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On the Vibration Analysis of the Floating Elastic Body Using the Boundary Integral Method in combination with Finite Element Method

by

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Abstract

In this research the coupling problem between the elastic structure and the fluid, specially the hydroelastic harmonic vibration problem, is studied. In order to couple the domains, i.e., the structural domain and the fluid domain, the boundary integral method (direct boundary integral formulation) is used in the fluid domain in combination with the finite element method for the structure. The boundary integral method has been widely developed to apply it to the hydroelastic vibration problem. The hybrid boundary integral method using eigenfunctions on the radiation boundaries and the boundary integral method using the series form image-functions to replace the even bottom and free surface boundaries in case of high frequencies have been developed and tested. According to the boundary conditions and the frequency ranges the different boundary integral methods with the different idealizations of the fluid boundaries have been studied.

Using the same interpolation functions for the pressure distribution and the displacement the two domains have been coupled and using Hamilton principle the solution of the hydroelastic system have been obtained through the direct minimizing process.

It has become evident that the finite-boundary element method combining with the eigenfunction method or the image-function method give good results in comparison with the experimental ones and the other numerical results by the finite element method.

1. Introduction

All the systems in ship and offshore engineering contain certain amount of interactions, i.e., the coupling between structure and fluid. In many cases of various practical purposes it can be assumed that one system does not influence the other simultaneously. Typical examples of these behaviors are the hydrodynamic forces on the massive offshore platform or the motion of pressure vessel.

On the other hand the systems composed of structure and fluid, for instance, structures such as thin steel offshore structure or large steel pressure vessel with relative large openings may have to be considered simultaneously. Systems such as these are said to be coupled at any time, i.e., the behavior of one influences the other and vice versa[20, 21].

The vibration problem of the floating elastic body is the coupling one between the elastic structure and the fluid. In general the ship sturctural vibration has been analysed by use of the strip theory and the beam theory, in which the hydroelastic coupling effect is neglected by using the hydrodynamic added mass of

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the two-dimensional rigid section using conformal mapping technique or the sink-source distribution method. It is only one possible way in this two dimensional method to consider the three dimensional correction factor (J-factor) for the three dimensional flow around the vibrating body. Because of the theoretical limitation this method cannot cover the three dimensional problem, i.e., the hydroelastic coupled vibration one and more over that with the special fluid boundaries[4, 9, 15, 16, 18].

The first numerical analysis of the hydroelastic coupling problem is the work of Zienkiewicz by using the finite element method [20], in which the two formulations, Lagrangian or Eulerian formulation, have been proposed. Although this method could give a new way to handle the hydroelastic coupling problem, there remain many problems, for example, setting the radiation boundary to cut the infinite fluid domains and needing too many degrees of freedom in order to idealize the fluid domains with the finite elements.

The boundary integral methods based on the direct weighted residual formulations (BIM) have been developed recently by the research group in UK [5] and they became very popular in the engineering because of the economic calculating possibilities and the exactness of the solution for the domain problems[1].

The sink-source distribution method (SDM) is one of the boundary integral method, where the Green function, which satisfies the boundary conditions on the free surface, bottom and radiation boundaries, is used. Therefore it is a boundary value problem of Neumann type with only one kinematic body boundary condition. It is also called indirect boundary integral method, because the first unknown variables are the sink-source densities, which have no physical meaning. Therefore it is impossible to deal with the boundary value problem of mixed type, for example, arbitrary solid boundaries and it contains also a lot of numerical difficulties to calculate the fluid behavior in high frequencies[7, 8, 10, 14].

The direct boundary integral method (BIM) has been developed as a general tool to solve the boundary value problem of mixed type. Yeung and Mei have applied this method to the hydrodynamic motion analysis of the floating structure [14] and Yeung has developed the hybrid method using the eigenfunctions on the radiation boundaries [19]. But in case of hydroelastic vibrating structure, i.e., if the vibration of the structure makes the relatively small radiating waves, it is a natural question, how many degrees of freedom we need to idealize the boundaries of the problem.

The first work to apply the indirect sink-source distribution method to the hydroelastic vibration problem is that of P. Kallef [11]. He has applied the simple Green function 1/r for the boundary value problem of Neumann type, i.e., he has neglected all other boundary conditions except the kinematic one on the wetted body boundary.

In this paper the general BIM and the hybrid BIM (HBIM) have been tested in the high frequency ranges. As Neumann has soon explained, the mirror effect can be successfully applied in the ideal cases of $\omega\to 0$ and $\omega\to\infty$. This image-function method can be used to analyse the vibration problem in combination with the general BIM or the HBIM.

The hydroelastic vibration is also a function of eigen mode shapes of the structure as well as the frequencies and the other boundary conditions. In order to couple the two domains the continuity and the compatibility between the two domains are required. Therefore the same elements and the same shape functions are used for the fluid and the structure. The hydrodynamic velocity potential functions in fluid is defined using the same unknown displacement vectors, which are used in the structural finite elements.

All the energies have been integrated into a Hamilton functional with the unknown displacement vectors and using the Hamilton principle the solution can be found at the stationary state of this functional.

2. A brief review of the theory on the boundary value problem

2.1. The boundary value problem

It is assumed that the fluid is ideal, the oscillation amplitudes are small with respect to the dimensions

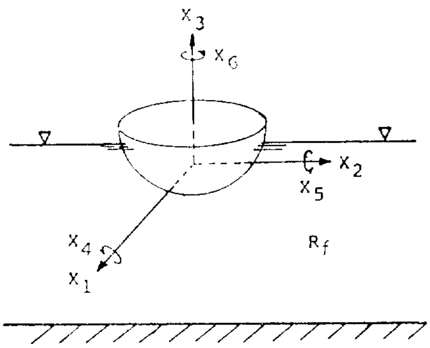


Fig. 1 Coordinate System

of the body, only, the first order effects are considered and the waves are harmonic and linear. (Airy type)

The six possible degrees of freedom are defined as three translations and three rotations at each point on the wetted structural surface. They are indicated by:

displacements: $\bar{U}(x,t)=\mathrm{Re}[U(x)\exp(i\omega t)]$

velocities : $\bar{V}(x,t)=\mathrm{Re}[-i\omega U(x)\exp(-i\omega t)]$

accelerations : $\bar{a}(x,t)=\mathrm{Re}[-\omega^2 U(x)\exp(-i\omega t)]$ (2.1)

where x is the coordinate vector and ω is the frequency.

The solution of the flow problem can be described by using the velocity potential in the fluid domain:

$$\bar{\phi}(x,t)=\mathrm{Re}[\phi(x)\exp(-i\omega t)] \quad (2.2)$$

$$\bar{\phi}(x,t)=\bar{\phi}_w+\bar{\phi}_s+\bar{\phi}_a=\bar{\phi}_w+\bar{\phi}_s+\sum_{j=1}^{6}\bar{\phi}_{aj}$$

where

$\bar{\phi}_w$=elementary wave potential (Airy type)

$\bar{\phi}_s$=scattered potential

and

$\bar{\phi}_{aj}$=radiation potential ($j=1,\cdots,6$)

And the following Laplace equation must be fulfilled:

$$\nabla^2\phi=0 \text{ in the fluid domain Rf} \quad (2.3)$$

The boundaries are composed of wetted vibrating

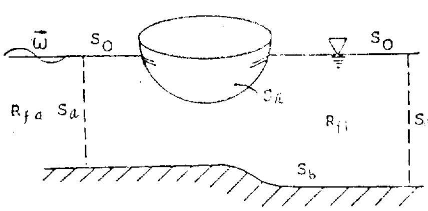


Fig. 2 Definition of domains and boundaries

body surface(S_k), free surface(S_o), rigid surface(S_b) and control surface at infinity.

In order to have a finite length of the total boundary the control surface at infinity should be replaced by radiation surface(S_a) at finite distance from the body.

$R_f=R_{fi}+R_{fa}$ in the fluid domain

$S=S_k+S_b+S_o+S_a$ on the boundaries

After linearization the boundary conditions can be described as follows:

Three homogeneous boundary conditions:

1) bottom condition:

$\partial\phi/\partial n=0$ on S_b (2.4.1)

2) free surface condition:

$\partial\phi/\partial z-\omega^2\phi/g=0$ on S_o (2.4.2)

3) radiation condition (approximated Sommerfeld type):

$\partial\phi/\partial n-ik\phi=0$ on S_a (2.4.3)

where k is the wave number, which is the solution of the following dispersion relation:

$k\tanh(kH)=\omega^2/g$ (H=water depth, g=gravitational acceleration)

One non-homogeneous boundary condition:

4) kinematic condition on the wetted body surface:

$\partial\phi(x)/\partial n=V(x)\cdot n(x)$ on S_k (2.4.4)

where

$V(x)$=velocity vector at a point x

$n(x)$=unit normal vector at a point x into the fluid domain

Through this non-homogeneous kinematic boundary condition the two systems can be coupled and the mode dependency of the hydroelastic system can be obtained.

