

A Method for Propagating Fuzzy Concepts through Fuzzy IF-THEN-ELSE Rules

Doohyun Kim⁺
Younghwan Lim^{**}
Jin H. Kim^{*}

ABSTRACT

This paper presents a method for propagating fuzzy concepts through fuzzy IF-THEN-ELSE rules. A fuzzy IF-THEN-ELSE rule consists of a set of fuzzy condition and conclusion pairs. These pairs assumed to contain informations about a fuzzy mapping from fuzzy concepts of condition parts to the fuzzy concepts of conclusion parts.

Conventionally, vectors are used to define fuzzy concepts and matrices are used to define a fuzzy mapping between fuzzy conditions and conclusions. This approach, however, does not satisfy the existing condition property, i.e., when a fuzzy input data exactly matches to a fuzzy condition, fuzzy output data should be mapped to a corresponding fuzzy conclusion.

Alternatively, we propose a parameterized approach in which every fuzzy concept is described by a parameterized standard function, including fuzzy conditions and fuzzy conclusions. A fuzzy IF-THEN-ELSE rule takes the parameterized fuzzy concept as an input, and produces a standard function with new parameters as an output. New parameters are determined by a parameterwise interpolation. That is, each output parameters are determined by interpolating parameters of the same class contained in fuzzy conclusions. Obviously, the proposed scheme always satisfies the existing condition property.

1. Introduction

In our daily life, we often make such an inference as the following form:

Rule: If a man is tall then his weight is heavy.

Fact: This man is very tall.

Consequence: This man's weight is very heavy.

A special feature of the inference is that there are fuzzy concepts such as 'tall', 'very tall', 'heavy'

* : Korea Advanced Institute of Science and Technology
** : Electronics and Telecommunications Research Institute
+ : Electronics and Telecommunications Research Institute(Current Address)

and 'very heavy'. Such an inference pattern is called Generalized Modus Ponens. Devising Fuzzy Set[13] and Fuzzy Logic[16, 17], Dr. Zadeh formalized Generalized Modus ponens as a composition rule of inference. And some researchers[12, 18] who are interested in expert systems have studied on incorporating this inference rule into an uncertainty management of expert systems.

A special feature of the Generalized Modus Ponens is to make use of only one rule. One rule, however, is too insufficient for expert systems to reach reasonable conclusions for various facts. If the Fact in the above example is replaced by 'This man is very short', then what conclusion can a system produce? To produce a reasonable conclusion, the system should have more detailed information about the relationship between domains of height and weight.

An IF-THEN-ELSE rule is a sort of rule which contains more information about the relationship of two different domains. Particularly if an IF-THEN-ELSE rule contains fuzzy concepts, then this rule is called a fuzzy IF-THEN-ELSE (in abbreviation, 'FITE') rule. The following example shows an inference pattern which makes use of the FITE rule :

Example 1.

FITE Rule:

IF a man is tall THEN his weight is heavy

ELSE IF a man is short THEN his weight is light

ELSE IF a man is about 170cm THEN his weight is about 65kg

ELSE IF a man is about 180cm THEN his weight is about 75kg

Fact: This man is about 175cm.

 Consequence: This man's weight is about 70kg.

where 'tall', 'short', 'about 170cm' and 'about 180cm' are fuzzy concepts.

Such an inference pattern is called FITE inference. If a system can do FITE inference, then this system will produce more reasonable conclusions.

By the way, there is a necessary property that every system performing the FITE inference should satisfy. This property is called existing condition property (in abbreviation, 'ECP'), i.e., when the fact exactly match to one of the antecedent proposition, the conclusion should be mapped to the corresponding consequent proposition. This property is based on our strong intuition. As an example, if the fact is 'this man is about 170cm', then the conclusion must be 'this man's weight is about 65kg', in the Example 1.

Several researchers[2, 5, 6, 10] proposed some methods for the FITE inference. A common feature of those method is that vectors are used to define fuzzy concept and matrices are used to define fuzzy mapping between fuzzy antecedence and consequence. This approach, however, has some shortcomings: difficulty to define a fuzzy concept, low precision, and more importantly the violation of the existing condition property.

Alternatively, we propose a parameterized approach in that every fuzzy concept is described by parameterized standard function, including fuzzy conditions and fuzzy conclusions. A fuzzy IF-THEN-ELSE rule takes parameterized fuzzy concept as input, and produces a standard function with new

parameters as output. The new parameters are determined by parameterwise interpolation. That is, each output parameters are determined by interpolating the parameters of the same kind contained in fuzzy conclusions. Obviously, the proposed scheme always satisfies the ECP.

