S_2^T$,

where

$$S_1 = \begin{pmatrix} 0&0&0 \\ 1/T&0&0 \\ 0&0&0 \\ 0&1/T&0 \\ 0&0&0 \\ 0&0&1/T \end{pmatrix} \quad S_2 = \begin{pmatrix} 1&0&0 \\ 1/T&0&0 \\ 0&1&0 \\ 0&1/T&0 \\ 0&0&1 \\ 0&0&1/T \end{pmatrix}$$

and R_1, R_2 are obtained from(4).

Now U_κ is also unknown but will be estimated in the sequel. If the target is not maneuvering, the target motion can be well modeled by simplifying the maneuvering model, i.e., $U_\kappa = 0$ in(1). The filter that uses the simplified model is called the Simplified Kalman Filter(SKF). The filtered estimate $\hat{X}_{\kappa+1/\kappa+1}$ can be expressed by using $\hat{X}_{\kappa/\kappa}$ and $Z_{\kappa+1}$:

$$\begin{aligned}\hat{X}_{\kappa+1/\kappa+1} &= A\hat{X}_{\kappa/\kappa} + B_\kappa U_\kappa + K_{\kappa+1}[Z_{\kappa+1} - H(A\hat{X}_{\kappa/\kappa} + B_\kappa U_\kappa)] \\ &= D_{\kappa+1} A\hat{X}_{\kappa/\kappa} + K_{\kappa+1} Z_{\kappa+1} + D_{\kappa+1} B_\kappa U_\kappa \end{aligned} \tag{9}$$

where $D_{\kappa+1} = [I - K_{\kappa+1} H]$.

Using the similiar idea to the innovations process,[2] a sequence is defined by use of the filtered estimate

$$\tilde{Z}_\kappa = Z_\kappa - H\hat{X}_{\kappa/\kappa}. \tag{10}$$

This residual sequence $\{\tilde{Z}_k\}$ has the important properties[2),3)] that it is a zero mean white Gaussian noise if the initial state and the two noises are Gaussian. Its covariance becomes

$$E\{\tilde{Z}_\kappa \tilde{Z}_\kappa^T\} = H P_\kappa/_\kappa H^T + R_\kappa = \omega_\kappa. \tag{11}$$

4. Estimator of the Input[3]

As shown in the previous section, the Kalman filter equation requires an U_κ, which is unknown, but should be estimated. When $U_\kappa=0$ in(9), we denote the estimate of the SKF by $\bar{X}_{\kappa/\kappa}$. Suppose that prior to time t_κ no maneuvers occur such that $\hat{X}_{\kappa/\kappa}=\bar{X}_{\kappa/\kappa}$ and the target now undergoes a maneuver with a sequence of inputs U_κ, $U_{\kappa+1},\cdots$, $U_{\kappa+n-1}$,. The Kalman filter(9), which is linear, will continue to give, at times $t_{\kappa+1}$, $t_{\kappa+2},\cdots,t_{\kappa+n}$, the estimates:

$$\hat{X}_{\kappa+1/\kappa+1}=D_{\kappa+1}A\hat{X}_{\kappa/\kappa}+K_{\kappa+1}Z_{\kappa+1}+D_{\kappa+1}B_\kappa U_\kappa$$
$$=\bar{X}_{\kappa+1/\kappa+1}+D_{\kappa+1}B_\kappa U_\kappa$$
$$\hat{X}_{\kappa+1/\kappa+2}=D_{\kappa+2}A(D_{\kappa+1}A\hat{X}_{\kappa/\kappa}+K_{\kappa+1}Z_{\kappa+1}+D_{\kappa+1}B_\kappa U_\kappa)+K_{\kappa+2}Z_{\kappa+2}+D_{\kappa+2}B_{\kappa+1}U_{\kappa+1}$$
$$=\bar{X}_{\kappa+2/\kappa+2}+D_{\kappa+2}AD_{\kappa+1}B_\kappa U_\kappa+D_{\kappa+2}B_{\kappa+1}U_{\kappa+1}$$
$$\vdots$$
$$\hat{X}_{\kappa+n/\kappa+n}=\bar{X}_{\kappa+n/\kappa+n}+\sum_{j=0}^{n-2}\Big[\prod_{i=0}^{k}(D_{\kappa+n-i}A)D_{\kappa+n-1-j}B_{\kappa+n-2-j}U_{\kappa+n-2-j}\Big]+D_{\kappa+n}B_{\kappa+n-1}U_{\kappa+n-1} \quad (12)$$

