IMPLICATIONS OF DIFFERENT MARKET NOTIONS ON ASSET PRICING MODELS WITH UNCERTAIN INFLATION

By

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I. Introduction

The primary function of the capital market is to provide the economy with appropriate criteria to allocate scarce resources efficiently. This objective can be realized by the price mechanism whereby firms make production-investment decisions and consumers make consumption-investment decisions. Thus, finance theories focus on the following two points: 1) to build valid models of equilibrium asset pricing, 2) to test whether the models explain real phenomena adequately.

Different market notions play an important role in finding a proper model and testing it. While concepts of a perfect capital market and a complete capital market are related to the former, empirical works concerning the efficient market are developed for the latter.

This paper has dual objectives. First, this study is concerned with presenting the relationship among three different market notions and providing views regarding what conditions are necessary and sufficient for each market. For this purpose characteristics of markets are described and four distinctive equilibrium models (M.M's risk class, CAPM, Time-state prefrence, and Option pricing) are reexamined in the light of different market notions. Second, this paper presents implications of these notions on works related to asset pricing models incorporating uncertain inflation and shows the applicability of the extended CAPM to test capital market efficiency. These works include Roll's stochastic consumption model, Friend, Landskroner, and Losq's (hereafter

FLL) infinitesimal planning model, Merton's continuous time model with stochastic life-time consumption-investment opportunities, Long's multi-beta model, and Breeden's single aggregate consumption model.

In section II sufficient conditions and necessary conditions for each market are described. In addition, the relationship among different markets are presented. In section III I review several extended CAPMs , and discuss the implications of different market notions on the asset pricing models which explicitly take into account the influence of uncertain inflation. The final section presents the conclusion of this paper.

II Perfect markets, complete markets, and efficient markets

1. Sufficient conditions and necessary conditions for each market

A. Perfect market

The notion of perfect capital market (heafter PCM) is one of the most frequently used concepts in finance theory. For example, the four eqilibrium models under certainty are developed on the basis of PCM. The perfect market is a purely idealized situation which allows us to explain real phenomena with limited number of variables. It also provides desirable theorems for financial and investment decisions. Under PCM two separation theorems hold. First, a firm's investment decision is independent of financing decision. Second, management can make investment decision on the basis of market value maximization rule regardless of individual shareholder's taste.

Market conditions for perfect markets are relatively subjective. In a broad sense, assumptions of PCM are as follows:

1). Market are frictionless, i.e., there are no brokerage fees, taxes, or other transactions costs.

2). All assets are perfectly divisible.

3). In securities market all investors are price takers.

4). Information is costless and is equally received by individuals.

5). No firm is large enough to affect the consumption opportunity set facing consumers.

6). All individual rationally behave to maximize their expected utility.

7). There exists an open exchange market for borrowing and lending at the same rate of interest.

8). Short sales of all assets are allowed.

9). Unlimited lending and borrowing at the riskless rate.

Even though the set of assumption of PCM is required to derive an equilibrium pricing model, it does not show any relationship among variables of interest. Hence, it is more resonable to say that the concept of PCM is just supplemental apparatus for an asset pricing model. We can easily find many different sets of assumptions of PCM. Fama (10) requires assumptions 1), 3), and 4) to prove that there is a unique interest rate under certainty. The market setting in the earlier works of mean-variance madels done by Sharpe (30), Lintner (19), and Mossin (24) (hereafter S-L-M) includes all conditions except assumptions 4) and 5). To derive an intertemporal CAPM Merton (22) added one more assumption to standard assumptions of PCM that trading in assets takes place continuously in time. He maintains that the added condition almost follows directly from assumption 1). While Fama (10) explictly includes assumptions 4) and 5) as PCM condition for derivation of CAPM, Long (20) defines PCM by conditions 1), 2), and 3), and separate 4) from it for his multi-beta CAPM.

In the time-state preference model (hereafter TSP) the equilibrium price of contigent claims is obtained based on the notion of PCM. For example, Myers (25) assumes that security markets are perfect in the sense that there are no transactions costs, there is costless information about the distribution of returns on cotingent claims, and investors are price takers. He describes the impact of restriction on short-sales on asset pricing. To apply the implication to the asset pricing model which takes into account stochastic consumption and investment opportunities, Breeden (4) implicitly assumes the economy with PCM by mentioning assumptions 1), 2), 3) and 8). Dybvig and Ingersoll (7) also combine conditions 1), 2), 3), 8), and 9) to prove that the standard mean-variance separation theorem obtains in a complete market only if all investors have quadratic utility.

