

# 고장벡터 모델링에 의한 프로세스 고장 검출필터의 설계 및 응용

## A Process Fault Detection Filter Design by Fault Vector Modelling Approach and an Application

李 起 常\* · 裴 相 旭\*\*  
(Kee-Sang Lee · Sang-Wook Bae)

### Abstract

A Detection Filter that can be used for the Detection and Isolation of process faults is proposed by the use of fault vector modelling, and is applied to DC Motor fault detection.

The proposed detection filter is a new one in a view point that its outputs are the estimates of fault variables(or linear combination of them) while all the existing filters estimate the state of process. By this properties, the process fault detection systems with this filter can be constructed in very simple structure.

Besides the simplicity of structure and design procedure, the filter has an useful feature that various types of fault can be estimated via the filter by choosing appropriate fault models.

### 1. Introduction.

Recently, Process Fault Detection problem has been received considerable attention due to the increasing demand on the system reliability and safety, where a fault is considered to be a nonpermitted deviation of characteristics which leads to inability to fulfill the intended purpose.

In general, a Fault Detection Scheme consists of two stages: residual generation and decision making. In the first stage, measured variables are processed to form residuals. And the residuals are monitored to make failure decision and to identify the fault types in the second stage.

Since the 1970's, various fault detection schemes have been reported and the work to date can be classified into two classes according to the method of residual generation. The one is the state estimator based scheme where State Observers or Kalman Filters are used for generating the residual.<sup>1)</sup> And the other is the scheme based on parameter Identification and theoretical modelling.<sup>2)</sup>

The present study is related to the class 1. And the purpose of this paper is to propose a Process Fault Detection Filter that generate the fault vector directly, but not states of the system, by the use of fault vector modelling and to apply the filter to the process fault detection.

While the previous approaches such as GLR<sup>3)</sup>, MLE<sup>4)</sup> can be used for the detection of abrupt jumps only, the proposed filter can be used for various types of fault: jump, ramp and exponent etc. And the detection of fault can be performed by simple threshold test.

The Detection Filter is a new one since the out-

\*正 會 員 : 檀國大 工大 電氣工學科 · 士博

\*\*正 會 員 : 檀國大 大學院 電氣工學科

接受日字 : 1987年 2月 12日

1次修正 : 1987年 5月 26日

puts of the filter are the estimates of fault variable itself while all the existing Detection Filters estimate the states of the process.<sup>5)</sup>

The organization of this paper is as follows. In the next section, problem description is given. And mathematical modelling of the fault vector is treated in section III. In section IV, design method of detection filter is proposed. In section V, the proposed detection filter is applied to the armature controlled DC Motor system for the detection and classification of probable process faults. Finally, in conclusion, possible extension and applications are discussed.

## 2. Problem Descriptions.

Consider the following dynamic system described by

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where  $x(t) \in R^n$ ,  $u(t) \in R^r$  are the state vector, input vector and  $y(t) \in R^p$  is measurement output vector. And A, B and C are appropriate dimensional constant matrices.

With the possible occurrence of a plant or actuator fault, eq(1) can be modified as eq(3).

$$\dot{x}(t) = Ax(t) + Bu(t) + Qw(t) \quad (3)$$

In eq(3), Q is the (n, q) dimensional matrix that represents the channel through which fault vector affects the process characteristics. And  $w(t) \in R^q$  is the fault vector that may be expected to occur during the process operation. Thus, the  $Qw(t)$  possibly means Bias Fault, parameter fault  $\Delta Ax(t)$  and/or actuator fault  $\Delta Bu(t)$  or  $B\Delta u$ , etc.

Then, our problem is reduced to design a filter that can directly generate the estimates of fault vector,  $f_i(t) = [i\text{-th row}(Q)]w(t)$ , where the faults may occur at completely unknown instants and to design a Decision Logic and, if possible, to find the source of the fault.

To design a Detection Filter that can generate the estimates of fault, fault must be represented by the filter output or linear combination of states of the filter. And for the filter to have the property, the system dynamic equation must be modified to have the faults as state variables.

Therefore, design procedure of the filter contains the mathematical modelling of the faults that may occur.