The remaining part of this paper is organized as follows. In Chapter 2, we define more rigidly the FITE rules, the FITE inferences and the ECP. In Chapter 3, there will be some surveys of the previous methods with analysis, and in Chapter 4, a standard membership grading function with four parameters is defined. In Chapter 5, a new FITE inference method based on interpolation is devised, and in Chapter 6, some examples will be shown to demonstrate that our method at least satisfies the ECP.

2. Fuzzy IF-THEN-ELSE(FITE) Inference

2.1 Rule-based Expert Sytem

In [1], Buchanan and Duda provided an excellent introduction to the principles of rule-base expert systems. As noted by them, the fundamental building blocks constructing the knowledge base of rule-based expert systems are propositional statements of the following form :

The <attribute> of <object> is <value>

For an example,

The height of John is 180 cm.

The weight of Kim is 60 kg.

One can combine the <attribute> and <object> into a variable. That is, in the above example, the height of John and the weight of Kim can be considered variables. Using this notation, fundamental building blocks of rule-based expert systems would be

$$V \text{ is } A$$

where V is a variable, and A is its current value.

In the systems based on a symbolic pattern matching such as MYCIN[11] and PROSPECTOR[4], values of variables are left as symbols. That is, in the fact

The height of John is tall,

there in no attempt to give any meaning to the value, tall. A matching to determine a fireability of rules is carried out at a level of a symbolic structure.

Leaving the value in this form, however, provokes some problems.

- a. When two people, a knowledge engineer and a human expert, use a same word, they might have a different meaning in mind.
- b. When a people want to express his knowledge, there might be no proper word.
- c. When a matching to determine a fireability of rules is attempted, there might be a closeness

in the meaning, even though no exact matching is occurred in symbolic structures.

These problems lead us to provide meanings for values associated with variables to construct an inference engine making inferences on the basis of not symbolic structures but meanings.

2.2 Meaning Representation by Possibility Distribution

Introducing fuzzy subsets[13], fuzzy logic[16, 17] and theory of approximate reasoning[15], Dr. Zadeh attempted to represent the meaning of a propositional statement. His attempt is based on a theory of possibility distribution[14]. Assume that X is a set of objects. A fuzzy subset A of X is a subset in which the membership grade for each $x \in X$ is an element in the unit interval $[0, 1]$. Let's denote this membership grading function $\mu_A(x)$, Then a propositional statement such as

The height of John is tall

has the effect of associating the variable, height of John, with a possibility distribution[14].

Assume we have the proposition

V is A

where A is a value. We can express A as a fuzzy subset of a base set X , that is, the set of all values the variable V can assume. For example, if A is tall, then we can express A as a fuzzy subset of a base set $X=[130, 250]$, that is, an interval of height. Then the proposition in turn induces a possibility distribution, π_V over the set X such that

$$\pi_V(x) = \mu_A(x)$$

where $\mu_A(x)$ is the membership grade of x in A . $\pi_V(x) = \mu_A(x)$ is interpreted as the possibility that $V=x$ given the fact, V is A .

In a rule-based expert system, the fundamental components of the rules are conditional statements of the following form:

if V_1 is A , then V_2 is B

Propositions of this type also induce possibility distributions. If the sets X and Y are base sets of V_1 and V_2 respectively, then the above conditional statement induces a conditional possibility distribution $\pi_{V_2|V_1}$ over $X \times Y$ such that

$$\pi_{V_2|V_1}(x, y) = \min(1, 1 - \mu_A(x) + \mu_B(y))$$

Thus the meanings of facts and rules can be represented by possibility distributions.

2.3 Definition of the FITE Inference

Definition. The FITE inference is of the following form:

FITE rule:

```

IF  $\pi_U(x) = \mu_{A_0}(x)$  THEN  $\pi_V(y) = \mu_{B_0}(y)$ 
ELSE IF  $\pi_U(x) = \mu_{A_1}(x)$  THEN  $\pi_V(y) = \mu_{B_1}(y)$ 
    ELSE IF.....
        ELSE IF  $\pi_U(x) = \mu_{A_n}(x)$  THEN  $\pi_V(y) = \mu_{B_n}(y)$ 
Fact:  $\pi_U(x) = \mu_{A'}(x)$ .
-----
Concn.:  $\pi_V(y) = \mu_{B'}(y)$ .

```

where U and V are variables, and A_i 's, A' , B_i 's, and B' are fuzzy subsets of the base sets X , X , Y and Y respectively, and $x \in X$, $y \in Y$.

That is, the FITE inference is to produce the new possibility distribution, $\pi_V(y)$, as a conclusion, depending on the possibility distributions in the FITE rule.

