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Adaptive Linear Predictive Coding of Time-varying Images Using Multidimensional Recursive Least-squares Ladder Filters

Nam, Man K.*
Kim, Woo Y.*

Abstract

This paper presents several adaptive linear predictive coding techniques based upon extension of recursive ladder filters. A 2-D recursive ladder filter is extended to a 3-D case which can adaptively track the variation of both spatial and temporal changes of moving images. Using the 2-D/3-D ladder filter and a previous farme predictor, two types of adaptive predictor-control schemes are proposed in which the prediction error at each pel can be obtained at or close to a minimum level. We also investigate several modifications of the basic encoding methods. Performance of the 2-D/3-D ladder filters, their adaptive control schemes, and variations in coding methods are evaluated by computer simulations on a real sequence and compared to the results of motion compensation and frame differential coders. As a validity test of the ladder filters developed, the error signals for the different predictors are compared and evaluated.

* HQs, ROKA.

1. Introduction

Image sequence processing and dynamic scene analysis are currently exciting application areas of multidimensional system theory. Particularly, digital image coding in the time domain is required in broad applications including teleconferencing, remote sensing, medical image processing, and many others for important industrial/military purposes. One of the major recent developments in time-varying image coding is the use of mathematical models describing the motion of objects. Motion considerations have become more and more important, specifically for low bit-rate coding. These needs force one to improve the current techniques having relatively low efficiency in predictive, transform, and interpolative coding.

Several methods have been proposed for coding of moving images. A simple and original coding method, which has been proposed for video-telephone and video-conference applications, is conditional replenishment [1]. Several modifications of this method are described in a survey by Haskell [2]. Displacement estimation algorithms for TV coding were first proposed in [3]-[4]. Recently developed is a motion compensation technique by Netravali and Robbins [5] where the translational estimate is recursively adjusted at every pel. The performance of the technique with its variations are discussed in [6]-[7].

Another approach to motion considerations is adapative linear prediction using pels in both present and previous frames neighboring the encoded pel and adapting the coefficients to minimize a prediction error function. The techniques of linear predictive coding in spatial/temporal domains exploit statistical correlation within the image data of intra/interframes to permit differential signalling. Adaptive intra/interframe prediction [8] is important in this context. This method is based on previously transmitted reconstructed pels with only the quantized prediction error coded and transmitted. Motivated by this approach, a goal of this paper is to design an efficient algorithm of linear prediction for coding of moving images at low-bit rates by applying a 2-D recursive ladder filter.

2. Three-Dimensional Ladder Filtering for Image Coding

In coding time-varying images for transmission over a digital channel, it is well known that the required bit rate can be significantly reduced by removing various redundancies that exist in the signal. Linear predictive coding is an efficient method of transmitting these sample values. In this predictive method we are interested in finding out the "best" way to make use of previously scanned pels to predict the currently scanned pel. It is known that for a large class of natural scenes, transmission bit rate reduction is more successful with adaptive, combined intra/interframe predictors. Motivated by this, we now devise a 3-D recursive ladder form for coding of moving images in which the algorithm adaptively tracks spatial/temporal variations within a sequence of images.

2.1 3-D Autoregressive Modeling

To begin, we define a discrete, finite, sample process Y as

$$Y = [y(k,j,t); (k,j,t) \in N^3]. \tag{2.1}$$

An image sequence is considered to be a three-dimensional (3-D) random process of intensity elements y(k, j, t) with integer spatial coordinates (k, j) and integer temporal coordinate or frame number t. We use the causal quarter-plane model defined in the previous section in order to implement the 2-D recursive ladder algorithm in the 3-D image sequence case. For recursive filtering of 3-D Markov processes, we must introduce the "past" of such a process. With the definition of the past, we will obtain a 3-D filtering structure representing the sequence of frame differences in the spatial domain. The past of the point y(k, j, t) will be the set (see Fig. 1):

$$\begin{aligned}&[y(k\text{-}1,j,t), y(k,j\text{-}1,t), y(k\text{-}1,j\text{-}1,t), y(k,j,t\text{-}1), y(k\text{-}1,j,t\text{-}1),\\ &y(k,j\text{-}1,t\text{-}1), y(k\text{-}1,j\text{-}1,t\text{-}1)].\end{aligned} \tag{2.2}$$

The 3-D filter is to provide a recursive estimation $\hat{y}(k, j, t)$ of y(k, j, t) with respect to the observations given in (2.2).