And the fault model is combined with original system equation(3). Then a detection filter can be constructed for the Combined(Augmented) System.

At first, mathematical modelling of fault is presented, then filter design procedure for the Augmented System is described.

## 3. Mathematical Model of a Fault.

Since the performance of the proposed Detection Filter is largely dependent on the mathematical model of the fault, it should be modelled so that the resultant filter gives good estimates the faults.

Fortunately, it can be modelled by eq.(4), due to the fact that the faults occurred in physical process frequently possess the waveform structure such as jump (step), ramp and exponential type etc.

$$w(t) = \sum_{i=1}^m c_i \phi_i(t) \quad (4)$$

where  $\phi_i(t)$ s are some basis functions and  $c_i$ s are unknown coefficients and m is finite number which can be determined.

And methods of finding the dynamic representation of  $w(t)$  can be classified into two cases: when experimental data for each expected fault is available and when there is no data on fault.

CASE[A]: Experimental Data (Waveform structure) is given.

In this case,  $\phi_i(t)$ s can be selected from the experimental data and  $w(t) = \sum_{i=1}^m c_i \phi_i(t)$  can be considered as a primitive of following differential equation

$$\frac{d^m w}{dt^m} + a_m \frac{d^{m-1} w}{dt^{m-1}} + \dots + a_2 \frac{dw}{dt} + a_1 w = \delta(t) \quad (5)$$

where  $\delta(t)$  is an impulse function and  $a_s$  are coefficients of the equation. And this can be represented by an observable phase variable canonical form of eq.(6).

$$\begin{aligned} \dot{z}(t) &= Dz(t) + E \delta(t) \\ w(t) &= hz(t) \end{aligned} \quad (6)$$

with

$$D = \begin{bmatrix} -a_m & 1 & 0 & \dots & 0 \\ -a_{m-1} & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix} \text{ and } h = [1 \ 0 \ \dots \ 0]$$

$$\begin{bmatrix} -a_2 & 0 & 0 & \dots & -1 \\ -a_1 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (7)$$

$$E = [0 \ 0 \ \dots \ 1]^T$$

where D, E and h are (m,m), (m,1) and (1,m) dimensional matrices, respectively.

CASE[B] : No Experimental Data is given.

In many cases, experimental data for fault condition cannot be obtained since the system with some fault may be operating in critical environment. In this case, a fault may be modelled as Taylor series about t as follows.

$$w(t) = c_0 + c_1 t + \dots + c_{m-1} t^{m-1} \quad (8)$$

Then, the parameters of eq.(6) can be obtained as

$$D = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, h^T = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ 0 \\ 0 \end{bmatrix} \text{ and } E = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ 1 \end{bmatrix} \quad (9)$$

where D, h and E are (m, m), (1, m) and (m, 1) dimensional matrices.

By the use of suggested mathematical models, the resultant Detection Filter gives good estimates for almost all of the fault that may occur in many practical processes by choosing appropriate m. For example, m=1 for Jump type fault, m=2 for ramp type and exponential type fault are sufficient order.

If  $\underline{w}(t)$  is q dimensional vector, then i-th fault is represented as

$$\begin{aligned} \dot{z}_i(t) &= D_i z_i(t) + E_i \delta(t) \\ w_i(t) &= h_i z_i(t) \end{aligned} \quad (6')$$

where  $z_i$  is  $m_i$  dimensional vector and  $D_i$ ,  $h_i$  and  $E_i$  are  $(m_i, m_i)$ ,  $(1, m_i)$  and  $(m_i, 1)$  dimensional matrices. And fault vector can be represented as

$$\begin{aligned} \dot{z}(t) &= \underline{D}z(t) + \underline{E}\delta(t) \\ \underline{w}(t) &= \underline{H}z(t) \end{aligned} \quad (10)$$

where  $z(t) \in \mathbb{R}^M$  where  $M = \sum_{i=1}^q m_i$  and  $\underline{D}$ ,  $\underline{E}$  and  $\underline{H}$  are  $(M, M)$ ,  $(M, q)$  and  $(q, M)$  dimensional matrices respectively.