Equation(12) gives us an insight that the bias developed due to target maneuvers will be removed by an addition of a correction term to the estimate of the SKF. Now we make one approximation to estimate the unknown deterministic input. The target moves under a constant acceleration, i.e., $U_{\kappa+n}=U$ for $n=0,1,\cdots,m-1$. Even though $U_{\kappa+n}$ are not constant over the interval, the estimator will give the best constant estimate for the different inputs in the least square sense. Equation(12) can be rewritten as

$$\hat{X}_{\kappa+n/\kappa+n}=\bar{X}_{\kappa+n/\kappa+n}+\Big\{\sum_{j=0}^{n-2}\Big[\prod_{i=0}^{j}(D_{\kappa+n-i}A)D_{\kappa+n-1-j}B_{\kappa+n-2-j}\Big]+D_{\kappa+n}B_{\kappa+n-1}\Big\}U. \quad (13)$$

Let

$$\bar{Z}_{\kappa+n}=Z_{\kappa+n}-H\bar{X}_{\kappa+n/\kappa+n} \quad (14)$$

and from(10)

$$\tilde{Z}_{\kappa+n}=Z_{\kappa+n}-H\hat{X}_{\kappa+n/\kappa+n}, \quad (15)$$

then from (13), (14), and (15)

$$\bar{Z}_{\kappa+n}=H(\hat{X}_{\kappa+n/\kappa+n}-\bar{X}_{\kappa+n/\kappa+n})+\tilde{Z}_{\kappa+n}$$
$$=H\Big\{\sum_{j=0}^{n-2}\Big[\prod_{i=0}^{j}(D_{\kappa+n-i}A)D_{\kappa+n-1-j}B_{\kappa+n-2-j}\Big]+D_{\kappa+n}B_{\kappa+n-1}\Big\}U+\tilde{Z}_{\kappa+n}. \quad (16)$$

The matrix form of(16) for $n=1,2,\cdots,m$, namely for moving data window m, is given by

$$Y=FU+e, \quad (17)$$

where

$$Y=\begin{pmatrix}\bar{Z}_{\kappa+1}\\ \bar{Z}_{\kappa+2}\\ \vdots\\ \bar{Z}_{\kappa+m}\end{pmatrix},$$

$$F=\begin{pmatrix}HD_{\kappa+1}B_\kappa\\ H(D_{\kappa+2}AD_{\kappa+1}B_\kappa+D_{\kappa+2}B_{\kappa+1})\\ \vdots\\ H\Big\{\sum_{j=0}^{m-2}\Big[\prod_{i=0}^{j}(D_{\kappa+m-i}A)D_{\kappa+m-1-j}B_{\kappa+m-2-j}\Big]+D_{\kappa+m}B_{\kappa+m-1}\Big\}\end{pmatrix}$$

and

$$e=\begin{pmatrix}\tilde{Z}_{\kappa+1}\\ \tilde{Z}_{\kappa+2}\\ \vdots\\ \tilde{Z}_{\kappa+m}\end{pmatrix}.$$

Y and e are both $3m\times1$ vectors and F is a $3m\times3$ matrix. Since R_κ is diagonal for all time, the $3m\times3m$ covariance matrix for e is found to be the diagonal matrix M as

$$M=E\{ee^T\}=\begin{pmatrix}\omega_{\kappa+1} & & & 0\\ & \omega_{\kappa+2} & & \\ & & \ddots & \\ 0 & & & \omega_{\kappa+m}\end{pmatrix}. \quad (18)$$

The unbiased least square estimate[10),11)] of $\hat{U}$ is

$$\hat{U}=(F^TM^{-1}F)^{-1}(F^TM^{-1}Y) \quad (19)$$

and its error covariance is

$$L=(F^TM^{-1}F)^{-1} \quad (20)$$

As long as detection occurrs, the estimate of the tracker is updated by the optimum input estimate $\hat{U}$ through

$$\hat{X}_{\kappa+m/\kappa+m}=\bar{X}_{\kappa+m/\kappa+m}+C_{\kappa+m}\hat{U}, \quad (21)$$

where

$$C_{\kappa+m}=\sum_{j=0}^{m-2}\left[\prod_{i=0}^{j}(D_{\kappa+m-i}A)D_{\kappa+m-1-j}\right.$$
$$\left.B_{\kappa+m-2-j}\right]+D_{\kappa+m}B_{\kappa+m-1}$$

and $\bar{\bar{X}}_{\kappa+m/\kappa+m}$ is the updated estimate of $X_{\kappa+m}$.