2.4 Existing Condition Property

Definiton. The Existing Condition Property(ECP) is that

$$\text{If } \mu_{A'}(X) = \mu_{A_i}(x) \text{ then } \mu_{B'}(y) = \mu_{B_i}(y), 0 \leq i \leq n.$$

That is, when the input statement, $U=A'$, is exactly with one of the statements, say $U=A_i$, in IF part of a FITE rule, the conclusion, $V=B'$, should be equal to the corresponding statement, $V=B_i$, of THEN part of the rule. In the Example 1, if the Fact is 'this man is about 170cm', then the Conc. must be 'this man is about 65kg'.

3. Conventional Methods

A common feature of conventional methods[2, 10] is that they used vectors to represent membership grading functions of fuzzy subsets. So, possibility distributions are also represented by vectors, and conditional possibility distributions are represented by matrices.

Cayrol, et al.[2] proposed a FITE inference method. They firstly construct an overall conditional possibility distribution, $\pi_{V|U}(x, y)$, from the fuzzy IF-THEN rules in FITE rule, applying the following formula:

$$\pi_{V|U}(x, y) = \max_{i=0, n} (\min(1, 1 - \mu_{A_i}(x) + \mu_{B_i}(y)))$$

where U and V are variables, and A_i 's, and B_i 's, are fuzzy subsets of the base sets X and Y respectively, and $x \in X$, $y \in Y$. And $\pi_{V|U}(x, y)$ is a conditional possibility distribution over $X \times Y$.

Then they apply max - min operator to produce the possibility distribution of the conclusion, $\mu_{B'}(y)$, as follows :

$$\pi_V(y) = \mu_{B'}(y) = \max_{x \in X} \min[\mu_{A'}(x), \mu_{V|U}(x, y)]$$

where U and V are variables, and A' and B' are fuzzy subsets of the base sets X and Y respectively, and $x \in X$, $y \in Y$. And $\mu_{V|U}(x, y)$ is a conditional possibility distribution over $X \times Y$.

The max-min operator, which Dr. Zadeh proposed to perform the Generalized Modus Ponens[15], is a sort of matrix operator where maximum and minimum operations are performed instead of addition and multiplication respectively. Masaki, et al.[10] also used this formula to design an inference architecture and to implement it on a chip with VLSI technology.

This formula, however, has a serious problem: this formula does not always satisfy the ECP. A counter example is revealed in [7].

4. FITE Inference using Standard Function

We have discussed a serious problem of conventional methods in that they can not always satisfy the ECP. The main reason is assumed in this paper that the membership grading function of a fuzzy subset is represented by a vector.

An alternative approach satisfying the ECP is developed in this paper. Firstly, we devise a Standard Membership Grading Function(in abbreviation, 'SMGF') with four parameters to define every fuzzy concepts in a FITE rule, facts, and conclusions. Then we will propose a FITE inference method using the S function and show several examples to reveal its features including the ECP.

4.1 SMGF with four parameters

Assume that a base set, X , is a set of continuous real numbers. Then X can be described with lower bound and upper bound.

Definition. L is the lower bound of a base set, and U is the upper bound of a base set.

Definition. Core Region, C , is subset of a base set $X(=[L, U])$ where membership grades of a fuzzy subset, A , are exactly one. That is,

$$C = \{x \mid \mu_A(x) = 1, x \in [L, U]\}$$

Definiton. Support Region, P , is the subset of a base set $X(=[L, U])$ where membership grades of a fuzzy subset, A , are greater than zero. That is,

$$P = \{x \mid \mu_A(x) > 0, x \in [L, U]\}$$

Definition. A SMGF with four parameters, S function, devised in this paper is defined as follows:

$$S(x; \alpha, \beta, \gamma, \delta) = \begin{cases} 1 & \text{if } \beta \leq x \leq \gamma, \\ 2 \left[\frac{x-\alpha}{\beta-\alpha} \right]^2 & \text{if } \alpha < x < \frac{\alpha+\beta}{2}, \alpha \neq \beta, \\ 1-2 \left[\frac{x-\beta}{\beta-\alpha} \right]^2 & \text{if } \frac{\alpha+\beta}{2} \leq x < \beta, \alpha \neq \beta, \\ 1-2 \left[\frac{x-\gamma}{\delta-\gamma} \right]^2 & \text{if } \gamma < x < \frac{\gamma+\delta}{2}, \gamma \neq \delta, \\ 2 \left[\frac{x-\delta}{\delta-\gamma} \right]^2 & \text{if } \frac{\gamma+\delta}{2} \leq x < \delta, \gamma \neq \delta, \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where x, α, β, γ and $\delta \in [L, U]$.

Definiton. The parameter α is the lower bound of Support Region.

Definition. The parameter β is the lower bound of Core Region.

Definition. The parameter γ is the upper bound of Core Region.

Definition. The parameter δ is the upper bound of Support Region.