$$\begin{aligned} \underline{H} &= \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} & \underline{D} &= \begin{bmatrix} D_1 & 0 & \dots & 0 \\ 0 & D_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & D_q \end{bmatrix} \\ \underline{E} &= \begin{bmatrix} 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \end{aligned} \quad (11)$$

#### 4. Detection Filter Design by using Functional Observer Theory.

The proposed Detection Filter design procedure consists of two stages. In the first stage, the original system of eq.(3) and mathematical model of the fault variables (eq.(10)) are combined to construct an Augmented System as follows.

$$\dot{\underline{x}}(t) = \begin{bmatrix} A & QH \\ 0 & \underline{D} \end{bmatrix} \underline{x}(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ \underline{E} \end{bmatrix} \delta(t) \quad (12)$$

$$y(t) = [C \ 0] \underline{x}(t)$$

where  $\underline{x}(t) \in \mathbb{R}^{(n+M)}$  is augmented state vector

$$\underline{x}^t = [x^t, z_1^t, z_2^t, \dots, z_q^t] \quad (13)$$

and M is the order of fault vector model.

Now, the problem is reduced to design a filter that estimates  $w_i(t) = h_i z_i(t)$ ,  $i=1, 2, \dots, q$  for the augmented system of eq.(12), (13).

To design a filter, we assume that the pair (C:A) of the original system is completely observable. Then, the pair  $\left( [C \ 0] : \begin{bmatrix} A & QH \\ 0 & \underline{D} \end{bmatrix} \right)$  of the augmented system of

eq.(12) is also observable if and only if  $\text{rank} \begin{bmatrix} A & Q \\ C & 0 \end{bmatrix}$

$= n+q^9$  since the mathematical model of each fault variable was represented by observable canonical forms. And Detection filter for eq.(12), therefore, exists.

Although the well known observer design procedure may be applicable when the order of Au-

gmented System is low, it may not be applicable in many cases when the order of original system is high and /or the number of faults that may occur is large, because the order of the resultant filter is very high in these cases.

Therefore, it is desired, in general, to design a filter of very lower order than that of the state estimators. And the requirement can be achieved by restricting the variables to be estimated to a class of linear functionals of the states of augmented system of eq.(12).

For the detection purpose, q independent linear functionals of  $w_i$ s or equivalently  $x_i$ s can be selected as follows.

$$V(\underline{x})=K\underline{x} \tag{14}$$

where  $V=[v_1 \ v_2 \ \dots \ v_q]^t$  and  $K$  is  $(q,n+M)$  dimensional matrix whose first n column have all zero elements. Then, a detection filter that generate the linear combinations of fault variables can be constructed as eq.(15), (16).

$$\dot{s}(t)=F_s(t)+Jy(t)+Gu(t) \tag{15}$$

$$V(\underline{x})=K\underline{x}(t)=Ly(t)+Ps(t) \tag{16}$$

where  $s(t)$  is the state vector of the filter. And the parameters in eq.(15), (16) and the order of the filter should be selected so as to meet the conditions in Theorem(1). Here after, we use  $\underline{B}=\begin{bmatrix} B \\ O \end{bmatrix}$ ,  $\underline{C}=[C \ O]$  and  $\underline{A}=\begin{bmatrix} A & QH \\ O & D \end{bmatrix}$ , for brevity.

[Theorem, 1]

For the preselected  $F$  that has stable eigenvalues, if there exist  $T$  such that

$$FT-TA=JC, \ G=TB \tag{17}$$

$$\text{and } K=PT+LC \tag{18}$$

and if the pair  $(P:F)$  is completely observable, then  $V(\underline{x})$  is asymptotic estimates of  $K\underline{x}$ .

Proof of theorem, 1 is omitted since it can be seen from ref[6]

Although the Theorem provide the necessary and sufficient existence condition of the Detection Filter, design procedures are very complicated, specially when the estimation of multifunctional is required. For this reason, a simple algorithmic procedure is given below,

in the light of [7].

The problem can be described as follows:

Given the system of eq.(12), find the parameters of eq.(15), (16) and transformation matrix  $T$  such that Theorem.1 is satisfied. And the solution for multifunctional estimation may be given by sequential procedures that start with design of the filter for the first functional  $v_1(\underline{x})=K_1\underline{x}$  where  $K_1$  is 1st row of matrix  $K$  and the preselected stable diagonal matrix  $F_1$ .