Althogh many different previous pels could be used in 3-D filtering, the seven pels in (3.2) are used in this study since they are closely correlated with the currently scanned pel and are necessary to construct a causal quarter-plane AR model of order 1 (i.e., 7 pels is minimum for a 3-D AR model of order 1). The 3-D AR model of order 2 requires 17 pels which could provide better prediction where local stationarity exists, but it more than double of the complexity without a commensurate return. This situation is analogous to 1-D AR filters in which the first lag of the dependent variable has the highest explanatory power.

We thus design an adaptive linear predictor based on the previously scanned pels, which are nearby both spatially and temporally, of the form:

$$\begin{aligned}\hat{y}'(k,j,t) = \ & a(k,j,t;0,1,0)\,[y'(k,j\text{-}1,t) - y'(k,j\text{-}1,t\text{-}1)]\\ &+ a(k,j,t;1,0,0)\,[y'(k\text{-}1,j,t) - y'(k\text{-}1,j,t\text{-}1)]\\ &+ a(k,j,t;1,1,0)\,[y'(k\text{-}1,j\text{-}1,t) - y'(k\text{-}1,j\text{-}1,t\text{-}1)]\\ &+ a(k,j,t;0,0,1)\,y'(k,j,t\text{-}1).\end{aligned} \tag{3.3}$$

where a (;) are the 3-D prediction filter coefficients and y′ (, ,) represent previously transmitted and reconstructed pel values. Note that the coefficients a (?) are variant in both spatial and temporal domains. With no loss of generality we assume a(k,j,t;0,0,1) = 1 in (3.3) and define the frame-difference z(k,j,t) as