This is the well known boundary value problem of mixed type to fulfill the Laplace equation and the four boundary conditions.

2.2. Direct boundary integral formulation (BIM)

The direct boundary integral formulation is based on the simple Green function ϕ^*; $\ln(1/r)/2\pi$ or $1/r/4\pi$ for two or three dimensional potential problems. With the weighted residual integral statements this boundary value problem of mixed type can be integrated into one integral equation. The numerical errors in the domain and on the boundaries, which come from the approximation with the assumed function ϕ, have been weighted with the simple Green function ϕ^*, which is a fundamental solution of the Laplace equation. These have been integrated into one integral equation.

$$\int_{R_f} \nabla^2\phi\phi^* dR = \int_{S_k} (\partial\phi/\partial n - Vn)\phi^* ds + \int_{S_a} (\partial\phi/\partial n - ik\phi)\phi^* ds + \int_{S_b} (\partial\phi/\partial n)\phi^* ds + \int_{S_o} (\partial\phi/\partial n - \omega^2\phi/g)\phi^* ds \quad (2.5)$$

After performing the partial integration twice the following inverse integral equation is obtained.

$$C(x)\phi(x) + \int_{S_k+S_b} \phi(\xi)(\partial\phi^*/\partial n) ds + \int_{S_a} (\partial\phi^*/\partial n - ik\phi^*)\phi(\xi) ds + \int_{S_o} (\partial\phi^*/\partial n - \omega^2\phi^*/g)\phi(\xi) ds = \int_{S_k} \phi^* Vnds \quad (2.6)$$

where constant $C(x)$ is proportional to the solid angle at the singular boundary point "x" and ξ is the integral independent variable on the boundaries.

Comparing with the traditional sink-source distribution method the boundary conditions of mixed type, for example, arbitrary bottom boundary, canal wall etc., can be integrated into one integral statement. In addition the singular integrations in the domain and on the boundaries, $C(x)$ in Eqn. (2.6), can be accomplished using direct numerical integrations and therefore it is not a constant 1/2 like in SDM. Moreover the important difference from SDM is the integration of the velocity vectors on the body boundaries after weighting it with ϕ^*.

This direct formulation of BIM can be also obtained using Green's theorem, which has the following form:

$$\int_{R_f} (u\nabla^2 v - v\nabla^2 u) dR = \int_S (u\partial v/\partial n - v\partial u/\partial n) ds \quad (2.7)$$

If u and v are two harmonic functions in the fluid domain and we take $u=\phi$ and $v=\phi^*$, this equation becomes:

$$-Ci\phi i + \int_{\bar{S}_e} (\phi\partial\phi^*/\partial n - \phi^*\phi\partial/\partial n) ds = 0 \quad (2.8)$$

where

$\bar{S}_e = S - S_e$

S_e=boundary segment at a point "x" with the radius "e"

$C_i\phi_i$=the constant and the potential at the i-th singular point "x".

Inserting the boundary conditions in Eqn. (2.4) into Eqn. (2.8) the same integral formulation as Eqn. (2.6) will be obtained.

2.4. Hybrid boundary integral method (HBIM)

The potentials on the radiation boundary can be approximated as a superposition of undisturbed potentials $\phi_e(k, x)$ in the outer region R_{fa} using Fourier integral statement:

$$\phi(x) = \int_0^\infty A(k)\phi_e(k, x) dk \quad (2.9)$$

where $A(k)$ is the unknown modulation coefficients and $\phi_e(k, x)$ is the eigenfunctions on the radiation boundaries. The variables k and x are the wave number and the coordinate vector.

Wehausen and Laitone showed that $\phi_e(k, x)$ is a set of eigenfunctions, which satisfy the Laplace equation and the boundary conditions on the free surface as well as on the even bottom and represents the outgoing waves at infinity ($y=\pm\infty$) [17].

The eigenvalues k_o and ik_i are the roots of the transcendental equation:

$$k_o \tanh(k_o H) = \omega^2/g \quad \text{and}$$
$$k_i \tan(k_i H) = -\omega^2/g \ (i=1, 2, 3, \cdots) \quad (2.10)$$

Nothing that the eigenvalues are the discrete ones from Eqn. (2.10) and nothing the damping factors in the eigenfunctions, the integral in Eqn. (2.9) can be converted to a discrete summation with finite terms /24/. Substituting the boundary conditions on S_k, S_o and S_b into Green's integral equation (2.8) we obtain the following equation:

$$C(x)\phi(x) + \int_{S_k+S_b} \phi(\xi)(\partial\phi^*/\partial n) ds + \int_{S_o} (\partial\phi^*/\partial n - \omega^2\phi^*/g)\phi(\xi) ds + \int_{S_a} (\phi_e\partial\phi^*/\partial n - \phi^*\partial\phi_e/\partial n) ds = \int_{S_k} \phi^* Vnds \quad (2.11)$$

If we substitute the eigenfunctions into Eqn. (2.11), i.e., if we match the potential and the normal velocity of the inner regions with those of the outer regions on S_a, we obtain the integral formulation with the unknown coefficients on the radiation boundaries.

2.5. The image-function method

By choosing a fundamental solution ϕ^*, which

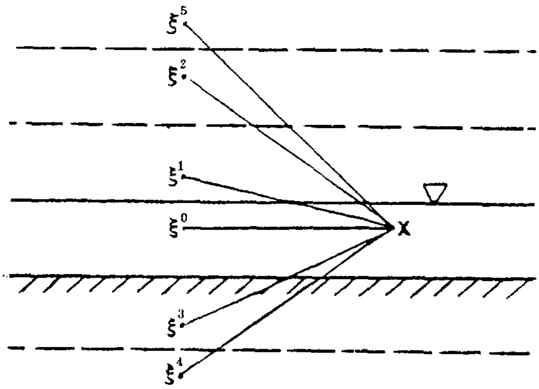


Fig. 3 Images of a source ξ^0

satisfies a certain boundary condition, we have not to discretize this surface.

For instance the horizontal bottom can be disregarded using the following image function, which fulfills the boundary condition on that.

$$\partial\bar{\phi}^*(\xi;x)/\partial n=0 \quad (2.12)$$
$$\bar{\phi}^*(\xi;x)=\phi^*(\xi;x)+\phi^*(\xi^3;x)$$

where ξ=the source point on the boundary, ξ^3=the reflected singular point on the bottom and x=the observing point on the boundary.

For the two extreme cases ($\omega\to 0$ and $\omega\to\infty$) the free surface condition (2.4.2) becomes:

$$\partial\bar{\phi}^*/on=0 \quad \text{on } S_o \text{ for } \omega\to 0 \quad (2.13)$$
$$\bar{\phi}^*=0 \quad \text{on } S_o \text{ for } \omega\to\infty$$

For the motions at infinite frequency and at the zero frequency the Green function ϕ^*, which satisfies the boundary conditions, can be generated by a series function, which represents the summation of the images with respect to the bottom and the free surface.

$$\bar{\phi}^*(\xi;x)=\phi^*(\xi;x)+\phi_1^*(\xi^1;x)+\sum_{n=1}^{N}(\phi_{2n}^*+\phi_{3n}^*+\phi_{4n}^*+\phi_{5n}^*) \quad \text{for } \omega\to 0$$
$$\bar{\phi}^*(\xi;x)=\phi^*(\xi;x)-\phi_1^*(\xi^1;x)+\sum_{n=1}^{N}(-1)^n(\phi_{2n}^*-\phi_{3n}^*+\phi_{4n}^*-\phi_{5n}^*) \text{ for } \omega\to\infty \quad (2.14)$$

where

$$\phi_{kn}^*=\phi_{kn}^*(\xi^k;x)$$

These Green function $\bar{\phi}_*$ yield good convergence with increasing N and these are therefore very efficient for the numerical evaluation of the hydroelastic vibration problem (See Fig. 4.6).