(solution for  $v_1(\underline{x})$ ):

Let  $p_1$  be  $(\mu_1-1)$  dimensional vector,  $[1 \ 1 \ \dots \ 1]$  where  $\mu_1$  is the observability index of the pair  $(\underline{C}:\underline{A})$ . Then,  $L_1$  and  $J_1$  are the solutions of following linear algebraic equation.

$$\begin{bmatrix} L_1 : J_1^t & J_2^t & \dots & J_{\mu_1-1}^t \end{bmatrix} \begin{bmatrix} \underline{C} \prod_{j=1}^{\mu_1-1} (\underline{A}-d_j I) \\ \underline{C} \prod_{j=2}^{\mu_1-1} (\underline{A}-d_j I) \\ \dots \dots \dots \\ \underline{C} \prod_{j=1}^{\mu_1-2} (\underline{A}-d_j I) \end{bmatrix} = K_1 \prod_{j=1}^{\mu_1-1} (\underline{A}-d_j I) \tag{19}$$

$$\text{Then } G_1=T_1\underline{B} \tag{20}$$

where  $T_1$  is a  $((\mu_1-1), (n+qM))$  dimensional transformation matrix with  $j$ -th row,  $T_{1j}$  of  $T_1$  is computed as follows.

$$T_{1j}^t = J_{1j}^t \underline{C} (\underline{A}-d_j I)^{-1}, \ j=1, 2, \dots, (\mu_1-1) \tag{21}$$

The propriety of the parameters can be proved by simple substitution of them to the conditions in Theorem.1 and is omitted.

(solution for  $v_j(\underline{x}); j=2, 3, \dots, q$ );

Then same procedure is applied to the triple  $[\underline{A}, \underline{B}, C_2]$  for the functional  $v_2=K_2\underline{x}$  and to the triple  $[\underline{A}, \underline{B}, C_j]$  for the functional  $v_j=K_j\underline{x}$ , where  $C_j^t = [\underline{C}^t; T_1^t, \dots, T_{j-1}^t]$ .

When this procedure is applied, the order of the filter to estimate  $v_j$  is less or equal to that of the filter to estimate  $v_{j-1}$  since  $\mu_j \leq \mu_{j-1}$ . And the order of the overall filter is  $N$ , where  $N = \sum_{j=1}^q (\mu_j - 1)$ .

Then, resultant Detection Filter that generate  $q$ -independent linear combinations of fault variables can be constructed as following equation

$$\dot{s} = \begin{bmatrix} F_1 & 0 & 0 & \dots & 0 \\ J_{21} & F_2 & 0 & \dots & 0 \\ J_{31} & J_{32} & F_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ J_{q1} & J_{q2} & J_{q3} & \dots & F_q \end{bmatrix} s + \begin{bmatrix} J_{11} \\ J_{22} \\ J_{33} \\ \vdots \\ J_{qq} \end{bmatrix} y + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ \vdots \\ G_q \end{bmatrix} u \quad (22)$$

and

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_q \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_q \end{bmatrix} y + \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_q \end{bmatrix} s \quad (23)$$

where  $s$  is  $N$  dimensional state vector of the filter and  $J, P$  are  $(M, \sum_{j=1}^{q-1} (\mu_j - 1) + p)$ ,  $(q, N)$  dimensional matrices with  $J_{ji}=0, P_{ji}=0$  for  $j < i$ . And  $F, G$  and  $L$  are  $(N, N)$   $(N, r)$  and  $(q, p)$  dimensional matrices respectively.

Since the state vector of the augmented system of eq( 12 ) contains not only the process state vector but also the fault vector, the state vector of the detection filter,  $s(t)$ , is also the linear combination of them:

$$s = T\bar{x} = T \begin{bmatrix} x \\ z \end{bmatrix}$$

Therefore, the filter output  $V(\bar{x})$  may be chosen as any linear combination of  $\bar{x}$ , while only the combination of  $z$  may be selected as  $V(\bar{x})$  for detection purpose.

