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ON SCHRÖDINGER SEMIGROUPS AND RELATED L^p-PROPERTIES

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Open questions (1) Does $e^{-tH(\vec{a})}$ map $L^\infty(R^n)$ to continuous function?
(2) To function with $L^2_{loc}(R^n)$ gradients?
(3) Are the integral kernels continuous?

These questions are given by B. Simon. Then, we obtain the following theorems which are partial solutions of above questions.

THEOREM 1. Let $V_- \in K_n$, $V_+ \in K_n^{loc}$ and let $\vec{a} \in L^2_{loc}$. Then for every $t>0$ and $1 \leq p \leq q \leq \infty$, $e^{-tH(\vec{a})}$ is a bounded from $L^p(R^n)$ to $L^q(R^n)$.

THEOREM 2. Let $V \in K_n^{loc}$ with $V \geq 0$. If λ is a bounded H^1_{loc}-function and $\vec{a} = i\vec{\nabla}\lambda$, then then for all $\phi \in L^\infty(R^n)$ and any $t>0$, $e^{-tH(\vec{a})}\phi$ has a distributional gradient in $L^2_{loc}(R^n)$.

THEOREM 3. Let $V \in K_n^{loc}$ with $V \geq 0$. If λ is a continuous H^1_{loc}-function and $\vec{a} = i\vec{\nabla}\lambda$, then $e^{-tH(\vec{a})}$ has a continuous integral kernel.

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