The brief configuration of the input estimator is shown in Fig. 2.

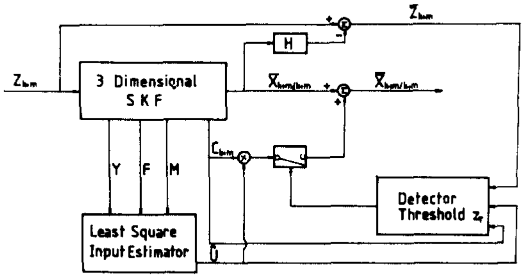

Fig. 2. Input estimator at t_{k+m}.

As we expect, the correction through the equation(21) will not only remove the bias developed in $\bar{X}_{\kappa+m/\kappa+m}$ but also increase the covariance of estimation errors. So, the update of the estimate should be performed when maneuvers are detected. While the target doesn't maneuver, nonzero $\hat{U}$ is due to the effects of the two noises. Such small $\hat{U}$ should be ignored. Using the results obtained in the Appendix B, the following covariance matrix with the positive definite $C_{\kappa+m}LC^T_{\kappa+m}$ term is obtained :

$$E\{(X_{\kappa+m}-\bar{\bar{X}}_{\kappa+m/\kappa+m})\ (X_{\kappa+m}-\bar{\bar{X}}_{\kappa+m/\kappa+m})^T\}$$
$$=P_{\kappa+m/\kappa+m}+C_{\kappa+m}LC^T_{\kappa+m}. \qquad (22)$$

5. Detection of the Maneuver

Detection of a target requires the choice of a threshold and a moving data window. These quantities are chosen by considering tradeoffs among the probability of false alarm P_F, the probability of detection P_D, savings of computation time, and accuracy of the least square estimate. Now we assume that all the statistics related to our detection problem are Gaussian. As shown in the previous section, if the detector correctly detects the maneuver, the estimate of the SKF is updated by the input estimate $\hat{U}$. Our detection scheme is that the norm of $\hat{U}$, $\|\hat{U}\|_\infty$ is first found and the optimum test for the corresponding component of $\|\hat{U}\|$ is made. The idea behind this detection scheme is that if $\|\hat{U}\|$ is small, the actual U is not only small but aslo $\bar{Z}_\kappa$ is small. Let the subscript x denote vectors or scalars correspondiag to $\|\hat{U}\|$. Surely, x will represent one of the following: r(ange), b(earing or e(levation). Detection of the bias at time t_{q+m} based on the multiple observation reduces to the following hypothesis test:

H_0: no maneuver occurs: $r_\kappa=(\bar{Z}_\kappa)_x$ (23)

H_1: maneuver occurs: $r_\kappa=(\bar{Z}_\kappa)_x-(C_\kappa\hat{U}_\kappa)_x$

for $k=q+1,\cdots,q+m$ (24)

where $\hat{U}_\kappa$ is the optimum estimate of U at t_κ. From the above expressions, $\boldsymbol{r}$, $\boldsymbol{c}$, and $\boldsymbol{v}$ are defined as

$$\boldsymbol{r}=\begin{pmatrix} r_{q+1}\\ r_{q+2}\\ \vdots\\ r_{q+m}\end{pmatrix},\qquad \boldsymbol{c}=\begin{pmatrix} -C_{q+1}\hat{U}_{q+1}\\ -C_{q+2}\hat{U}_{q+2}\\ \vdots\\ -C_{q+m}\hat{U}_{q+m}\end{pmatrix}_x,$$

and

$$\boldsymbol{v}=\begin{pmatrix} \bar{Z}_{q+1}\\ \bar{Z}_{q+2}\\ \vdots\\ \bar{Z}_{q+m}\end{pmatrix}_x$$