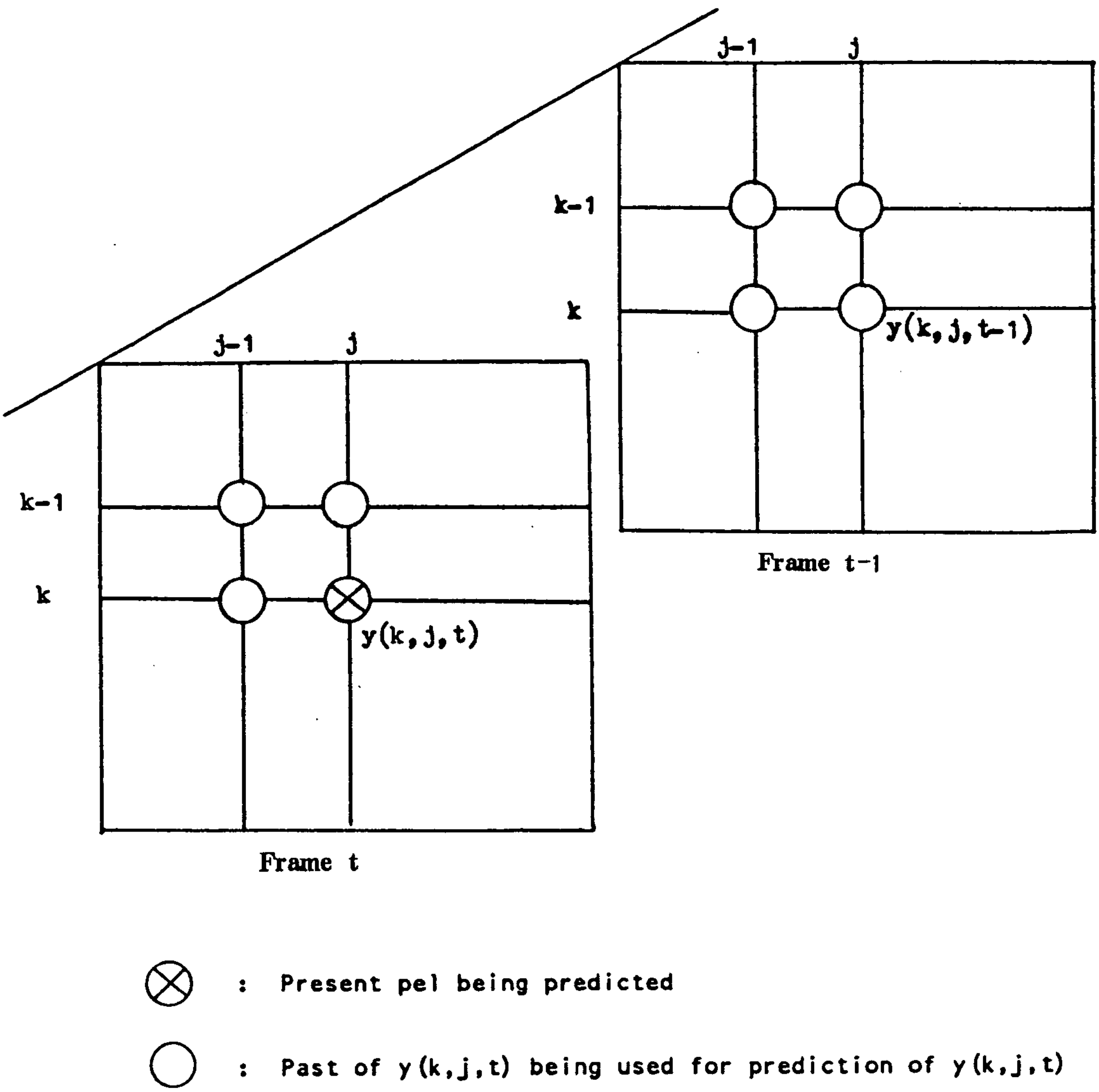


Fig. 1. Configuration of pels for 3-D AR modeling. Past of y(k,j,t) consists of scanned pels, closely correlated with y(k,j,t) in both present and previous frames. These provide a structural basis for 3-D ladder predictor with order one.

$$z(k,j,t) = y'(k,j,t) - y'(k,j,t-1). \tag{2.4}$$

From (2.3) – (2.4) we then have a 3-D frame-difference prediction filter of the form:

$$\hat{z}(k,j,t) = a(k,j,t;1,0,0)z(k-1,k,t) + a(k,j,t;0,1,0)z(k,j-1,t) + a(k,j,t;1,1,0)z(k-1,j-1,t). \tag{2.5}$$

Equation (3.5) is now in the form of an AR model of order 1 in three dimensions. This 3-D AR model will now be used for the design of a ladder filter to make one-step ahead predictions of the frame-differences in all two successive frame pairs.

2.2 3-D Recursive Ladder Filtering Algorithm

We motivate the use of the 2-D recursive least-square ladder algorithm presented in [18] to obtain a one-step ahead prediction of the frame-difference in the pel-domain. Based on (2.5) the prediction error for the frame-difference at pel coordinates (k,j,t) for the first quarter-plane region is given by

$$\begin{aligned} e(k,j,t) &= z(k,j,t) - \hat{z}(k,j,t) \\ &= z(k,j,t) - \sum_{p,q \in Q_1} a(k,j,t;p,q,0)\, z(k-p,j-q,t), \end{aligned} \tag{2.6}$$

where $Q_1 = [(p,q): [(1,0), (0,1), (1,1)]]$. Similarly, the second, third, and fourth quarter-plane prediction errors are, respectively, defined as

$$f(k,j,t) = z(k-1,j,t) - \sum_{p,q \in Q_2} b(k,j,t;p,q,0)\, z(k-p,j-q,t), \tag{2.7a}$$