Definition. Core Region, C , is redescribed as a subset of $[L, U]$ where

$$S(x; \alpha, \beta, \gamma, \delta) = 1.$$

This region is actually the subset $[\beta, \gamma]$ of $[L, U]$, according to the equation(1).

Definiton. Support Region, P , is redescribed as a subset of $[L, U]$ where

$$S(x; \alpha, \beta, \gamma, \delta) > 0.$$

This region is actually the subset $[\alpha, \delta]$, according to the equation (1).

Adjusting the four parameters, we can describe various shapes of membership grading functions with S function as shown in Fig. 1.

But the S function can not generate complex shapes which have more two core and support regions as shown in Fig. 2.

If we devise a SMGF having more parameters than S function defined above, then we can adjust more complex shapes. The main subject of this paper, however, is not to determine how many parameters are most appropriate, but to propose a FITE inference method satisfying the ECP with a SMGF. Therefore we remain this problem as a further research topic.

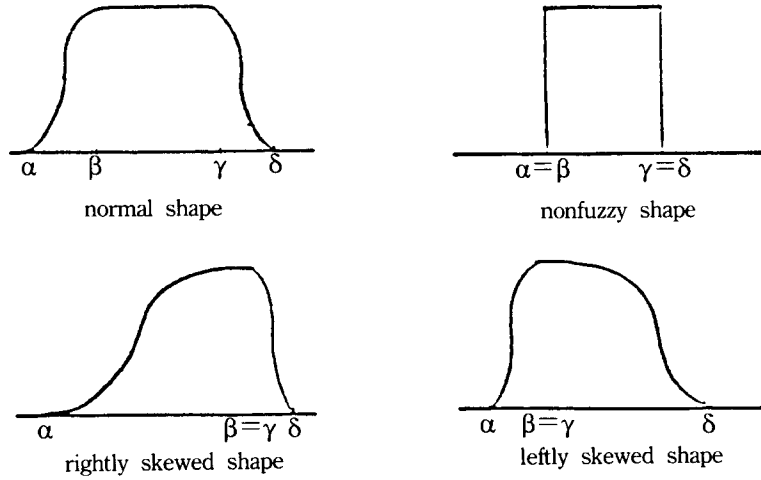


Fig. 1. Examples shapes that S function typically generates

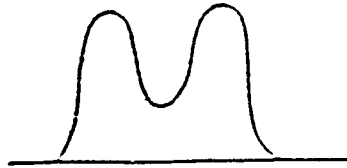


Fig. 2. a complex shape

4.2 FITE Inference and Existing Condition Property Revisited

If we use the S function, the possibility distribution of a statement, V is A , can be represented simply as the following form:

$$\pi_V(x) = \langle a, b, c, d \rangle$$

where A is a fuzzy subset of base set X , and is defined as $S(x; a, b, c, d)$. Accordingly, FITE inference is interpreted as follows:

FITE rule:

IF $\pi_U(x) = \langle a_0, b_0, c_0, d_0 \rangle$ THEN $\pi_V(y) = \langle a_0', b_0', c_0', d_0' \rangle$,
 ELSE IF $\pi_U(x) = \langle a_1, b_1, c_1, d_1 \rangle$ THEN $\pi_V(y) = \langle a_1', b_1', c_1', d_1' \rangle$
 ELSE
 ELSE $\pi_U(x) = \langle a_n, b_n, c_n, d_n \rangle$ THEN $\pi_V(y) = \langle a_n', b_n', c_n', d_n' \rangle$.

Fact: $\pi_U(x) = \langle a, b, c, d \rangle$.

Conc.: $\pi_V(y) = \langle a', b', c', d' \rangle$

In the same manner, the ECP is redescribed that

if $a=a_i$ and $b=b_i$ and $c=c_i$ and $d=d_i$
then $a'=a'_i$ and $b'=b'_i$ and $c'=c'_i$ and $d'=d'_i$.

5. FITE Inference Based on Parameterwise Interpolation

5.1 FITE Inference as an Extension of Interval Mapping

Assume that we have two linear functions, $y=f_l(x)$ and $y=f_u(x)$, which describe lower bound and upper bound of y , given a point x . Given these two functions, in order to calculate an interval $[y_1, y_2]$ corresponding to an interval $[x_1, x_2]$, we shall calculate, firstly, each interval

$$[y_{l1}, y_{u1}] = [f_l(x_1), f_u(x_1)] \text{ and}$$

$$[y_{l2}, y_{u2}] = [f_l(x_2), f_u(x_2)],$$

and then adjust a final interval

$$[y_1, y_2] = [\min(y_{l1}, y_{l2}), \max(y_{u1}, y_{u2})].$$

This procedure is mathematically correct and fits our intuitions. We call this procedure an interval mapping.