To derive option valuation formula Black and Scholes (3) presents ideal conditions in the maket for the stock and option. Among them assumptions 1), 2), 3), 4), 7), 8) and 9) are relted to PCM.

As shown in above examples, it is evident that the notion of a PCM is purely subjective one. Thus, the necessary conditions and sufficient conditions depend on individual's definition of PCM which faciliates the derivation of a specific equilibrium model with key variables.

B. The complete market

By the contribution of Arrow (2), Debreu (6) and Hirshleifer (17), analytical formula called T model is developed. To find general eqiuibrium prices of contingent claims it is assumed that the economy has a complete market, i.e., the number of independent securities (N) is equal to the number of states (S). If the capital market is complete, security valuation is straighforward because the value of a security is regared as portfolio of pure securities. Thus Arrow (2) pointed out that an inadequate number of markets in contigent claims would be a source of inefficiency. Ross (28), Arditti and John (1), however, proved that financial markets can achieve allocational efficiency by the single efficient fund or portfolio of the primitive assets on which options can be written, although the number of securities is less than that of states. In this point the modification of the sufficient condition for the complete capital market (hereafter CCM) is justified. That is, either N is equal to S ,or the existence of options on a efficient fund or portfolio can be a sufficicent condition for CCM. Dybvig and Ingersoll (7) define the market which satisfies the former condition as " relatively weakly complete market ". Ingersoll (18) combines these two conditions and describes that if there is a security for each possible state,then the market is complete. I would like to agree with Ingersoll, and say that one of sufficent conditions for CCM is that there is one to one correspondence between state and state security. In my view point, one of following conditions can play a role of the sufficient condition individually.

1). There are as many assets with linearly independent returns as there are states.

2). If not, investors can form options on asingle efficient fund or portfolio of securities.

3). There is a state security for each possible state.

Then what are necessary conditions for it? In other words, what kinds of implications can we infer from CCM ? It depends on the methodology of individual researcher. Generally speaking, the notion of CCM plays role of the fundamental assumption on which a general equilibrium model is derived. In this sense CCM should gurantee the existence of an equilibrium. Then, what is the necessary conditions for the general equilibrium ? At this stage, I would like suggest following conditions.

1). The probability that state s will occur judged by investor k is not zero.

2). The value of a security is the sum of the present value of the contigent returns of pure securities.

3). A state security exists for each possible state.

If the probability is zero, for some investors but not all, then they are willing to sell an infinite number of this state relatively weakly complete market ". Ingersoll (18) combines these two conditions and describes that if there is a security for each possible state,then the market is complete. I would like to agree with Ingersoll, and say that one of sufficent conditions for CCM is that there is one to one correspondence between state and state security. In my view point, one of following conditions can play a role of the sufficient condition individually.

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If the probability is zero, for some investors but not all, then they are willing to sell an infinite number of this state securities and there is no equilibrium. This restriction does not necessarily mean that the investors have homogeneous expecation regarding the probability distribution of state. Instead it just imposes least constraint on heretrogeneous expectations. The second condition is self evident in the sense that any security is regareded as a contract to pay an amount which depends on the state which actually occurs. The general equilibrium model also implies the third condition.

It is notable that while we need only one of sufficient conditions described above to have CCM, we can infer the set of three conditions simultaneously from CCM.

C. The efficient capital market

Fama (9) defines that a market in which prices always "fully reflect" available information is called "efficient". The efficient capital market (hereafter ECM) implies that market prices of individual security adjust very quickly to new information about the economy, about financial market, about the specific company involved. It seems that the efficient market principle really tells us only that we may treat the observed prices of securities as equilibrium one. As long as the security prices adjust instantaneously to all relavent information at any time and are assumed to be in equilibrium, they provide accurate criteria for resource allocation. That is, firms can make appropriate investment and production decisions to maximize their market value, and individuals determine optimal consumption and investment decisions to maximize their expected utility of consumption.