2.6. Conversion to algebraic equations

In order to couple the two domains we approximate the unknown variables with the same shape functions, as in FEM, through which the compatibility and the continuity between the two domains can be fulfilled. As in FEM we use the isoparametric shape functions [3], i.e., the same shape functions for the potential and the differential of that.

$$\phi(x)=N_i(\underset{\sim}{x})\phi_i \quad \text{and}$$
$$\partial\phi(x)/\partial n=N_i(\underset{\sim}{x})(\partial\phi/\partial n)^i(i=1,\cdots, NOP_e) \quad (2.12)$$

where

NOP_e=the number of nodes in one element

$\underset{\sim}{x}$=the generalized coordinate

$N_i(\underset{\sim}{x})$=the isoparametric shape function.

In this paper the isoparametric one-dimensional element of one-to-three variable nodes and the isoparametric two-dimensional element of three-to-eight variable nodes are used [3]. In the general boundary integral method (BIM) there are the following two typical integrals, which can be converted into algebraic equations:

$$\int_S\phi^*(\xi;x)\phi(x)ds(x)=\sum_{i=1}^{NEL}\int_{S_e^i}\phi^*(\xi;x)N_i(\underset{\sim}{x})ds(\underset{\sim}{x})\phi_i=G_{ji}\phi_i$$
$$\int_S(\partial\phi^*(\xi;x)/\partial n)\phi(x)ds(x)=\sum_{i=1}^{NEL}\int_{S_e^i}(\partial\phi^*(\xi;x)/\partial n)N_i(\underset{\sim}{x})ds(\underset{\sim}{x})\phi_i=H_{ji}\phi_i \quad (2.16)$$

In the hybrid boundary integral method (HBIM) the unknown potentials on S_a would be described by the eigenfunction expansion:

$$\phi(x)=\sum_{i=0}^{N}d_i\phi_e^i(x) \text{ and } \partial\phi(x)/\partial n=\sum_{i=0}^{N}d_i(\partial\phi_e(x)/\partial n)^i \quad (2.17)$$

where N=the number of eigenfunctions on the radiation boundary,

$\phi_e^i(x)$=the i-th eigenfunction and

d_i=the coefficient according to $\phi_e^i(x)$

Using this definition the integrations on the radiation boundaries can be redefined as follows:

$$\int_{S_a}\phi^*(\xi;x)(\partial\phi_e(x)/\partial n)ds(x)$$
$$=\sum_{i=1}^{N}\int_{S_a}d_i(\partial\phi_e/\partial n)^i\phi^*(\xi;x)ds(x)$$
$$=d_iQ_i(\xi;S_a) \qquad (2.18)$$
$$\int_{S_a}(\partial\phi^*(\xi;x)/\partial n)\phi_e(x)ds(x)$$
$$=\sum_{i=1}^{N}\int_{S_a}d_i\phi_e^i(x)(\partial\phi(\xi;x)/\partial n)ds(x)=d_iP_i(\xi;S_a)$$

With this definition the boundary integral formulation can be written as the algebraic equation:

$$(H-\omega^2G^0/g-ikG^a)\phi=G^kF^k \text{ in BIM}$$
$$[H^k|H^b|H^0-\omega^2G^0/g|P_k-Q_k]\begin{Bmatrix}\phi_k\\ \phi_b\\ \phi_0\\ d_k\end{Bmatrix}$$
$$=G^kF^k \qquad \text{in HBIM} \qquad (2.19)$$

or

$$AX=B \qquad (2.20)$$

where A is the system matrix, X is the unknown vector, which involves unknown potentials or unknown coefficients, B is the right side vector and the indices k, b, o, a indicate the corresponding boundaries, S_k, S_b, S_o and S_a.

If we use the image-function method, the corresponding boundary integrals can be eliminated.

2.7. The solution of the algebraic equation

The above alegbraic equation (2.20) can be devided into two parts, real and imaginary parts:

$$\begin{bmatrix}A_r & A_i\\ -A_i & A_r\end{bmatrix}\begin{Bmatrix}\phi_r\\ \phi_i\end{Bmatrix}=\begin{bmatrix}G^kn_k & 0\\ 0 & G^kn_k\end{bmatrix}\begin{Bmatrix}V_r\\ V_i\end{Bmatrix}^k \qquad (2.21)$$

where

$A_r=R_e(A)$, $A_i=I_m(A)$, and
$V_r=R_e(V)$, $V_i=I_m(V)$.

Using this equation we can obtain:

$$\phi_r=A_m^kV_r^k-A_d^kV_i^k$$
$$\phi_i=A_d^kV_r^k+A_m^kV_i^k \qquad (2.22)$$

where

$$A_m^k=Ar^{*-1}G^kn_k,\ A_d^k=Ar^{*-1}A_iAr^{-1}G^kn_k$$
$$Ar^*=A_r+A_iA_r^{-1}A_i \qquad (2.23)$$

In order to solve the hydroelastic vibration problem we can think of a harmonic vibrating body (radiation problem). Without restriction of the general theory we can define the real velocity vector, i.e., $V_i=0$ for $t=0$. With this assumption we can write:

$$\phi_r=A_mV_r \text{ and } \phi_i=A_dV_r \qquad (2.24)$$

With this potential distributions we can obtain the pressure distribution using Bernoulli equation.

$$P(x)=\rho A_ma_r+\rho\omega A_dV_r \qquad (2.25)$$

where

ρ=the fluid density.
a_r=the real part of the acceleration vector

3. The analysis of the hydroelastic system

To analyse the interaction between two systems, fluid and structure, two system variables, potential and displacement, are used to define each system (Eulerian definition). The coupling of the two systems can be accomplished by satisfying the continuity and the compatibility conditions between two ones. The fluid motion and the structural motion on the wetted body boundary can be defined as dependent on each other through the kinematic body boundary condition. Therefore the pressure can be described with the unknown displacement. With these two system parameters the corresponding energies can be easily formulated and integrated into a Hamilton functional. Throught the Hamilton principle the stationary state of the total energy can be easily obtained by direct minimizing process.

The Hamilton's functional is defined:

$$A=\int_{t_1}^{t_2}(T_k-T_p)dt \qquad (3.1)$$

where $T_k=T_s+T_f+W$=the kinetic energy of the structure(T_s)+that of the fluid(T_f)+the work due to external forces(W) and T_p=the potential energy of the structure.

We can find the solution of the coupling problem in the following condition:

$$\delta A=0 \qquad (3.2)$$

3.1. The formulation of the energy functionals

3.1.1. The kinetic energy of the structure

$$T_s=1/2\int_{R_s}\rho_k(V_s)^2dR$$
$$=1/2\int_{R_s}\omega^2\rho_kU_iU_jdR\ \exp\ (-i2\omega t) \qquad (3.3)$$

where

ρ_k=the density of the structural component

R_s=the volume of the structure.

V_s=the velocity vector of the structural component

Using the same interpolation function as in the fluid domain we obtain:

$$T_s=1/2\omega^2\sum_{m=1}^{NEL}U_i\int_{R_m}\rho_k N_i N_j\ dR\ U_j\ \exp(-i2\omega t)$$
$$=1/2\omega^2\sum_m U_i M_m U_j\ \exp(-i2\omega t) \qquad (3.4)$$

or

$$T_s=1/2U^T MU\ \exp(-i2\omega t)$$

where

NEL=the number of the structural elements,

R_m=the m-th element volume,

$M_m=\int_{Rm}\rho_k N_i N_j dR$=the mass matrix of the m-th structural element (3.5)

$M=\Sigma M_m$=the mass matrix of the structure,

$U=\Sigma U_m$=the displacement vector of the structure

3.1.2. The potential energy of the structure

The potential energy of the structure is conserved as the strain energy. It can be quoted from the formulation of the general finite element method:

$$T_P=1/2\int_{Rs}\sigma^T\varepsilon dR=1/2\int_{Rs}\varepsilon^T c\varepsilon dR \qquad (3.6)$$

where σ=the stress tensor, ε=the strain tensor and C=the elasticity tensor.

In the linear elastic system the strain tensor can be defined as follow:

$$\varepsilon_m=B_m U \qquad (3.7)$$

where B_m=the strain-displacement operator tensor.

Using this notation we can write:

$$T_P=1/2(\sum_m U_i\int_{R_m}B_m^T CB_m dRU_j)\exp(-i2\omega t)$$
$$=1/2(\sum_m U_i K_m U_j)\exp(-i2\omega t)$$
$$=1/2U^T KU\ \exp(-i2\omega t) \qquad (3.8)$$

where

$K_m=\int_{R_m}B_m^T CB_m dR$=element stiffness matrix

$K=\sum_m K_m$=system stiffness matrix.