By the way, the interval $[y_l, y_u]$ is a special type of fuzzy subset of which membership grading function is

$$\pi_{[y_l, y_u]}(y) = \begin{cases} 1 & y \in [y_l, y_u] \\ 0 & \text{otherwise,} \end{cases}$$

Thus we may describe any intervals as fuzzy subsets defined by S function. In this case, however, four functions, $f_a(x)$, $f_b(x)$, $f_c(x)$ and $f_d(x)$, are needed for each parameters. These four functions are the parameter mapping functions that can be acquired from FITE rule by a parameterwise interpolation.

Thus we interpret that FITE inference is a fuzzy mapping extended from the interval mapping. That is, when an input is given with four parametric values, a, b, c and d , the support region $[a, d]$ of the input is mapped into a support region $[a', d']$ of the output through the mapping function $f_a(x)$ and $f_d(x)$, and the core region $[b, c]$ is mapped into a core region $[b', c']$ through the mapping function $f_b(x)$ and $f_c(x)$.

5.2 Inference Procedure

The FITE inference procedure based on the idea of fuzzy mapping is divided into three steps. Fig. 3. shows this procedure overally.

First Step: The mapping functions for each parameter, that is, $f_a(x)$, $f_b(x)$, $f_c(x)$ and $f_d(x)$, are acquired from a FITE rule with parameterwise interpolation. For example, a mapping function, $f_a(x)$, for the a parameter is acquired from the pairs (a_i, a'_i) by interpolation (we assume that a FITE rule parameterwisely satisfies the functionality, i.e. one-to-one or many-to-one mapping).

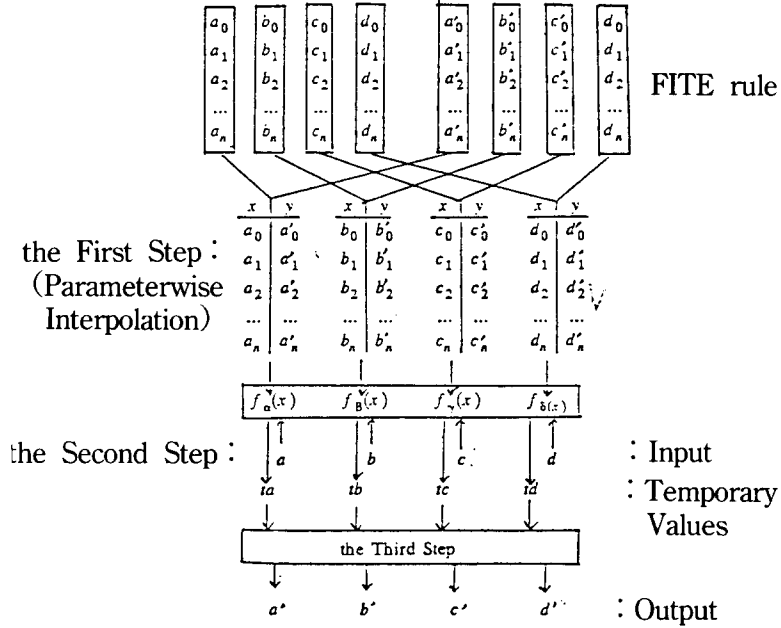


Fig. 3. FITE Inference Procedure

Second Step: When input parameters, for example, a, b, c and d for each parameters, are given, temporary values, ta, tb, tc and td , are calculated by applying the acquired parameter mapping functions as follows:

$$\begin{aligned}
 ta &= \min\{amin, dmin\} \\
 tb &= \min\{bmin, cmin\} \\
 tc &= \max\{bmax, cmax\} \\
 td &= \max\{amax, dmax\}
 \end{aligned}$$

Where the $amin$ and $dmin$ are minimum values of $f_a(x)$ and $f_d(x)$ in $[a, d]$, and $amax$ and $dmax$ are maximum values $f_a(x)$ and $f_d(x)$ in $[a, d]$, respectively. The $bmin$ and $cmin$ are minimum values of $f_b(x)$ and $f_c(x)$ in $[b, c]$, and $bmax$ and $cmax$ maximum values of $f_b(x)$ and $f_c(x)$ in $[b, c]$, respectively.

Third Step: Finally, parametric values, a', b', c' and d' , are adjusted as follows:

if $[ta, td] \cap [tb, tc] = \emptyset$
 then No_Possibility_Distribution
 else $a' \leftarrow ta$,
 $b' \leftarrow$ lower bound of $[ta, td] \cap [tb, tc]$,
 $c' \leftarrow$ upper bound of $[ta, td] \cap [tb, tc]$,
 $d' \leftarrow td$.