$$g(k,j,t) = z(k-1,j-1,t) - \sum_{p,q \in Q_3} c(k,j,t;p,q,0)\, z(k-p,j-q,t), \tag{2.7b}$$

$$h(k,j,t) = z(k,j-1,t) - \sum_{p,q \in Q_4} d(k,j,t;p,q,0)\, z(k-p,j-q,t), \tag{2.7c}$$

where

$$Q_2 = [(p,q): (0,0), (0,1), (1,1)], \tag{2.7d}$$

$$Q_3 = [(p,q): (0,0), (1,0), (0,1)], \tag{2.7e}$$

$$Q_4 = [(p,q): (0,0), (1,0), (1,1)]. \tag{2.7f}$$

The calculation of the prediction error for the frame-difference at spatial coordinates (k,j) using pel-by-pel recursions on a given frame follows exactly the procedure of the 2-D recursive ladder case except that the 3-D recursive ladder filter is order one and uses the frame-difference values as the sample data. Based on the 2-D recursive ladder filter, the recursive equation of the frame-difference prediction z(k,j,t) at (k,j,t) is given by

$$\hat{z}(k,j,t) = K^f(k,j,t)f(k-1,j,t) + K^g(k,j,t)g(k-1,j-1,t) + K^h(k,j,t)h(k,j-1,t), \quad (2.8)$$

in which $K^f(k,j,t)$, $K^g(k,j,t)$, and $K^h(k,j,t)$ are reflection factors corresponding to respectively $f(k,j,t)$, $g(k,j,t)$, and $h(k,j,t)$. Note that the frame-difference prediction $\hat{z}(k,j,t)$ is exactly the prediction error for the currently scanned pel at (k,j,t). Consequently, we have an expression for the prediction of the pel y(k,j,t) from the view of the receiver as follows:

$$\hat{y}'(k,j,t) = y'(k,j,t-1) + \hat{z}(k,j,t) \quad (2.9)$$

The 3-D recursive ladder filtering algorithm for an image sequence is summarized as follows:

1. Compute 3-D ladder parameters using the 2-D ladder filter.
2. As current pel (k,j,t) is being processed, y(k,j,t) is received.
3. Compute frame differences at (k,j,t):
   $z(k,j,t) = y(k,j,t) - y'(k,j,t-1)$.
4. Obtain an estimated frame-difference $\hat{z}(k,j,t)$.
5. Use $\hat{z}(k,j,t)$ to predict the current pel:
   $\hat{y}'(k,j,t) = y'(k,j,t-1) + \hat{z}(k,j,t)$.
6. Repeat steps 1 – 5 for all pels in frame t.
7. Repeat steps 1 – 6 for (t+1)th frame.

3. Adaptive Predictor-control Schemes

In this section, we design two types of adaptive predictor selection schemes using 2-D/3-D ladder filtering combined with interframe coding in an attempt to minimize prediction error at each pel location (k,j,t). A frame of a scene in a given image sequence can be segmented into unchanged and changed areas, (a pel is determined to be changed if $|y(k,j,t)-y(k,j,t-1)| > 3$). The unchanged areas consist of a stationary background with very small frame-to-frame differences whereas changed areas are caused by moving objects. Clearly, the best predictor function for unchanged areas is the previous frame predictor. Intraframe prediction is very efficient for changed areas [14] so that several adaptive predictors could be proposed. Here, either a 2-D ladder predictor or a 3-D ladder predictor or a previous frame predictor is selected depending on surrounding signal changes. Variations of the coding schemes are given by an adaptive switching control.