which are all $m\times 1$ vactors. It has always been known that the measurement error covariance R_x is uncoupled. One valid assumption can be made that $R_x=R$ for all time. If we suppose that the target is in a well-defined track long enough before any maneuver occurs so that the steady state value of $P_{q/q}$ can be chosen to compute ω_q in (11), then we might take, to a good approximation, the $m\times m$ covariance matrix of $\boldsymbol{v}$ as

$$E\{\boldsymbol{v}\boldsymbol{v}^T\}=\begin{pmatrix}\omega_q & & & 0\\ & \cdot & & \\ & & \cdot & \\ 0 & & & \omega_q\end{pmatrix}_x=\boldsymbol{\omega}. \qquad (25)$$

Then the likelihood ratio and likelihood ratio test are

$$L(\boldsymbol{r})=\mathrm{EXP}[-1/2(\boldsymbol{r}-\boldsymbol{c})^T\boldsymbol{\omega}^{-1}(\boldsymbol{r}-\boldsymbol{c})+1/2\boldsymbol{r}^T\boldsymbol{\omega}^{-1}\boldsymbol{r}] \quad (26)$$

and

$$L(\boldsymbol{r}) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \quad (27)$$

repestively and where λ is the threshold of the test. Taking logarithms on the both sides of (26), (27) and substituting (26) into (27), equation (27) can be written

$$\boldsymbol{c}^T\boldsymbol{\omega}^{-1}\boldsymbol{r} \underset{H_0}{\overset{H_1}{\gtrless}} \ln\lambda+1/2\boldsymbol{c}^T\boldsymbol{\omega}^{-1}\boldsymbol{c}. \quad (28)$$

If a normalized scalar sufficient statistic is defined as

$$\Lambda(\boldsymbol{r})=\boldsymbol{c}^T\boldsymbol{\omega}^{-1}\boldsymbol{r}/(\boldsymbol{c}^T\boldsymbol{\omega}^{-1}\boldsymbol{c})^{1/2}, \quad (29)$$

the likelihood ratio test thus becomes

$$\Lambda(\boldsymbol{r}) \underset{H_0}{\overset{H_1}{\gtrless}} \ln\lambda(\boldsymbol{c}^T\boldsymbol{\omega}^{-1}\boldsymbol{c})^{-1/2}+1/2(\boldsymbol{c}^T\boldsymbol{\omega}^{-1}\boldsymbol{c})^{1/2}=Z_T \quad (30)$$

The detection operation therefore consists in determing $\Lambda(\boldsymbol{r})$ and comparing it with the threshold Z_T. If $\Lambda(\boldsymbol{r})$ exceed Z_T, it is decided that H_1 is the true hypothesis: otherwise, it is decided that H_0. We now can obtain the two expressions about P_F and P_D from the conditional densities of the sufficient statistic(29) conditioned upon the hypothesis H_0 and H_1:

$$P_F=\int_{Z_T}^{\infty}f_{\Lambda|H_0}(\Lambda|H_0)d\Lambda=\mathrm{erf}[-Z_T] \quad (31)$$

$$P_D=1-\int_{-\infty}^{Z_T}f_{\Lambda|H_0}(\Lambda|H_0)\,d\Lambda=1-\mathrm{erf}[Z_T-(\boldsymbol{c}^T\boldsymbol{\omega}^{-1}\boldsymbol{c})^{1/2}] \quad (32)$$

In using the Neyman-Pearson criterion[13),14)] subject to the constraint

$$P_F=\alpha, \quad (33)$$

the threshold Z_T and the probability of detection P_D can be obtained from (31), (32), and (33). However, it should be noted that the P_D depends not only upon the threshold Z_T but also $\boldsymbol{c}$, i.e., U_x so that P_D will increase with increases in U_x. Thus, this means that we need to specify the lower bound of P_D by using the minimum U_x that must be detected because the computed P_D for the given value α is exact only for one sample period. If any U_x greater than $(U_x)_{min}$ is observed, it will give a P_D higher than the lower bound. So, the steady-state value of $P_{q/q}$ and K_q are used since the estimation and detection is needed most when the gain of the SKF is small.