The No_Possibility_Distribution can be strictly expressed as subnormal possibility distribution[15]. For more detailed explanation, [7] can be referred.

A main idea of interpolation is that if a parameter, say α , of a fact is exactly match to any one of a_i 's, say a_1 , in a FITE rule, then the parameter α of conclusion became a'_1 . Otherwise, the parameter α of conclusion is approximated on behalf of the pairs, (a_i, a'_i) ' s.

This idea led us to use the parameterwise interpolation to meet the existing condition property. Obviously the existing condition property is satisfied.

5.3 Interpolation Methods

To get parameter mapping functions, say $f(x)$, interpolation[3] is performed. A main idea of using interpolation is that if the value of a parameter, say α , of the Fact is exactly match to any one of a_i 's, say a_1 , in the FITE rule, then the value of the parameter α of the Conc. is to be a'_1 .

Otherwise, we approximate on behalf of the pairs, (a_i, a'_i) ' s. This idea led us to use the parameterwise interpolation to meet the existing condition property. Obviously the property is satisfied.

There is one assumption under the main idea. This assumption is that a FITE rule is consistent. What we mean by the consistency is the satisfaction of functionalities, that is, one-to-one or many-to-one mapping, for each parameter in a FITE rule. We define the consistency of a FITE rule in more details as follows :

Definition. Consistency of a FITE rule

$$\begin{aligned} & \text{if } a_i = a_j, \text{ then } a'_i = a'_j, \\ & \text{and if } b_i = b_j, \text{ then } b'_i = b'_j, \\ & \text{and if } c_i = c_j, \text{ then } c'_i = c'_j, \\ & \text{and if } d_i = a_j, \text{ then } d'_i = d'_j, \\ & \text{where } i \neq j, \text{ and } 0 \leq i, j \leq n. \end{aligned}$$

Of course, this assumption is for the acquisition of mapping functions.

There are many interpolation methods, and any of them can be used to acquire parameter mapping functions. In this section, piecewise linear interpolation, Newton method and Lagrange method are compared with each others in the view of time complexity and accuracy. Detailed explanations of those methods can be referred in [2, 7]

The piecewise linear interpolation is very time efficient. Excluding the time needed to search proper interval, only 2 multiplications/divisions and 4 additions/subtractions. This is a reason why the piecewise linear interpolation is popular. But this method is the worst in the accuracy of approximation. Newton method is less time efficient than the piecewise linear interpolation. Excluding the time to obtain the coefficients, it takes at least n multiplications/divisions and $2n$ additions/subtractions. Lagrange method is the worst in time efficiency. It takes at least $2(n+1)$ multiplications/divisions and $3n+2$ additions/subtractions. Both of Newton and Lagrange method are alike in the accuracy, but much more precise than the piecewise linear interpolation.

Although those interpolation methods differ from each other in time efficiency and accuracy of

approximation, all of them at least satisfy the ECP. So, we may use any of those interpolation methods, according to situations. If a time efficient method is necessary, then piecewise linear interpolation may be proper. On the other hand, if we need to use precise method, then we may choose either the Newton method or Lagrange method.

5. Examples

Some examples using our method appear in this section. Piecewise linear interpolation is used for the simplicity and time efficiency of calculation. Although we use piecewise linear interpolation whose accuracy is poor, it is enough to demonstrate that our method satisfies the ECP. Parameters of fuzzy subsets are elicited on the basis of our subjective intuitions.

Let's consider the Example 1 again. If the parameters of the fuzzy subsets are defined as follows:

$$\begin{aligned} tall &= \langle 175, 185, 250, 250 \rangle, \quad heavy = \langle 68, 80, 200, 200 \rangle, \\ short &= \langle 130, 130, 155, 165 \rangle, \quad light = \langle 30, 30, 50, 60 \rangle, \\ about\ 170cm &= \langle 168, 170, 170, 172 \rangle, \quad about\ 180cm = \langle 178, 180, 180, 182 \rangle, \\ about\ 65kg &= \langle 64, 65, 65, 66 \rangle, \quad about\ 75kg = \langle 74, 75, 75, 76 \rangle, \\ where\ HEIGHT &= [130, 250], \quad WEIGHT = [30, 200], \end{aligned}$$

then the FITE rule is expressed as:

$$\begin{aligned} & \text{IF } \pi_U(x) = \langle 175, 185, 250, 250 \rangle \text{ THEN } \pi_V(y) = \langle 68, 80, 200, 200 \rangle \\ & \text{ELSE IF } \pi_U(x) = \langle 130, 130, 155, 165 \rangle \text{ THEN } \pi_V(y) = \langle 30, 30, 50, 60 \rangle \\ & \text{ELSE IF } \pi_U(x) = \langle 168, 170, 170, 172 \rangle \text{ THEN } \pi_V(y) = \langle 64, 65, 65, 66 \rangle \\ & \text{ELSE IF } \pi_U(x) = \langle 178, 180, 180, 182 \rangle \text{ THEN } \pi_V(y) = \langle 74, 75, 75, 76 \rangle \end{aligned}$$