Since the definitional statement of ECM is so general, empirical works relating ECM are usually classified into three classes. In the weak form test of market efficiency, the relavent information is the historical prices. In the semi-strong form hypothesis, prices are tested whether they fully reflect currently publicly available information. Hypothesis test of strong form efficiency is concerned with the price adjustment process to information sets which are available to monopolistic group of investors. Most of tests regarding ECM are interested in investigating the price behavior of individual security. Moreover, they are done on the basis of the assumption that market equilibrium can be described in terms of following expected return or fair game model.

$$E(p_{j,t+1} \mid I_t) = [1 + E(r_{j,t+1} \mid I_t)] P_{j,t}$$

where I denotes a set of information fully reflected in P. We can safely say that the random walk model is an extension of the fair game efficient markes model because the former simply adds the stochastic process generating returns. The expected return model is also regared as more gereral than the submartingale model in the sense that it allowes the poissibility of negative expected return while it rules out any excess return.

It is worthwhile to enumerate conditions which faciliate the rapid adjustment of prices to revelent information.

According to Fama (9), the sufficient conditions for ECM are:

1). There are no trading costs.

2). Decision information is costless and available to all investors at the margin.

3). There is concurrence by all investors at the margin regarding the implication of information for the current price and the distribution of future prices.

Although above conditions are sufficient, they are not necessary to ensure market efficiency. In other words, market efficiency does not necessarily imply that three conditions are completely met. While the existence of significant transactions costs, heretrogeneous expectation among investors, and limited accessbility to costy information may be potential sources of inefficiency, any violation of the sufficient conditions does not rule out the possibility that the market prices fully reflect relevant information.

What types of condition can be drawn from ECM? I would like to suggest following two necessary conditions for it. First, no investor can consistently make better evaluations of available information. This condition is derived from the definitional statement of ECM. Regardless of trading costs, information costs, and different expectation across investors, ECM implies that nobody can be consistently superior to evauate the implications of information. Second, the utility function of investors has common feature. That is, the utility function is charaterized by the constant relative risk aversion. Considering the fact that the notion of ECM depends on the value of information, and that individual's valuation of information is governed by the expected utility maximization rule, the shape of investor's utility is the primary factor to determine the value of information with different messages. However, the problem is to find the way how the market aggregates different individuals' taste. If the value of information ,whether it is zero or positive, is fully reflected in the price of securities, thie implies that the market successfully aggerates the implication of any information across individuals. As many theoretical works including Friend and Blume (14) proved, the only charateristic of the utility function which faciliates aggeration is constant relative risk aversion. Utility functions exhibit such property are the power utility functions and logarithimic function.

2. Relationship among different market notions

Even though there are three distinctive notions of market some similarities and dissimilarities can be found. First of all, the notion of PCM is idealized world in which the partial equilibrium price of individual asset can be formulized. Modigliani and Miller (23) utilize PCM and homogeneous risk class to find the equilibrium relation between the expected return and the market price of risky asset. Based on the relationship, they propose the famouse three propositions related to the corporate finance and investment decision. S-L-M's CAPM is also regarded as the product of PCM although it requires some other ad hoc assumptions like quadratic utility function and normality. If investors with quadratic utility function make decisions to maximize expected utility of wealth at the end of one period, and they agree on the mean-variance of returns on risky assets, the PCM aggregates the optimal portfolio decisions of investors and gererates a unique equilibrium price of each risky asset. Nonstandard forms of CAPM which incorporates uncertain inflation, heretrogeneous expectations, non-marketable assets, missing assets, and international securities also assume the conditions of PCM.

It is well known that the general equilibrium approach like TSP is developed on the condition that the capital market is perfect. Moreover, B-S option pricing model (hereafter OPM) can not be exceptional for borrowing the convenient concept of PCM. Considering the fact that the conditions of PCM are fundamental assumptions for equilibrium pricing models, the PCM itself is the necessary and sufficient condition for the security pricing model. If perfect markets play so important role why are we concerned with the complete market? The reason is that even though CAPM suggests the compact relationship between the expected return and risk, it can not succesfully explain the simultaneous determination of asset prices. Thus, more analytical tools are required for describing the real world. Another reason for the necessity of CCM is that allocational efficiency of the capital market has attracted a great deal of interest from welfare economists. According to Arrow (2) the capital market can achieve Pareto efficiency only if the market is complete. In other words, as long as the sufficient conditions are satisfied, a unique set of Walasian equilibrium prices can be determined. As the First Theorem of Welfare ecomomics tells us that any competitive general equilibrium is Pareto efficient, we can infer that the notion of CCM is the precondition for the allocational efficiency as a capital market whole. The existence of financial institutions can be justified because they contribute to make the capital market complete and every market participant will be better off without hurting somebody's interest. Varian (31) shows excellent discussions on the general equilibrium and Pareto efficiency.