First, we denote the 3-D recursive ladder predictor by

$$\hat{y}1'(k,j,t) = \hat{y}'(k,j,t), \quad (3.1)$$

where $\hat{y}^1(k,j,t)$ is the predicted value from by the 3-D ladder algorithm presented in Section 3 For an intraframe prediction, the 2-D recursive ladder predictor for the 2-D AR process form [18]

$$\hat{y}_2'(k,j,t) = a(k,j,t;0,1,0)y'(k,j-1,t) + a(k,j,t;1,0,0)y'(k-1,j-1,t) + a(k,j,t;1,1,0)y'(k-1,j-1,t) \tag{3.2}$$

is used. The parameter vector a(;) of the 2-D process defined in (3.2) is space-variant in order to track nonstationarities within each intraframe of the image sequence. We let

$$\hat{y}_3'(k,j,t) = y'(k,j,t-1) \tag{3.3}$$

be the previous frame predictor (or the interframe predictor). We now adopt an adaptive control scheme called "the activity function" [15]. The activity function A_i is the sum of magnitudes of the prediction errors for each pel in a small window of neighboring pels closely correlated with the current pel. To avoid the burden of additional control information, the causal neighborhood shown in Fig. 2 is used. The predictor function which gives the smallest activity value is chosen for prediction. The basic selection rule is as follows:

$$\hat{y}^1(k,j,t) = y_i'(k,j,t) \text{ if } A_i = \min(A_1, A_2, A_3) \; i=1,2,3, \tag{3.4}$$

where

$$A_i = \sum_X |y'(X,t) - y_i'(X,t)| \qquad i = 1,2,3 \tag{3.5}$$

and X = [1,2,3,4] as shown in Fig. 2.

The selection of the predictor for a pel from the above three predictors could be made by choosing the one giving the lowest prediction error. This alternative selection rule is as follows:

$$\hat{y}'(k,j,t) = \hat{y}_i^1(k,j,t) \text{ if } E_i = \min(E_1, E_2, E_3) \; i = 1,2,3 \tag{3.6}$$

where

$$E_i = \left| y(k,j,t) - \hat{y}_i'(k,j,t) \right| \; i = 1,2,3. \tag{3.7}$$

In scheme (3.6), the transmitter must send the index of the predictor chosen by the above selection rule. This is an additional transmission requirement but it is negligible since the index is one of three levels. Also, this scheme provides the minimum value of prediction errors for three predictors and the total information for transmission will be small. The predictive coding system block diagram is shown in Fig. 3 where three predictions at every incoming sample are formed: the 2-D ladder prediction, the 3-D ladder prediction, and the previous frame prediction. Both 3-D ladder and previous frame predictors share the same frame memory. By subtracting a prediction value $\hat{y}^1$ from the input sample y, a prediction error e is formed. This prediction error is quantized and coded for transmission.

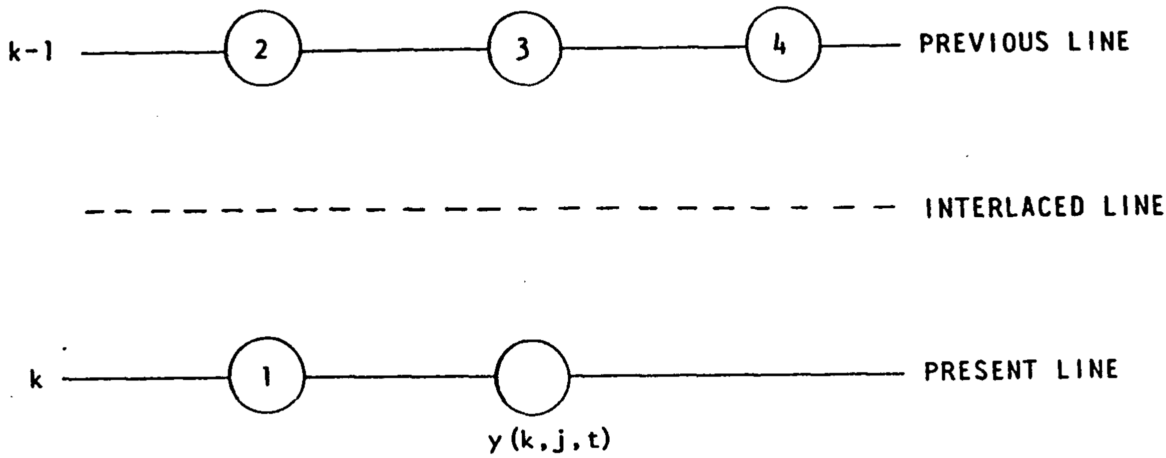


Fig. 2. Causal neighborhood of y(k,j,t) for adaptive predictor-control schemes. Pel number 1, 2, 3, and 4 represent y(k,j-1,t), y(k-1, j-1,t), y(k-1,j,t), and y(k-1,j+1,t), respectively.