From the above FITE rule, four tables are acquired for each parameters. Fig. 4. shows these tables.

table for α		table for β		table for ρ		table for δ	
x	y	x	y	x	y	x	y
130	30	130	30	155	50	165	60
168	64	170	65	170	65	172	66
175	68	180	75	180	75	182	76
178	74	185	80	250	200	250	200

Fig. 4. Tables for each parameter

These tables are used to acquire parameter mapping functions by interpolation.

Example 2. Let's assume the fuzzy concept, tall, is defined as

$$tall = \langle 130, 185, 250, 250 \rangle,$$

although it does not meet our commonsense. Then functionality is not satisfied in the table for parameter α . This is an example of inconsistency of FITE rule. Fig. 5. shows this inconsistency.

table for α	table for β	table for ρ	table for δ																																
x y	x y	x y	x y																																
<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">130</td> <td style="padding: 2px 5px;">30</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">130</td> <td style="padding: 2px 5px;">68</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">168</td> <td style="padding: 2px 5px;">64</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">178</td> <td style="padding: 2px 5px;">74</td> </tr> </table>	130	30	130	68	168	64	178	74	<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">130</td> <td style="padding: 2px 5px;">30</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">170</td> <td style="padding: 2px 5px;">65</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">180</td> <td style="padding: 2px 5px;">75</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">185</td> <td style="padding: 2px 5px;">80</td> </tr> </table>	130	30	170	65	180	75	185	80	<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">155</td> <td style="padding: 2px 5px;">50</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">170</td> <td style="padding: 2px 5px;">65</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">180</td> <td style="padding: 2px 5px;">75</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">250</td> <td style="padding: 2px 5px;">200</td> </tr> </table>	155	50	170	65	180	75	250	200	<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">165</td> <td style="padding: 2px 5px;">60</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">172</td> <td style="padding: 2px 5px;">66</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">182</td> <td style="padding: 2px 5px;">76</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 5px;">250</td> <td style="padding: 2px 5px;">200</td> </tr> </table>	165	60	172	66	182	76	250	200
130	30																																		
130	68																																		
168	64																																		
178	74																																		
130	30																																		
170	65																																		
180	75																																		
185	80																																		
155	50																																		
170	65																																		
180	75																																		
250	200																																		
165	60																																		
172	66																																		
182	76																																		
250	200																																		

Fig. 5. An example of an inconsistent FITE rule

It may seem too strict to satisfy the condition of consistency. But, based on our intuition, this condition should be satisfied, and the satisfaction of this condition does not seem to be over demanding to knowledge engineer.

Example 3. Let's consider again the example 1. The fact was

$$\pi_U(x) = \langle 173, 175, 175, 177 \rangle .$$

According our inference procedure, the first and second steps give temporary parametric values by applying piecewise linear interpolation.

$$f_\alpha(173) = 64 + \frac{68-64}{175-168} (173-168) = 66.86$$

$$f_\beta(175) = 65 + \frac{75-65}{180-170} (175-170) = 70$$

$$f_\gamma(175) = 65 + \frac{75-65}{180-170} (175-170) = 70$$

$$f_\delta(180) = 66 + \frac{76-66}{182-172} (180-172) = 74$$

$$ta = 66.86, tb = 70, tc = 70, td = 74.$$

From those temporary values, the third step generates a possibility distribution:

$$\pi_V(y) = \langle 66.86, 70, 70, 74 \rangle .$$

Example 4. IF the input is

$$\pi_U(x) = \langle 168, 170, 170, 172 \rangle .$$

which exactly match to one of possibility distributions, that is, a man is about 170cm, then the first and second step produce temporary values:

$$ta = 64, tb = 65, tc = 65, td = 66.$$

And the third step generates a possibility distribution

$$\pi_V(y) = \langle 64, 65, 65, 66 \rangle .$$

This is a possibility distribution induced from the statement, a man is about 65kg, in THEN part. This example shows that our method satisfy the existing condition property. In fact, our method always satisfies the property by the idea of using interpolation.

Example 5. In the example 4, the possibility of the point 72kg, $\pi_V(72)$, is obtained by applying the equation(1) as follows:

$$\pi_V(72) = S(72 ; 66.86, 70, 70, 74) = 0.5.$$

Our method gives possibility values not only for this point but also for any other points since we have S function filled with parameters after FITE inference has done.