The definitional statement of ECM is less restrictive than that of PCM because the former can be achieved even when some of conditions for the latter are not satisfied. It seems that ECM is confined to the degree of the adjustment of prices to information. The notion of ECM is also related to allocational efficiency. Since, in an ECM, prices always fully reflect avialable information, they are approate signals for resource allocation and are crucial for directing the flow of funds. While the notions of PCM and CCM are directly related to the equilibrium pricicig model of securities, ECM is nothing to do with-asset pricing. Instead efficient market principles really tells us only that we may treat the observed prices of securities as equilibrium prices. To exploit the principle to test the proposition about valuation, we may usually invoke a specific model of equilibrium. For an individual security an expected return model is frequently used to describe the equilibrium conditions. Although the family of expected models--fair game or random walk is useful apparatus for testing the weak form hypothesis of single security, it tells nothing about average returns of different securities. The mean- variance form of valuation expression is particularly convenient for formulating testable propositions on intersicurity returns. I will discuss the application of CAPM for testing market efficiency detailly in section III.

Since any test of market eficiency implies testing joint hypotheses regarding equilibrium model and prices adjustment to the subset of all information, the more detailed test of the latter can be done only on the basis of the development of the former. Fama (8) also concludes that " when the process generating equilibrium expected returns is better understood (....), we will have a more substantial framework for more sophisticated intersecurity tests of market efficiency".

III. The impact of uncertain inflation on asset pricing

1. Different ways of incorporating uncertain inflation into CAPM

Several authors (Merton, Roll, Long, Chen and Boness (C&B), FLL, and Breeden) have attempted to incorporate the influence of uncertain changes in commodity prices on security returns and prices. If security prices do fully reflect the information about relative changes in commodity prices or general level of inflation, it is possible to investigate the impact of these inflations on the equilibrium prices of securities. In this sense the standard S-L-M CAPM has been the subject of extension.

Depending upon different assumptions associated with the behavioral aspect of investors and upon different viewpoint on time horizon, various types of the extended CAPM are developed. While Roll (26) derives a discrete time model of consumer demand for money, goods, and assets in an economy that has only one future date. Merton (22) presents a continuous time model based on life-time consumption-investment decisions. Long (20) derives capital market equilibrium for a discrete multi-period economy where future prices of consumption goods are uncertain and investment opportunities of common stocks and default-free bills of different maturities are available to consumers. C&B (5) and FLL (15) assume that the objective of investors is to maximize the expected value of a utility function whose only arguement is real consumption (wealth) at the end of infinitesimal planning horizon. Breeden (4) attempts to provide an aggregate consumption-beta asset pricing model in a multi-goods continuous time economy with uncertain consumption goods price and stochastic investment opportunities.

2. Related works and assumptions

A. Roll's model

Roll (26) derives following equilibrium model which takes into account the influence of the general price level on security prices

$$r_j = r_f + (r_m - r_f)b(R_j) - [\,Cov(r_j, c) - Cov(r_m, c)b(R_j)\,] / c$$

where r is nominal return, R denotes real return,

$c = 1/(1+\text{inflation rate})$, and $b(R_j) = Cov(R_j, R_m)/Var(R_m)$

This equation tells that to compensate for uncertainty in the inflation rate additional risk premia are required to the expected nominal rate of return. He also shows that in the case of non-stochastic inflation above equation reduces to the standard equilibrium model. Roll is the first one who pointed out that relation between nominal interest rates and expected rates of commodity price inflation is no longer the simple Fisherian equation. Relating to perfect markets conditions, he assumes that there exists an asset that is riskless in real term. Other conditions like frictionless market, price taker, and divisibility remain unchanged. In addition to the assumptions for the standard CAPM, it is assumed that there exists a composite consumer good whose price (c) can be used as a proxy for all commodity price.

B. Merton's model

Based on all the assumptions of PCM and the assumption of continuity of time, Merton (22) presents very analytical view of intertemporal capital market equilibrium. He asserts that investors act so as to maximize the expected utility of life-time consumptions. Namely

$$\text{Max } E \left[\int U(c(s),s)ds + B(w(T),T) \right]$$

where s,c,w, U, B, and T denote state varible, consumption, wealth, utility, bequest function, and at the time of death respectively.