3.1.3. The kinetic energy of the fluid

$$T_f=1/2\int_{S_k}\bar{U}(-\bar{P})nds$$
$$=1/2\int_{S_k}U(-P)nds\ \exp(-i2\omega t) \qquad (3.9)$$

where P=the hydrodynamic pressure as a functions of the displacement vector, n=the unit normal vector on the wetted structural surface into the fluid.

In the isoparametric interpolation we use the same shape function to approximate the pressure distribution as for the structural displacement:

$$P=N_i P_i=N_i(\rho_f A_m a+\omega\rho_f A_d V) \qquad (3.10)$$

where A_m and A_d are the given matrices in equation (2.23).

Using this definition we obtain:

$$T_f=-1/2\sum_m U_i\int_{S_m}N_i N_j nds$$
$$(\rho_f A_m a_j+\omega\rho_f A_d V_j)\exp(-i2\omega t)$$
$$=-1/2\sum_m U_i(-\omega^2\int_{S_m}\rho_f N_i N_j nds A_m$$
$$-i\omega_{S_m}\rho_f N_i N_j nds A_d)U_j\exp(-i2\omega t)$$
$$=1/2\sum_m U_i(\omega^2 M_h^m+i\omega D_h^m)U_j\exp(-i2\omega t)$$
$$=1/2(\omega^2 U^T M_h U+i\omega U^T D_h U)\exp(-i2\omega t) \qquad (3.11)$$

where the hydrodynamic added mass matrix M_h and the hydrodynamic damping matrix D_h are defined as follows:

$$M_h=\int\rho_f N_i N_j nds A_m$$
$$D_h=\int\rho_f N_i N_j nds A_d \qquad (3.11.1)$$

3.1.4. The work due to the external forces

The work due to the external forces can be written as follows;

$$W=\int_S\bar{U}\bar{f}ds=\sum_m U_i\int_{S_m}N_j f_j ds\ \exp(-i2\omega t)$$
$$=U^T f\ \exp(-i2\omega t) \qquad (3.12)$$

where

f=the surface force vector on the structure.

S_m=the surface element of the structure.

3.2. The Solution of the hydroelastic problem

Combining all the energies into Hamilton functional we obtain:

$$A=1/2\int_{t_1}^{t_2}(\omega^2 U^T MU+\omega^2 U^T M_h U+i\omega U^T D_h U$$
$$+U^T F-U^T KU)\exp(-i2\omega t)dt \qquad (3.13)$$

Because we are concerning about the harmonic problem, we can take one period as an integral interval. We can determine the stationary state of the functional by differentiating it with respect to the independent variable U.

$$\delta A=-\delta U^T(\omega^2 MU+\omega^2 M_h U+i\omega D_h U$$
$$+F-KU)=0 \qquad (3.14)$$

If follows the well known characteristic equation of the hydroelastic vibrating system.

$$(K-i\omega D_h-\omega^2(M+M_h))U=F \quad (3.15)$$

Concerning only about the eigenvalue problem we can take the following equation.

$$|K-\omega^2(M+M_h)|=0 \quad (3.16)$$

3.3. Some aspects in treating the hydroelastic coupling problem

1) The general classification of the frequency ranges

In treating the boundary value problem in Chap. 2 the two remarkable points must be pointed.

a) The frequency dependency of the hydrodynamic added mass comes from the boundary condition on the free surface. We can write this boundary condition as follows;

$$g/\omega^2(\partial\phi/\partial n)-\phi=0.$$

In BIM the integral of $\partial\phi/\partial n$ on S_o is not dependent on the element size but only on the solid angle at that singular point. Therefore it is evident that the frequency dependency of this boundary condition dies proportional to the square of the frequency. The higher the exciting frequency becomes, the more stable the hydrodynamic added mass becomes. (See Fig. 3.1)

b) After we used the image functions on the free surface and the bottom in region II and III, the frequency dependency of the hydrodynamic damping comes from the radiation boundary condition. If the wave number k becomes large, the real and imaginary parts of the system matrix become decoupled. Therefore the damping goes zero.

From these two facts we can devide the frequency

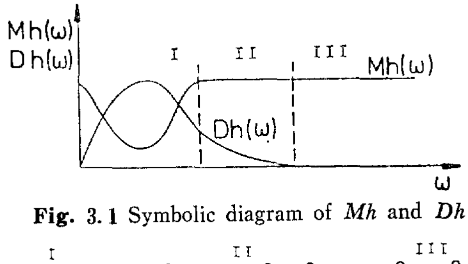


Fig. 3.1 Symbolic diagram of Mh and Dh

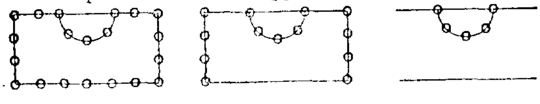


Fig. 3.2 Idealization of the boundaries

range into three parts (See Fig. 3.1). Corresponding to the frequency representing the hydrodynamic characteristics we can idealize the boundaries differently (See Fig. 3.2). Using small number of image functions on the free surface we don't have to ide alize this boundary in the frequency range II and III.

2) The symmetrization of the hydrodynamic matrix

Although we have used the isoparametric interpolation functions for the displacement and the pressure, the hydrodynamic matrices are asymmetric, because the matrix A_m and A_d in the Eqn. (2.23) is asymmetric. It comes from the fact, that two other types of functions, Green function ϕ^* for the weighting of the errors and the isoparametric approximation function for the potential ϕ, have been used.

In other words the potental function ϕ has been weighted with a singular function ϕ^* which is very strong concentrated at that singular point. Through this weighting the local distribution of the potential cannot be correctly described, and then the hydrodynamic matrices become asymmetric. But if we discretize the boundary very finely, the concentration divergies very rapidly. Therefore the asymmetry of the hydrodynamic matrix is not so strong and it can be symmetrized through the rational approximation.

In order to combine the BIM with the finite element method and in order to use the FEM codes to couple the two methods, in practical point of view, we must symmetrize the hydrodynamic matrices, because the solution procedure of the FEM can be applied only for the symmetric matrix.

The matrix can be symmetrized either by using the direct error minimizing process or by using energy conservation principle. The two methods give the same results /5/.

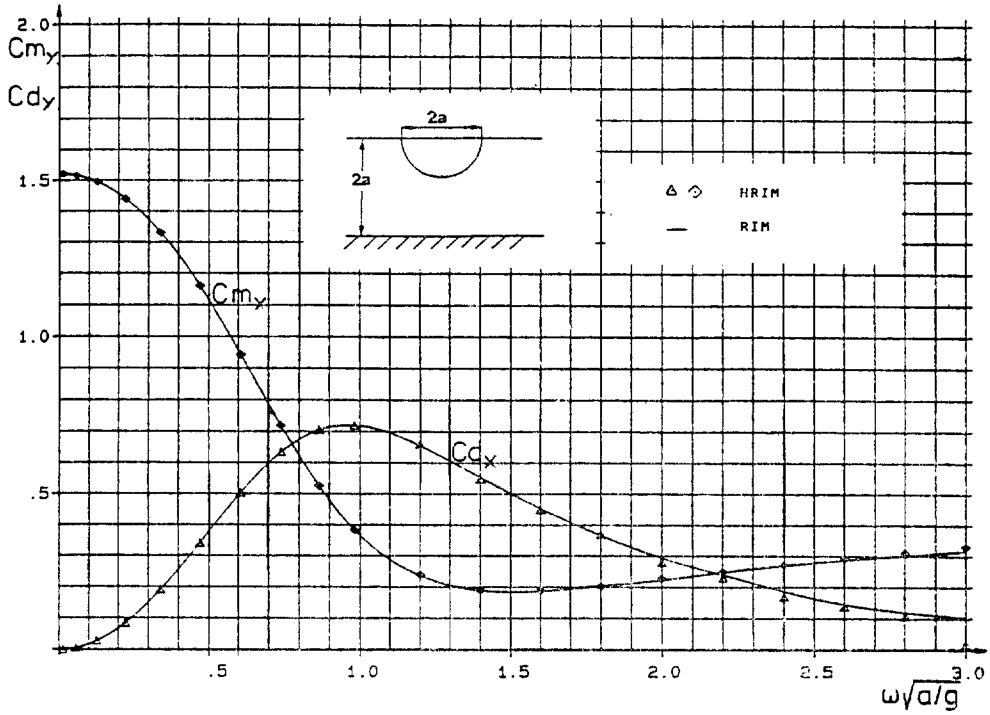
In this paper the direct error minimizing process has been used:

$$S=1/2((\bar{M}_{hij}-M_{hij})^2+(-M_{hji})^2) \quad (3.17)$$

where S is the square sum of the errors between the symmetric matrix $\bar{M}_{hij}$ and the asymmetric matrix M_{hij}. The symmetric matrix can be obtained to differentiate the square sum with respect to $\bar{M}_{hij}$.