7. Conclusions

A standard membership grading function(SMGF) with four parameters was devised, and this function was used to define membership grading functions of fuzzy concepts. And a fuzzy IF-THEN-ELSE(FITE) inference method based on parameterwise interpolation was also proposed. As a result, we showed that our method always satisfies the existing condition property which was not satisfied in conventional vectors and matrices approach. This property is surely satisfied in every sort of relationships between two domains.

As was seen through examples, however, it seems that our method is well applied to FITE rules which reveal nearly positive linear or negative linear relationships, when fuzzy input does not match to any of fuzzy conditions in IF-part. That is, our method does not guarantee the reasonable conclusions in every sort of relationships. To analyze how much reasonable our method is, we should find more properties of the FITE inference in addition to the existing condition property, and evaluate our method according to those properties. After this evaluation, it may be necessary to improve our method. However, we leave such concrete analysis as further research. In this paper, we evaluated our method, and compared with conventional methods only in a view of the existing condition property.

To be a more useful inference method, on the other hand, our method should be able to manage the linguistic hedges such as 'very', 'more or less', 'not' and so on. We might have employed more parameters. Our main consideration in this paper, however, was to devise a FITE inference method using SMGF and satisfying the existing condition property. Thus we leave these problems as next research topics.

References

1. B. G. Buchanan and R. O. Duda, "Principles of Rule-based Expert Systems," Lab. for Artificial Intelligence Research, Fairchild Camera, Palo Alto, CA, Fairchild Tech. Rep. no. 625, 1982.
2. M. Cayrol, H. Farrey and H. Prade, "Fuzzy Reasoning based on Multivalent Logics in the Framework of Production Rule System," *Int. Symp. Multi-valued Logics, IEEE*, 1980, pp.143-148.
3. S. D. Conte and Carl de Boor, *Element Numerical Analysis: an algorithmic approach*, McGraw-Hill, 1982, pp. 31-71.
4. J. Gaschnig, "PROSPECTOR: An Expert System for Mineral Exploration," *In Machine Intelligence*, Infotech State of Art Report, Series 9, no. 3, 1981.
5. E. Hisdal, "A fuzzy IF THEN ELSE relation with guaranteed correct inference," in R. R. Yager, Ed., *Recent Developments in Fuzzy Set and Possibility Theory*. Oxford and Newyork: Pergamon Press.
6. E. Hisdal, "The IF THEN ELSE statement and interval-valued fuzzy sets of higher type," *Int. J. Man-Machine Studies*, vol. 15, 1981, pp. 385-455.
7. D. H. Kim, "A Fuzzy IF-THEN-ELSE inference method based on Interpolation," *M.S. Thesis*, Dept. of Computer Science, KAIST, 1987.
8. J. Lebailly, R. Martin-clouaire and H. Prade, "Use of Fuzzy Logic in a Rule-based System in Petroleum Geology," To appear in *Artificial Intelligence: Applications of Qualitative Reasoning*, E. Sanchez, L.A. Zadeh, eds., Oxford and Newyork: Pergamon Press.
9. R. Martin-clouaire, "Efficient Deduction in Fuzzy Logic," *Int. Conf. on Information and Management of Uncertainty in Knowledge-based Systems*, Paris, June 30-July 4, 1986.
10. Masaki Togai and Hiroyuki Watanabe, "Expert System on a Chip: An Engine for Real-Time Approximate Reasoning," *IEEE Expert*, Fall, 1987, pp. 55-62.
11. E. H. Shortliffe, *Computer-Based Medical Consultation:MYCIN*, Elsevier, New York, 1976.
12. R. R. Yager, Apporximate Reasoning as a Basis for Rule-Based Expert Systems, *IEEE Transactions on Systems, Man, and Cybernetics*, vol. SMC-14, no. 4, July/August 1984.
13. L. A. Zadeh, "Fuzzy Sets," *Information and Control*, 8, 1965, pp. 338-353.
14. L. A. Zadeh, "Fuzzy Sets as a Basis for a Theory of Possibility," *Fuzzy Sets and Systems*, vol. 1, 1978, pp. 3-28.
15. L. A. Zadeh, "A Theory of Apporximate Reasoning" in *Machine Intelligence 9*, Hayes, Michie and Mikulich eds., New York: J. Wiley, 1979, pp. 149-194.
16. L. A. Zadeh, "Approximate Reasoning based on Fuzzy Logic," *IJCAI*, 1979, pp. 1004-1010.
17. L. A. Zadeh, "Inference in Fuzzy Logic," *Int. Symp. Multi-valued Logics, IEEE*, 1980, pp. 124-131.
18. L. A. Zadeh, "The Role of Fuzzy Logic in the Management of Uncertainty in Expert Systems," *Fuzzy sets and Systems*, vol. 11, 1983, pp. 199-227