It is notable that Merton utilizes the state dependent utility function. From the necessary condition of the objective function he derives following intertemporal asset pricing model which is function of the state variable.

$$r_j - r_f = \{[cov(j,M)-cov(j,n)cov(n,M)][SD(j)/SD(M)][1-cov(n,M)]\} \times (r_M - r_f) + \{[cov(j,n)-cov(j,M)cov(n,M)][SD(j)/SD(n)] \times [1- cov(n,M)]\} (r_n - r_f)$$

where r_n is the return on the portfolio which is perfectly negatively correlated with the risk free asset, SD(j) is the standard deviation of returns on asset j, and cov(j.M) denotes the correlation coefficient between return on asset j and return on market potfolio M.

Even though Merton illustrates the influence of changing investment opportunities on equilibrium price rather than changing consumption opportunities, his model is robust enough to evaluate both cases. In the latter case, an equilibrium relation can be modified as follows:

$$r_j - r_f = [Cov(j,m)/Var(M)](r_M - r_f) + [Cov(j,h)/Var(h)](r_h - r_f)$$

where r is return on the hedge portfolio which is perfectly correlated with the state variable.

This equation states that investors are compensated in terms of expected return, for bearing market (systematic) risk, and for bearing the risk of unfavorable shifts in the consumption opportunity set.

C. Long's model

Long (20) recognizes that the traditional CAPM is based on very restrictive assumptions that 1) the futures prices of all consumption goods are known with certainty, 2) the investment opportunities available to consumers do not change over time except possible in ways that are perfectly predictable. In Long's economy, neither assumption 1) nor 2) is characterized. While all the assumptions of PCM are intacted, changes in relative prices of consumption goods and uncertain interest rates are allowed. In this sense he provides a more complete view of the return generating process directly associated with uncertainty in price changes and future investment opportunities. Under the discrete time framework, he derives following expected return(r_j) model which is the function of short-term risk free rate (r_f), market rate of return (r_M), relative inflation rate (i_K), current price of consumption goods (p_K), current price of quasi futures contract (F_K), expected return on m period bill (r_m), market risk (b_{jM}), inflation risk (c_{jK}), and interest risk (d_{jm}). Symbolically

$$r_j = r_1 + b_{jM}(r_M - r_1) + \sum_{K=1}^{K} c_{jk}\left[(1+i_k) - (1+r_1)(F_k/p_k)\right] + \sum_{m=2}^{T} d_{jm}(r_m - r_1)$$

To get above equilibrium model, Long also utilizes the life-time state dependent utility function. Clearly if relative commodity prices and interest rates change, then the utility of consumption and future investment opportunities will be state dependent. Thus Long's work is regarded as the extension of the Merton's work. Moreover, above equation suggests the possibility of more adequate empirical research associated with market efficiency test. I will discuss this area section 3 of part III.

D. FLL's model and C&B's model

FLL (15) and C&B (5) present less rigorous analysis comparaing with methodologies cited above. However their models have one advantage over others in that they show the impact of general inflation on the asset pricing. Their models are also in contrast with Roll's zero beta CAPM since they directly incorporate uncertain inflation. Although they do not explicitly enumerate market conditions, it seems that their modls also require idealized conditions of PCM. Since they are interested in partial equilibrium model, the notion of complete market is unlikely related to the models. FLL's model and C&B's model are presented as follows in order:

$$r_j = r_f + Cov(r_j, i) + \frac{Cov(r_j, r_M) - Cov(r_j, i)/a}{Var(r_M) - Cov(r_M, i)/a} [r_M - r_f - Cov(r_M, i)]$$

$$r_j = r_f + \frac{Cov(r_j, r_M) - Cov(r_j, i)/a}{Var(r_M) - Cov(r_M, i)/a} (r_m - r_f)$$

where "a" denotes the ratio of wealth invested in the risky assets to total wealth as an economy whole. The difference between the equations derives because of two reasons: 1) different relationships between expected real returns and nominal returns, 2) FLL express all variables in continuous time. FLL argue that as long as the correlation between the rate of on the market and the rate of inflation is positive, the market price of risk is higher than that depicted in the standard CAPM. Moreover, they show that the risk of any asset is not only function of its covariance with market, but also a function of its covariance with the rate of inflation.