$$\bar{M}_{hij}=1/2(M_{hij}+M_{hji}) \quad (3.18)$$

3) An iteration method to solve the eigenvalue



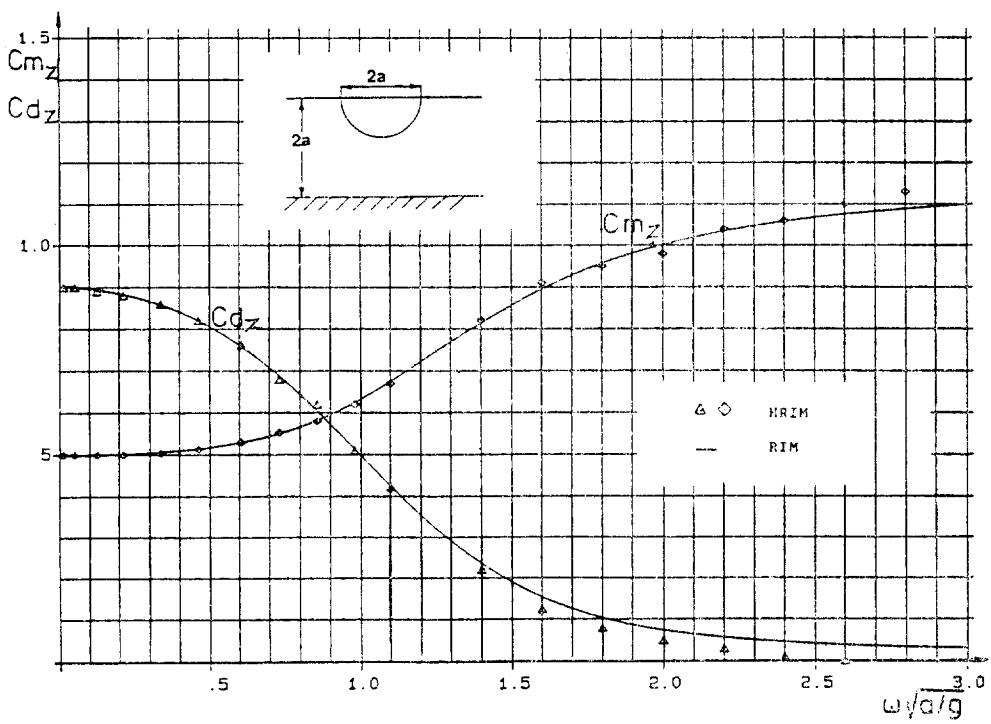
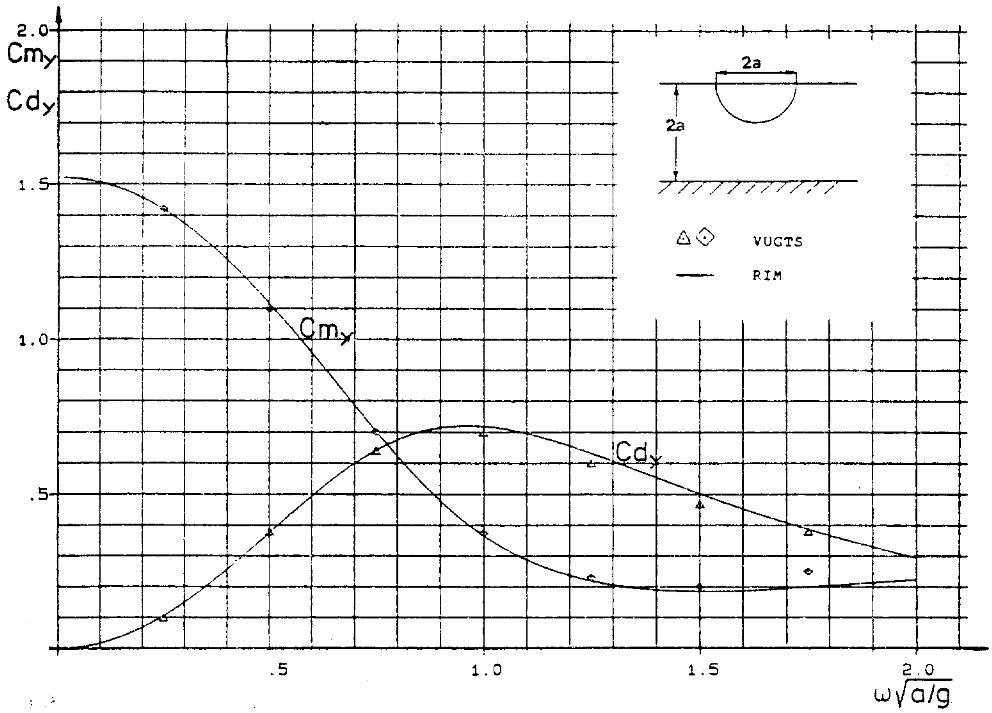


Fig. 4.1 C_m and C_d of a half circle calculated by BIM and HBIM in motion frequency range



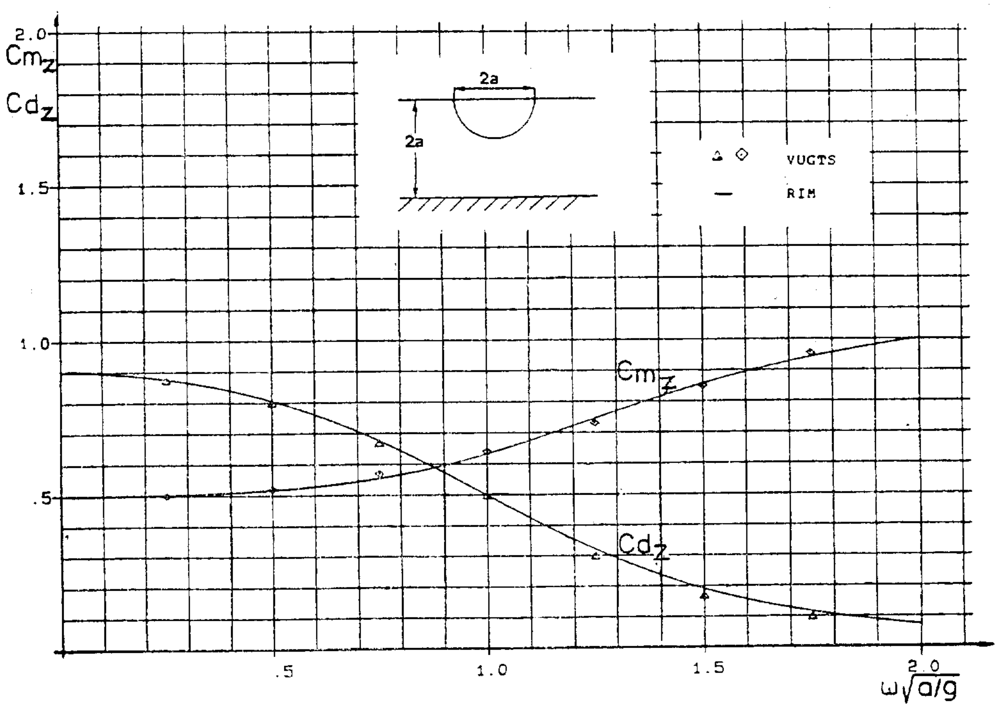


Fig. 4. 2 C_m and C_d of a half circle calculated by BIM and Vugts in motion frequency range

problem of the hydroelastic system

As far as we can not neglect the free surface boundary condition, the hydrodynamic matrix is implicitly dependent on the frequency. Therefore the characteristic equation becomes:

$$|K-\omega^2(M+M_h(\bar{\omega}))|=0 \tag{3.19}$$

Where ω is the eigen-frequency of the hydroelastic coupling system and $\bar{\omega}$ is the prescribed frequency to define the hydrodynamic system.