It has been mentioned earlier that the number m is a design parameter of the detector and estimator. Essentially, the m most recent residuals are examined to determine whether they differ significantly from the statistical description of their values that assumes no maneuvers. The number m greater than one will not only increase the accuracy of the input estimate but also prevent failure declarations due to a single unacceptable measurement. On the other hand, it is inappropriate to use a large number m since this will decrease the sensitivity to manenver occurrence as time progresses, along with an increase in the computation time. Hence, we might choose $m=5$ as the reasonable number of data points in the detection of a maneuver[3),4),5),10)]

6. Simulation and Results

The tracking scheme presented was implemented on a VAX/VMS computer using simulated data. For purposes of comparison, the estimate of the SKF without the maneuver detector was implemend in addition to that of the Kalman filter described in section 4, called the Modified Kalman Filter (MKF). Target trajectories generated in sensor coordinates are shown in Fig. 3 through Fig. 5. The following statistics are used:

$R=\mathrm{diag}(\sigma^2_r, \sigma^2_b, \sigma^2_e)$

where $\sigma_r=0.0183$km and $\sigma_b=\sigma_e=0.003$rad

$Q=\mathrm{diag}(\sigma^2_r, \sigma^2_b, \sigma^2_e)$

where $\sigma_r=0.183$m and $\sigma_b=\sigma_e=0.03$ mrad

T=2 sec

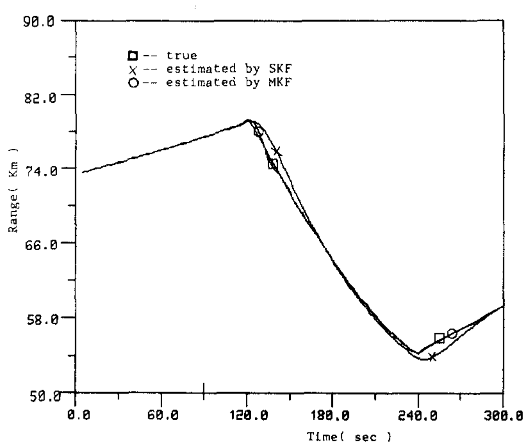

Fig. 3. Tracker Performance in range coordinate.

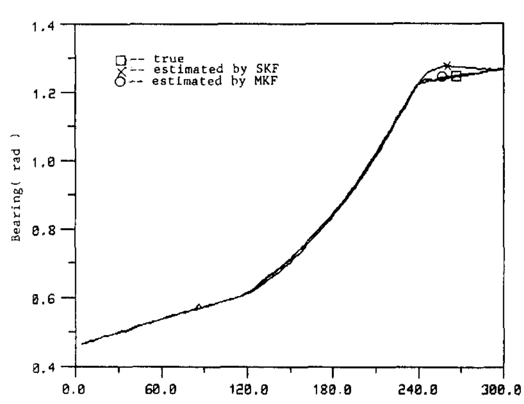

Fig. 4. Tracker performance in bearing coordinate.

and the simulated scenario is as follows.

The target initially flying at 0.1km/sec in speed is on a constant course for 112 seconds. At time $t=112$sec it begins to

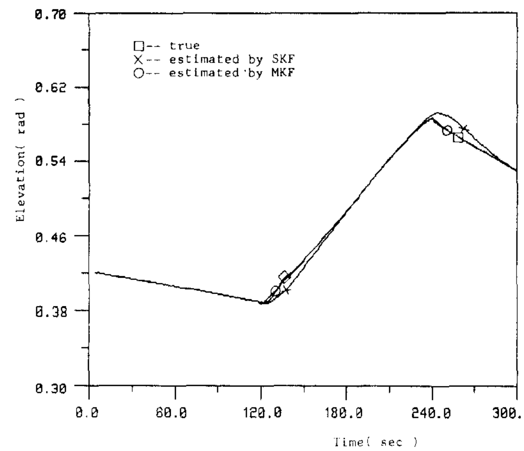

Fig. 5. Tracker performance in elevation coordinate.

maneuver with acceleration input +0.02 km/sec^2, at $t=120$sec commences a fast turn, and at $t=126$ sec completes its maneuvers. The target keeps a straightline track at 0.38km/sec in speed until it again starts its maneuvers with -0.02km/sec^2 at $t=232$ sec, making another fast turn at $t=240$sec, and finally completing its maneuvers at $t=246$ sec.