In this respect, the results of sec(iii) and (iv) can be considered as a parameter invariant functional observer or, equivalently, a functional observer for the systems with unknown disturbances (with waveform structure) that has not been previously reported.

### 5. An Application (DC Motor)

Consider an armature controlled DC Motor system driven by following equation

$$\begin{bmatrix} \dot{\omega}(t) \\ \dot{i}_a(t) \end{bmatrix} = \begin{bmatrix} -B/J & K_T/J \\ -K_e/L_a & -R_a/L_a \end{bmatrix} \begin{bmatrix} \omega(t) \\ i_a(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_a \end{bmatrix} e_a(t), y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega(t) \\ i_a(t) \end{bmatrix} \quad (24)$$

where parameters are given as follows

$$\begin{aligned} R_a &= 0.2813[\Omega], L_a = 0.007708[H] \\ J &= 0.133[Nmsec / rad], B = 0.00722[Nmsec / rad] \\ K_T &= 3.0531[Nm / A], K_e = 0.050885[V / rad sec] \end{aligned}$$

Here, we assumed two step type bias faults for brevity. Then, Augmented System is obtained as eq.(25).

$$\begin{aligned} \dot{\bar{x}}(t) &= \begin{bmatrix} -0.05429 & 22.9555 & 1 & 0 \\ -6.60155 & -36.4946 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \bar{x}(t) + \begin{bmatrix} 0 \\ 129.735 \\ 0 \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \bar{x}(t) \end{aligned} \quad (25)$$

where  $\bar{x}^t = [x^t, z^t]$  and  $u = e_a$ .

Since the detection filter must generate the fault variable, the output of the filter may be selected as

$$\begin{aligned} V_1(t) &= [0 \ 0 \ 1 \ 0] \bar{x}(t) \\ \text{and} & \\ V_2(t) &= [0 \ 0 \ 0 \ 1] \bar{x}(t) \end{aligned} \quad (26)$$

By applying the design procedures in previous section, two 1st order filters are constructed.

$$\begin{aligned} \dot{z}_1(t) &= -z_1(t) + [-.9457 - 22.9551]y(t) \\ \dot{z}_2(t) &= -6z_2(t) + [39.6093 \ 182.9673]y(t) - 778.412 u(t) \end{aligned} \quad (27)$$

$$\begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \quad (28)$$

For the simulation study, motor fault types are classified as following Table.

Table: Simulated motor fault.

failed comp.	parameter variation	[%]
Brush	$R_a$	+20
Load Shaft Support	$B, J$	+20, +20
Field Current	$I_f$	+5

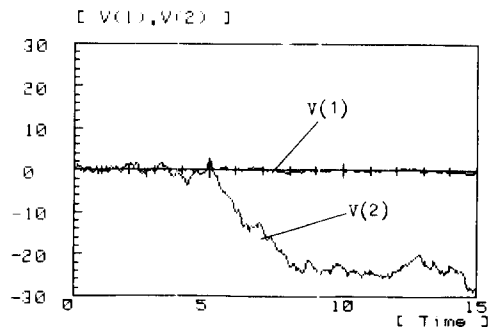


Fig. 1. Detection Filter Responses for Brush fault ( $R_a$ : 20% increase) ( $e_a = 75[V]$ ).

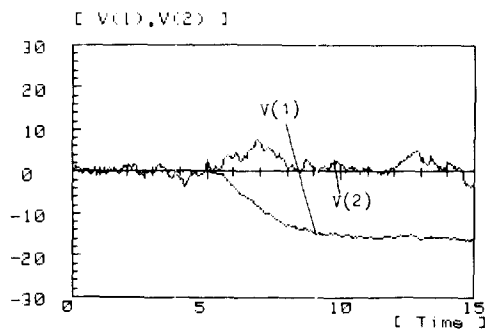


Fig. 2. Detection Filter Responses for Load change B, J: 20% increase) ( $e_a=75[V]$ ).