This is a nonlinear numerical eigenvalue problem. In this paper an iteraction method is proposed, where the frequency of the hydrodynamic system is prescribed as a given value. And it will be repeatedly iterated until the system eigenvalue becomes same as the prescribed one.

4. Calculated results

4.1. The hydrodynamic added mass and damping in wide frequency ranges

a) The hydrodynamic added mass and damping of a half submerged long cylinder section (H=2a) have

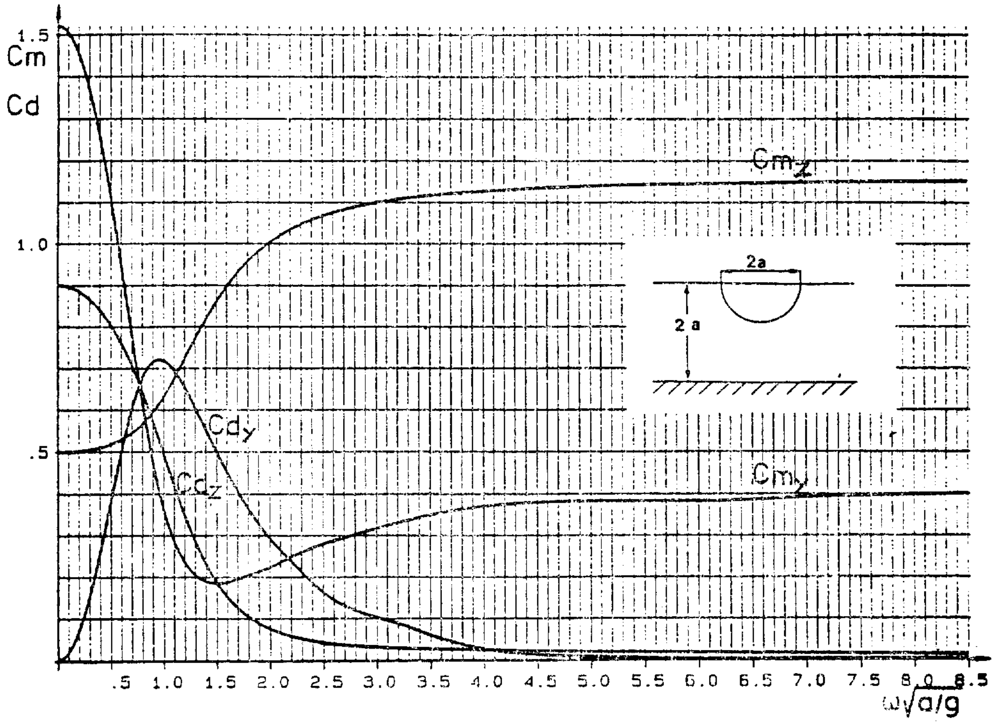


Fig. 4.3 C_m and C_d of a half circle by BIM up to the high frequency range

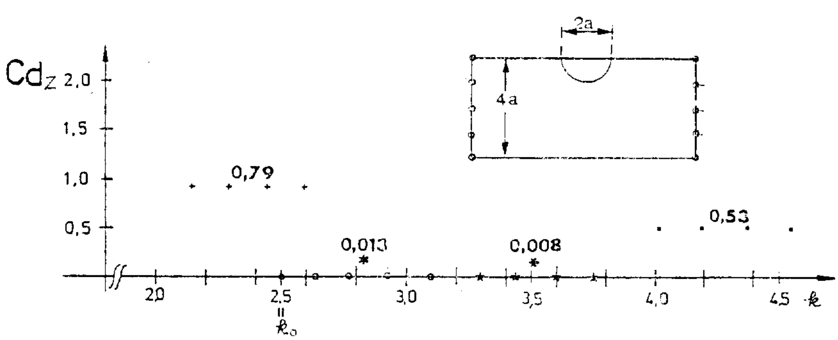


Fig. 4.4 The selection of the eigen-wave numbers to get the proper damping coefficient in high frequencies using HBIM

been calculated with the program RIM (based on BIM and HBIM). The results have been compared with those of singular-distribution method by HBIM in the motion frequency range (Fig. 4.1).

b) The results by BIM have been also compared with those by Vugts (Fig. 4.2).

c) These have been also calculated in high frequencies with the program RIM (Fig. 4.3). The same degrees of freedom as in the motion frequency range have been used.

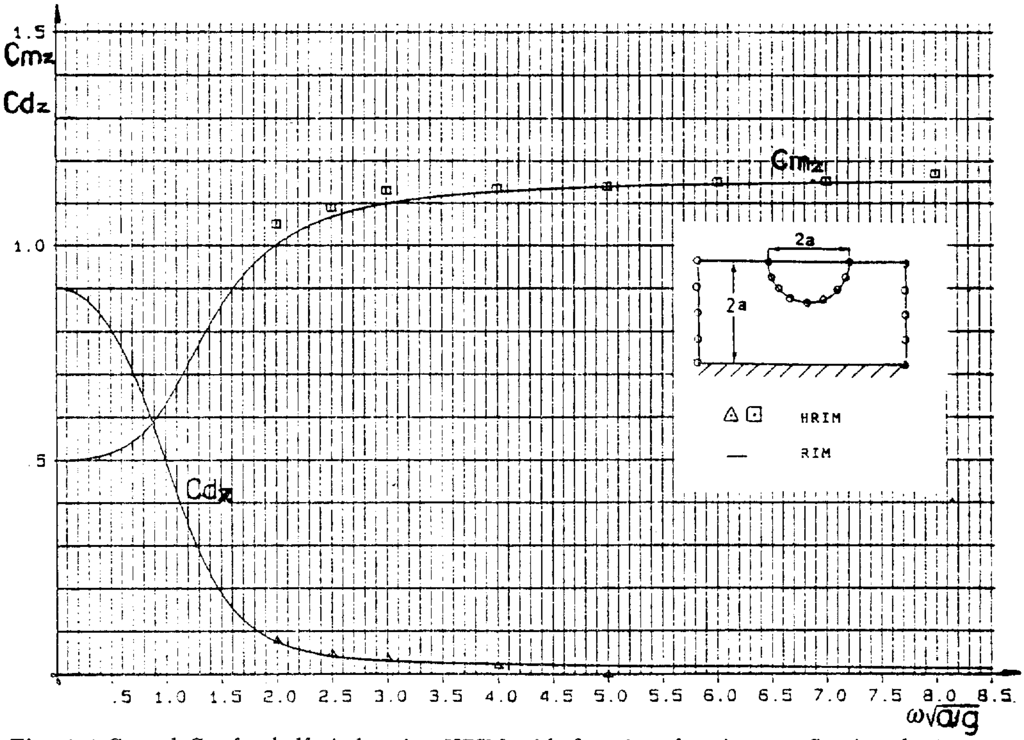


Fig. 4.5 C_m and C_d of a half circle using HBIM with five eigen-functions on S_a using the image functions on S_o and S_b