E. Breeden's model

Breeden (4) suggests that asset betas are measured relative to changes in the aggregate real consumption rate, rather than to the market. He supposes an economy where individuals behave as price takers in perfectly competitive, but possibly incomplete markets that are frictionless. He also assumes that investors are allowed to short-sell any assets with full use of the proceeds, and that they have identical beliefs in the probability of states of world. Thus he attempts to find an equilibrium asset pricing model under perfect but incomplete market. Based on these assumptions, he finds following eqilibrium expected excess return $(r_j - r_f)$ in terms of the ratio between consumption beta of asset returns (b_{jc}) and of any portfolio M's return (b_{Mc}), and the expected excess returns of the portfolio $(r_M - r_f)$. Namely

$$r_j - r_f = (b_{jc} / b_{Mc})(r_M - r_f)$$

If there is a security whose return is perfectly correlated with changes in aggregate consumption, then above equation reduces to

$$r_j - r_f = b(r_c - r_f)$$

where b is beta measured relative to that security's return and $r_c - r_f$ is the excess return of that security. As shown in the equation Breeden invented a very simple model which is almost same as S-L-M's standard CAPM, although it incorporates stochastic consumption investment opportunities, state dependent utility function, and life-time planning horizon.

3. Applicability of extended CAPMs for market efficiency test

If the two parameter model is an appropriate description of the process which determine equilibrium prices of any risky asset, the empirical test of capital market efficiency is subject to be generalized to verify the appropriateness of average returns on different securities. For this purpose, mean-variance approach has been the most popular technique used in the emprical research in finance.

Generally market efficiency test is done in the form of joint hypotheses that the equilibrium model adequately explains price structure in the real world and relevant information is rapidly reflected in prices. Most empirical works use the following Markowiz market model to test these hypotheses.

$$r_{j,t} = a_j + b_j r_{M,t} + u_{j,t}$$

As long as the model does not violate any assumptions of regression analysis, it provides very robust tool for testing any form of market efficiency. Specificaly, if correlation coefficient between u_t and u_{t-1} is equal to zero, and u_j is independent of r_M, and the value of the intercept term is not significantly different from $r_f(1-b_j)$, then the weak form hypothesis is accepted because the history of returns does not provide any insight. Fama and MacBeth (12) used CAPM to etimate expected return on a security. They then examined the correlation of execss return and found no significant correlation between the current residual and previous returns. From the equilibrium relation of S-L-M CAPM, we can estimate the expected rate of return on any security which is generated by the market. Thus, as long as any trading rules based on information can not consistently have greater profits than a buy-and-hold policy, the capital market is efficient in both semi-strong and strong sense. Fama, Fisher, Jensen, and Roll (11) investigated the effect of stock splits on security prices. Based on cummulative excess return which are computed from CAPM, they concluded that the performance after the announcement of split did not lead to excess return. It is the first of several studies that were directly related to testing of semi-strong form of market efficiency.

It is interesting to raise the question that whether the extended CAPM provides better insight into market efficiency test than the standard CAPM does. Roll's model suggests a number of possibilities that the incorporation of uncertain inflation may modify equilibrium analysis of asset prices. However, it does not provide testable propositions with regard to market efficiency because real return is not readily measurable. Merton's intertemporal CAPM with constant investment opportunity set is just continuous-time version of the security market line of the standard CAPM. However, Merton's model requires identification of state variable for empirical usages. Although FLL's model suggests interpretable relationship between the expected rate of return on a risky asset and inflation rate, the variable of the utility function is not realistic one. It is more reasonable to assume that utility of individual is dependent on consumption, not on the amount of wealth possessed.

If we test market efficiency with a extented CAPM derived under the continuous-time framework, the naive test of $r_f(1-b_j)$ could be misleading. Since continuous-time approach assumes that the distribution of continuously compounded rate of return follows a normal distribution, the random varible can be expressed as follows:

$$r_j dt = [m_j -(1/2)Var(r)]dt + s_j dz$$

where Var(r), s, m, denote instantaneous variance, standard deviation, and mean rate of return respectively. And dz represents the Winer process.