For our experiment, the probability of false alarm, P_F, was given by $P_F=2\times10^{-3}$ which led to $Z_T=2.87$. The filter was near steady state at $t=64$sec after the SKF was first put into operation.

In Fig. 3 through Fig. 5, we have shown the position estimates of the MKF and the SKF. The results shown in Fig. 3 through Fig. 5 show the MKF's superior tracking performance.

In order to give a good comparision, we computed the sum square residual errors which are the sum square of the differences between the true and estimated values. Table 1 gives the sum square residual errors. It is clear that the residual errors are quite small, especially for the MKF and a small difference as shown indicates the sat-

Table 1. Sum square residual errors between the true aud estimated values.

	SKF	MKF
Range	0.726914×10^{2}	0.262592×10^{1}
Bearing	0.204657×10^{-1}	0.964886×10^{-3}
Elevation	0.415042×10^{-2}	0.127342×10^{-3}

isfactory performance of the MKF. The next simulation, described by Fig. 6 through Fig. 8, shows the rms error in the MKF position estimate. It becomes apparent that during dectecting maneuvers the rms error increases but decreases during constant-speed, straight-line flight because the filter is settled by the dectector. This simulation also shows that the rms estimation errors are kept below the inherent measurement errors. This means that the MKF is able to maintain track with good tracking accuracy.

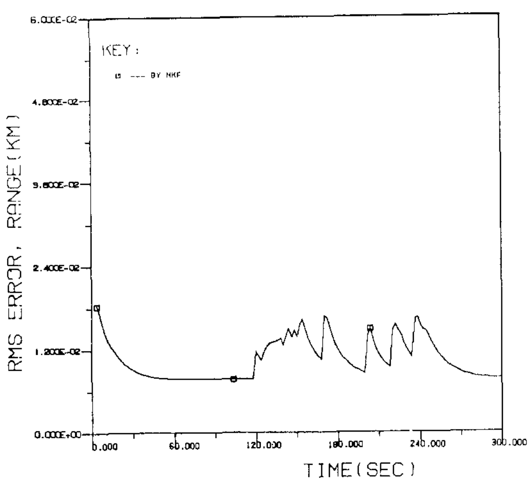

Fig. 6. rms error in range coordinate.

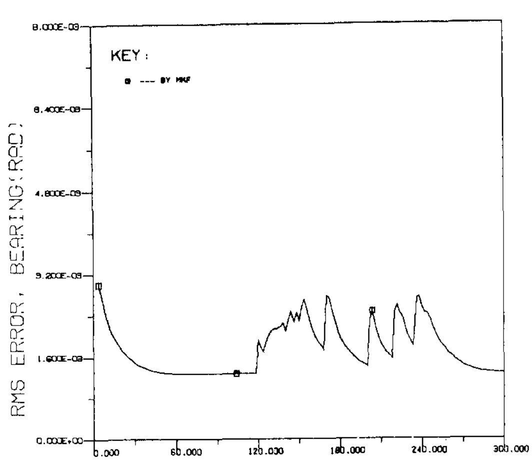

Fig. 7. rms error in bearing coordinate.

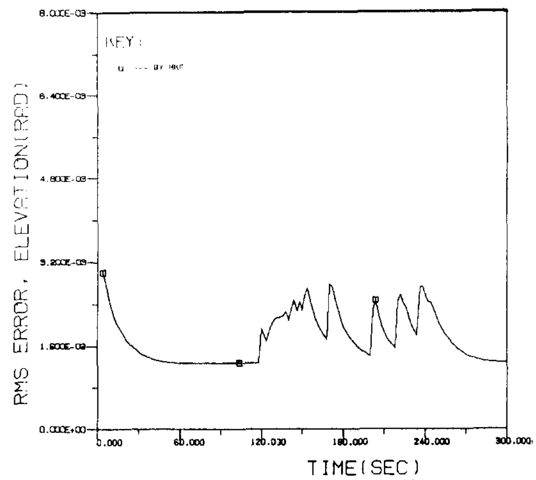

Fig. 8. rms error in elevation coordinate.

7. Conclusions

In this paper we have presented a tracking scheme which has given a good estimate of a target position in three dimensional space. This tracking scheme gives two advantages in addition to the several advantages which the scheme suggested by Chan et al. does:

First, by incorporating the least square input estimator and a detector, our baseline tracker produces a filtered estimate instead of a predicted estimate to ensure increased accuracy and the filtered estimate is updated when maneuvers are detected. Second, a detector based on multiple observation residuals are used to detect maneuvers to avoid maneuver declarations due to a single unacceptable observation.