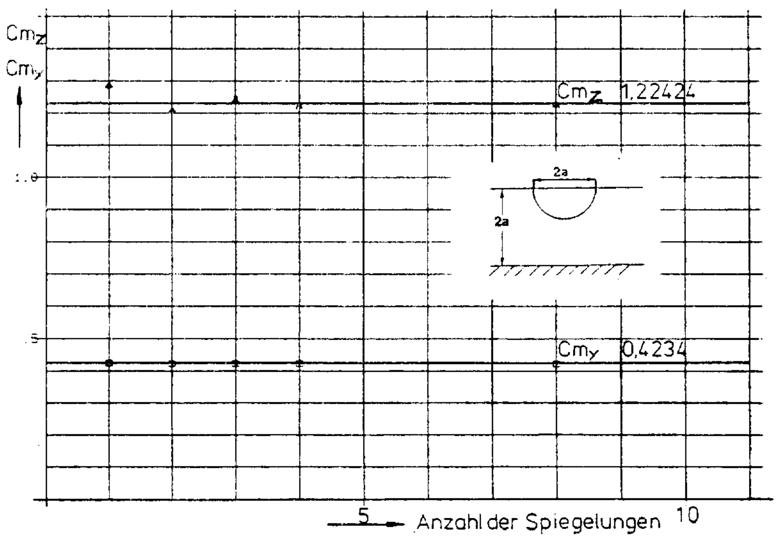


Fig. 4.6 Convergence test to select the number of image functions

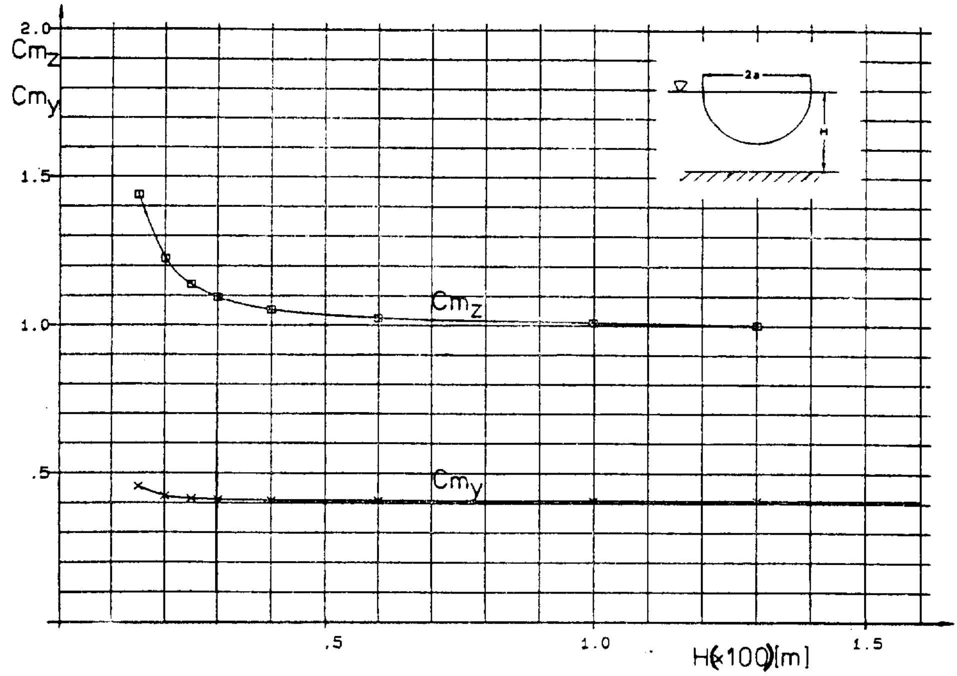


Fig. 4.7 C_{my} and C_{mz} of a half circle for various water depths in infinite frequency

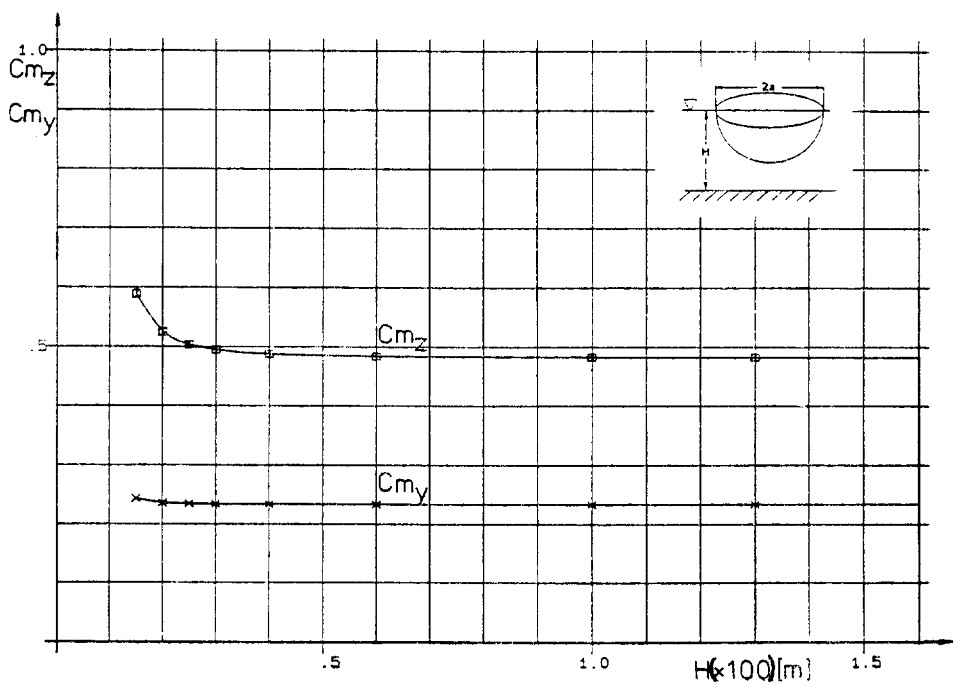


Fig. 4.8 C_{my} and C_{mz} of a half sphare for various water depths in infinite freequency

d) The calculation of the hydrodynamic damping in high frequency with four selected eigen wave-numbers have been demonstrated(Fig. 4. 4), and it becomes clear that the eigen wave-numbers for HBIM must be selected over the base wave-number k_o. The calculated results in high frequencies with HBIM have been shown in Fig. 4. 5, where only five eigenfunctions have been used to approximate the potential function on the radiation boundaries.

e) In using the image-function method for the bottom and the free surface in the infinite frequencies the convergence has been tested with increasing number of reflections (Fig. 4. 6). It is clear that the result converges very rapidly.

f) Two examples to calculate the hydrodynamic added mass using the image-function method have been shown in the Fig. 4. 7 for a half circle and in the Fig. 4. 8 for a half sphere, which are floating on the free surface.

In this paper the hydrodynamic added mass *Mh* and the hydrodynamic damping *Dh* are nondimensionalized as follows:

$$C_m = Mh/\rho\Delta, \quad C_d = Dh/\rho\Delta(A/g)^{0.5}$$

where Δ is the volume of the body, ρ is the fluid density, A is the characteristic half breadth($\equiv 10^m$) and C_m, C_d are the hydrodynamic added mass, and damping coefficients.

4. 3. The free vibration of a rectangular plate with one fixed edge under the free surface

The eigenfrequencies of a rectangular steel plate (20cm × 10cm × 0. 262cm), which is fixed along the shorter edge under the free surface (See sketch 4. 1), have been caculated with the program system NISA/RIM (based on the finite-boundary element method) and compared with those of the experimental ones by Lindholm /13/, who has given the results only by deep water (Table 4. 1). The results for the various water depth, which is calculated with NISA/RIM, have been tabulated in Table 4. 2. It is evident that

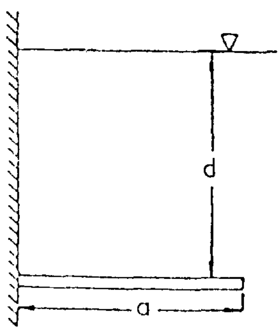
Table 4. 1 Comparison between the calculated results by NISA/RIM and the experimental ones by Lindholm ($d \to \infty$)

Mode	NISA/RIM			LINDHOLM		
	dry	wetted	Δ	dry	wetted	Δ
1R	14. 45	5. 633	0. 388	13. 8	—	—
M	—	—	—	12. 9	5. 1	0. 369
2R	62. 57	31. 31	0. 500	59. 3	—	—
M	—	—	—	58. 2	29. 8	0. 502
3R	96. 83	41. 06	0. 434	85. 9	—	—
M	—	—	—	80. 8	34. 4	0. 400
4R	211. 7	114. 09	0. 543	194. 0	—	—
M	—	—	—	189. 0	99. 1	0. 5108
5R	310. 85	154. 82	0. 498	—	—	—
6R	407. 0	235. 71	0. 579	—	—	—

R =calculated, M=mesured

Table 4. 2 The calculated results by NISA/RIM for varions water depths

d/a \ Mode	1	2	3	4	5	6
∞	5. 633	31. 31	42. 06	114. 09	154. 82	235. 71
1. 0	5. 645	31. 31	42. 08	114. 09	154. 84	235. 71
0. 5	5. 702	31. 34	42. 27	114. 13	155. 03	235. 73
0. 25	5. 948	31. 76	43. 55	115. 12	157. 29	236. 52
0. 125	6. 535	34. 15	47. 70	122. 70	169. 80	247. 82



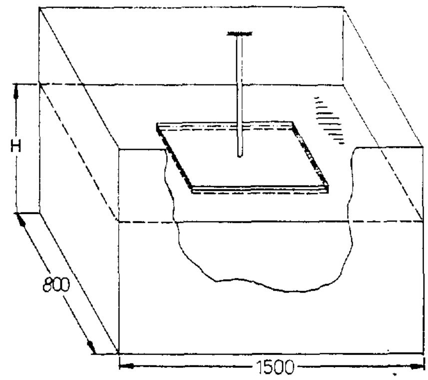
Sketch 4.1 The rectangular plate under S_o

the reduction factor is strong dependent on the eigen modeshapes, which have been shown in Fig. 4.9.