The market model is

$$r_j dt = a_j + b_j r_M dt + e_j dt$$

We will find $b_j = Cov(r_j, r_M)/Var(r_M) = Cov(r_j, r_M)dt/Var(r_M)dt$

This market model estimates desirable coeficient independent of the time interval. However, in the continuous time model, the intercept term a of above regression line possesses following statistical properties.

$$E(a_j) = \text{probability limit } a_j = E(r_j) - b_j E(r_M)$$

$$= [r_f(1-b_j)] + (1/2)[b_j Var(r_M) - Var(r_j)]dt$$

$$= [r_f(1-b_j)] + [(1/2)Var(r_M)b_j(1-b_j) - S_j^2]dt$$

where S^2 is the standard error of residuals

Thus, the usual test of market efficiency using $r_f(1-b_j)$ can not be an unbised and consistent one. Ingersoll (18) suggests the way to avoid this problem. To test the appropriateness of return on a security, the continuous-time equilivalent measure can be used.

$$\frac{\ln(\overline{1+dP_j/P_j}) - r_f dt}{b_j} \gtrless \ln(\overline{1+dP_M/P_M}) - r_f dt$$

If b>0 and the greater than sign applies, the security in question will yield higher return than the risk-adjusted market rate of return ,and vice versa.

It is noted that Breeden invented a very compact model which hedges against only aggregate consumption to determine the proper prices of risky securities. However, it is very difficult to verify the model empirically because of two reasons. First, it requires precise description of state variables. Second, the definition of the aggregate consumption is too broad.

With slight modification, we can use Long's model as tool for testing market efficiency. That is, the following equation is just extension of Markowitz market model.

$$r_{jt} = a_j + b_j r_M + \sum_{k=1}^{K} c_{jk} i_{kt} + \sum_{m=2}^{T} d_{jm} r_m + e_{jt}$$

It is also similar to Ross's (29) arbitrage pricing model in that i_K, r_M, and r_m can be thought as the factors in a K+T+1 factor model.

The most distinctive feature of Long's model is that it takes into account shifts in term structure of interest rates which most of previous researchers neglected. Furthermore, his model suggests another empirically testable hypothesis that portfolio of stocks and bills can be effectively used to hedge against relative price changes. This proposition shows contrast to Fama and Shewert's (13) finding that common stocks are negatively correlated with expected inflation, and unexpected inflation. Gay and Manaster (16) conclude that portfolio of stocks and bills do not provide any additional information about commodity price inflation beyond that contained in the return on a short-term bill.

IV. Conclusion

Considering the fact that the ultimate objective of finance theory is to find an appropriate model to explain the equilibrium price generating process, the notions of perfect market and complete market play an important role of providing desirable economic setting for the asset pricing model.

PCM presents purely idealized situation on which most of equilibrium models can be explained with a significant few variables. Unfortunately, there is no agreement on the definition of PCM. I enumerated the conditions for PCM in a broad sense, and showed how different combinations of these are constructed in defining PCM. Thus, in my viewpoint, the necessary and sufficient conditions are dependent on assumptions of PCM that individual researcher makes.

In contrast to the notion of PCM, CCM is well defined. Sufficient conditions for CCM are: 1) the number of securities with linearly independent returns is greater than the number of states, 2) othrewise, there exists single efficient fund on which options can be written, and 3) there is a state security for each possible state. One of three conditions is sufficient for CCM. I also suggested three implications which are drawn from general equilibrium conditions. Among them, the most important one is that the probability that a state will occur judged by individual is not zero.

To test capital market efficiency, a testable equilibrium model is required. The standard CAPM seems to provide a useful technique for it. However, The S-L-M CAPM has been the subject to be supplemented and extended because of its ad hoc assumptions regarding investment opportunity set. Thus, uncertain inflation should be incorporated to explain the behavior of invesors under stochastic consumption and investment opportunities.

Several authors have attempted to take into account the influence of inflationary uncertainty on the equilibrium generating process. Among them, FLL's model, Breeden's aggregate consumption model and Long's multi-beta model provide testable propositions. FLL's model shows the risk premium associated with the general inflation rate. The model can not be regard as a general model because the investment planning horizon is too short. Although Breeden's single aggregate consumption beta presents very convenient way of asset pricing, there are some difficulities in finding the aggregate consumption.

Relating to market efficiency test, it is suggested that for a continuous time model the usual test using intercept term of regression line could be misleading because the statistic is inconsistent and biased.

Long provides a more complete view of the return generating process directly associated with uncertainty in price changes and shifts in the yield curve. However, using specific commodity prices as state variables is not empirically useful.

It is evident that intertemporal asset pricing models that incorporate uncertain inflation improved the descriptive validity of the single- period model. Moreover, the extended CAPM suggests the possibility that inflation hedging portfolio can be constructed using stocks and bills. Although the advanced form of CAPM provides a great deal of insights into asset pricing process, we are looking forward to the appearance of the testable model which exlpains the impact of uncertain inflation on the equilibrium asset pricing more accurately.

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