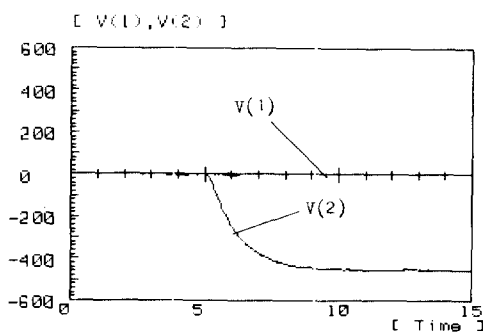


Fig. 3. Detection Filter Responses for Field circuit fault ( $I_f$ : 5% increase) ( $e_a=75[V]$ ).

For the above three cases, the fault is assumed to be occurred at  $t=5[\text{sec}]$ , while time reference was chosen to be the time when transient responses of the process is settled down. Simulation results are given in Fig.1, 2, 3. And the simulations are performed in the noisy environment where input, measurement noises are assumed to have zero mean and variance of 0.000225, 8, 3521 and 0.00004096, respectively.

Fig.1 shows the filter outputs when 20[%] variation of armature resistance is occurred at  $t=5[\text{sec}]$  due to brush fault. Fig.2 shows the estimates of  $V_1$  and  $V_2$  when 20[%] variation of Biscous Friction is occurred due to the abrupt Load change and/or shaft support fault. And the filter outputs for 5[%] variation of field current that may occur when field circuit is in abnormal operationg condition are shown in Fig.3.

The details of the threshold selection and of the isolation method are not described in here for brevity, since

it is not the main purpose of this study. However, the fault can be easily detected by simple threshold test and the isolation of the faults can also be performed by the logical operation of the informations provided by the filter and / or measuring instrument.

It is worth to note that  $V(\underline{x})$  may be generated even in the normal operating condition by the tolerable parameter perturbation and / or additional input,output noise. Therefore, threshold values must be carefully selected by considering the system characteristics such as tolerable bounds of parameter perturbation and effects of noise on the filter etc.

## 6. Conclusion.

A Detection Filter(DF) that can be used for direct generation of the process fault variables is proposed by introducing the method of mathematical modelling of unknown but probable fault variables. And with the Detection Filter, simple Threshold Test is sufficient for the Process Fault Detection(PFD) and Diagnosis since outputs of the filter are the fault variable itself, while state estimator based Fault Detection schemes require very complicated decision function and logic. Due to the structural simplicity and trivial computational burden, the PFD scheme with the proposed filter may be practically implemented in real time fashion.

In addition, various types of fault, for example, jump, ramp, exponent and some nonlinearities etc, can be estimated by constructing appropriate fault models. Finally, the Detection Filter is applied for the fault detection of armature controlled DC Motor System.

## References

- 1) A. S. Willsky, "A survey of design methods for failure detection in dynamic systems", *Automatica*, Vol. 12, pp.601-661, 1976.
- 2) R. Isermann, "Process Fault Detection Based on Modelling and Estimation Method-A Survey", *Automatica* Vol. 20, No. 4, pp.387-404, 1984.
- 3) A. S. Willsky and H. L. Johnes, "A Generalized Likelihood Ratio Approach to the Detection and Estimation of Jumps in Linear Systems", *IEEE Trans.*

- Automat. contr, AC-21, pp.108-112, Feb. 1976.
- 4) B. Friedland and S. Grabousky, "Estimating sudden changes of Biases in Linear Dynamic Systems", IEEE Trans. Automat. contr, AC-27, No.1, pp.237-240, 1982.
  - 5) J. E. White and J. L. Speyer, "Detection Filter Design by Eigensystem Assignment", Proc ACC, pp.156-163, 1985.
  - 6) J. E. Fortmann and D. Williamson, "Design of low-order observers for linear feedback control laws", IEEE Trans. Automat. contr, AC-17, No.3, pp.301-308, 1972.
  - 7) P. Murdoch, "Design of Degenerate Observers", IEEE Trans. Automat. contr, AC-17, pp.441-442, 1974.
  - 8) L. F. Paw, "Failure Diagnosis and Performance Monitoring", Marcel Dekker, 1981.
  - 9) J. O'Reilly, "Minimal-order observer for linear multi-variable systems with unmeasurable disturbances", INT. J. Contr, Vol. 28, No. 5 pp.743-751, 1978.