The reduction factor is defined as follows:

$\Delta = f(\text{wetted})/f(\text{dry})$

where f(wetted) and f(dry) are the eigenfrequency in water and in the air.



Sketch 4.2 Reservoir

4.4. The free vibration of a rectangular aluminium plate fixed at the center of the plate on the free surface

The eigenfrequencies of a rectangular aluminium plate (60cm×30cm×0.5cm), which is fixed at the

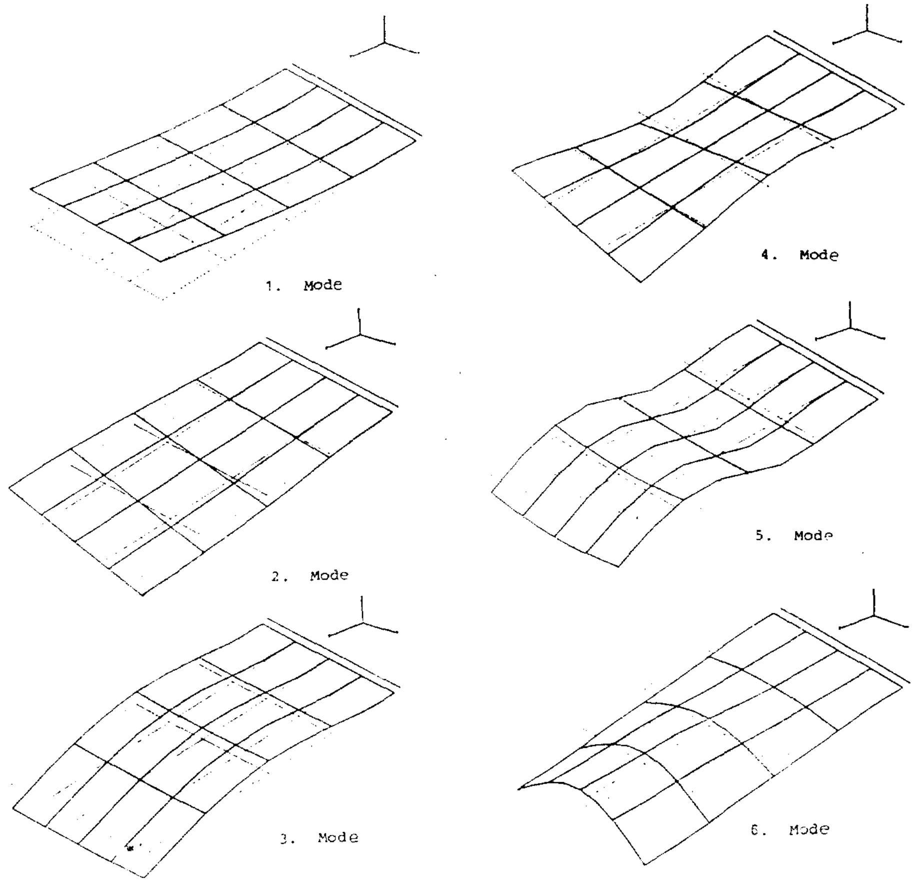


Fig. 4.9 The eigen modeshapes of a rectangular plate with one edge fixed under water

center with volting (See Scatch 4. 2), have been measured in a small reservoir (150cm×80cm×50cm), and calculated with NISA/RIM (See Fig. 4. 10). The material properties (the specific weight γ, elasticity module E, Poisson's ratio ν) are:

$\gamma=2700\text{kg/cm}^3$, $E=6.75\times 10\text{N/cm}^2$, $\nu=0.3$

The calculated and measured eigenfrequencies up to 6th mode has been tabulated in the Table 4. 3. But only the second mode in water has been compared with each other, because the experimental boundary condition for the antisymmetric mode can not be realised in the practice.

Table 4. 3 The calculated eigenfrequencies and the measured ones for various water depths

Mode / H(water-depth in cm)	1	2	3	4	5	6
7.5	13. 93	17. 64(18. 0)	38. 63	45. 56	64. 67	126. 34
10. 0	17. 01	18. 64(18. 5)	40. 60	47. 04	72. 27	130. 32
20. 0	19. 3	19. 6 (19. 5)	42. 1	48. 1	78. 95	132. 8
30. 0	20. 07	20. 42(19. 8)	42. 65	48. 34	81. 95	133. 43
40. 0	20. 19	20. 87(20. 0)	42. 66	48. 31	82. 91	134. 4

( ) : measured eigenfrequencies

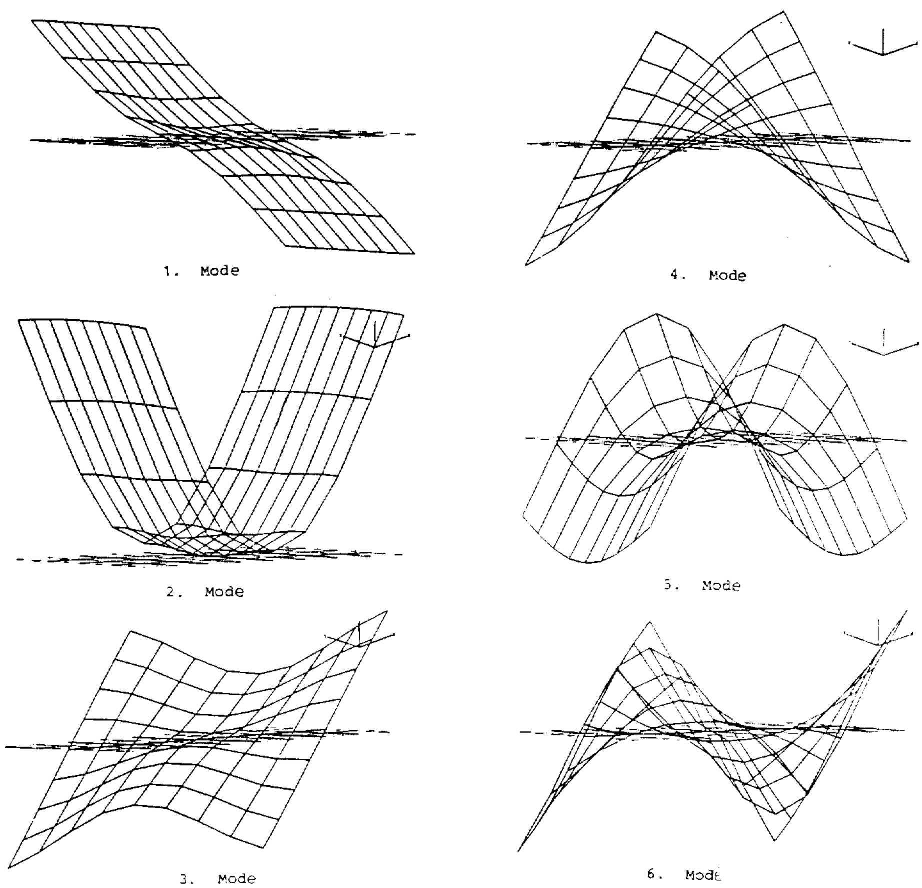


Fig. 4. 10 The eigen modeshapes of a rectangular plate fixed at the center under water

5. Conclusion

The boundary integral method combined with finite element method has been applied to analyse the fluid-structure coupling problem. In this paper the boundary integral method and the hybrid boundary integral method, which have been developed in recent years for the structural rigid motion, has been widely developed and tested for the application to the hydroelastic free vibration problem (program system NISA/RIM).

In contrast with the rigid motion in the low frequencies the elastic deformation of the structure should not be disregarded for the analysis of the hydroelastic system. In this case the hydrodynamic characteristics are also function of the structural mode shapes as the frequencies. Therefore the two systems, fluid and structure, must be handled simultaneously.

The calculated results show that the direct boundary integral method give very stable results in the whole frequency ranges, and it is not necessary to idealize the boundary finer corresponding to the increasing frequencies. The boundary condition on the free surface and the even bottom can be eleminated by using the series form image-functions (asymptotic Green function).

The hybrid boundary integral method give a good result with a few eigenfunctions on the radiation boundary in case of high frequency, because the spectrum in high frequency is concentrated near the forced vibration frequency.

Specially the image-function method can be well applied for the hydroelastic problem in high frequency, because in this case we need to idealize only the wetted body boundary with the fluid boundary elements.

The forced vibration analysis can be also accomplished through the hydroelastic coupling using the program system NISA/RIM.

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