Simulations show that the tracking scheme presented here can give a realistic solution to tracking problems for maneuvering or nonmaneuvering targets. In particular, if a typical target trajectories are known in practical applications, we can recommend reasonable choices of the threshold to improve the overall performance.

Appendix A

Initialization of the optimum state vector and the corresponding covariance of a tracking filter may be often taken while the maneuvers of the target are not well known, Therefore, we start the initialization under assumption of constant velocity, straight-line flight and with approximate equations(A-1) which provide satisfactory results for the majority of applications

$$\begin{aligned} \dot{r}_2 &\simeq (r_2 - r_1)/T \\ \dot{b}_2 &\simeq (b_2 - b_1)/T \\ \dot{e}_2 &\simeq (e_2 - e_1)/T, \end{aligned} \quad \text{(A-1)}$$

Equation(8) can be rewritten as

$$\begin{aligned} \hat{r}_{2/2} &= r_2 + v_r(2) \\ \hat{\dot{r}}_{2/2} &= \dot{r}_2 + [v_r(2) - v_r(1)]/T \\ \hat{b}_{2/2} &= b_2 + v_b(2) \\ \hat{\dot{b}}_{2/2} &= \dot{b}_2 + [v_b(2) - v_b(1)]/T \\ \hat{e}_{2/2} &= e_2 + v_e(2) \\ \hat{\dot{e}}_{2/2} &= \dot{e}_2 + [v_e(2) - v_e(1)]T. \end{aligned} \quad \text{(A-2)}$$

Expression in the matrix form of (A-2) becomes

$$X_2 - \hat{X}_{2/2} = S_1 V_1 - S_2 V_2,$$

$$S_1 = \begin{pmatrix} 0 & 0 & 0 \\ 1/T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1/T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/T \end{pmatrix} \text{ and } S_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1/T & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1/T & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1/T \end{pmatrix}$$

Hence,

$$\begin{aligned} P_{2/2} &= E\{(X_2 - \hat{X}_{2/2})\ (X_2 - \hat{X}_{2/2})^T\} \\ &= S_1 R_1 S_1^T + S_2 R_2 S_2^T. \end{aligned}$$

Appendix B

The least square estimate $\hat{U}$ is unbiased because $\boldsymbol{e}$ is also a zero mean, white sequence. Taking the expectation in both (21) and (13) with the time index change, we get

$$\begin{aligned} E\{\bar{\bar{X}}_{\kappa+m/\kappa+m}\} &= E\{\bar{X}_{\kappa+m/\kappa+m}\} + C_{\kappa+m} E\{\hat{U}\} \\ &= E\{\bar{X}_{\kappa+m/\kappa+m}\} + C_{\kappa+m} E\{U\} \\ &= E\{\hat{X}_{\kappa+m/\kappa+m}\}. \end{aligned}$$

Next we derive the error covariance in (22). From (13) and (21),

$$\bar{\bar{X}}_{\kappa+m/\kappa+m} = \hat{X}_{\kappa+m/\kappa+m} + C_{\kappa+m}(\hat{U} - U).$$

From (17), (19), and (20), we see that $\hat{U} - U = LF^T M^{-1} \boldsymbol{e}$.

Then,

$$X_{\kappa+m} - \bar{\bar{X}}_{\kappa+m/\kappa+m} = (X_{\kappa+m} - \hat{X}_{\kappa+m/\kappa+m}) - C_{\kappa+m} LF^T M^{-1} \boldsymbol{r}.$$

Hence,

$$\begin{aligned} &E\{(X_{\kappa+m} - \bar{\bar{X}}_{\kappa+m/\kappa+m})(X_{\kappa+m} - \bar{\bar{X}}_{\kappa+m/\kappa+m})^T\} \\ &= P_{\kappa+m} + C_{\kappa+m} LF^T M^{-1} E\{\boldsymbol{ee}^T\}(C_{\kappa+m} LF^T M^{-1})^T \\ &= P_{\kappa+m} + C_{\kappa+m} L C^T_{\kappa+m}. \end{aligned}$$

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