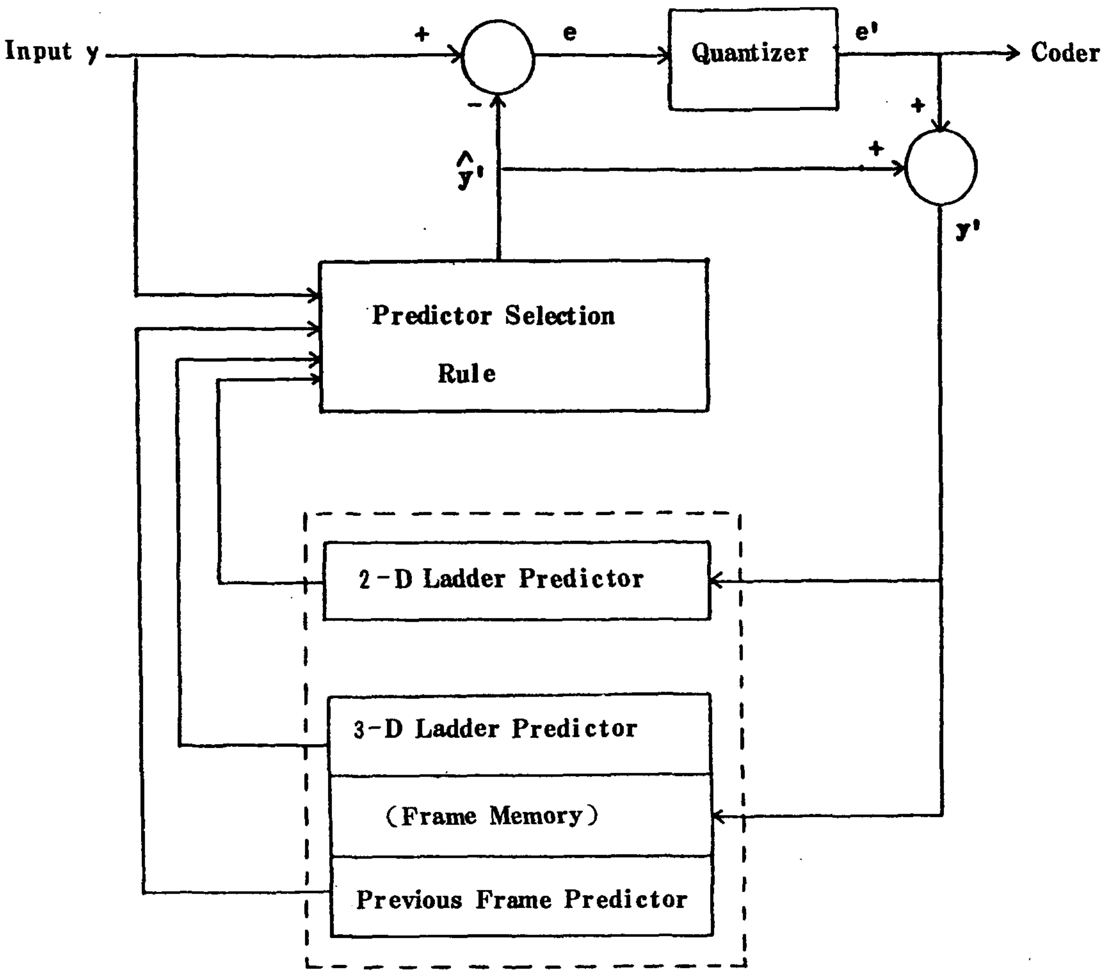


Fig. 3. Block diagram of adaptive ladder predictor-control scheme.

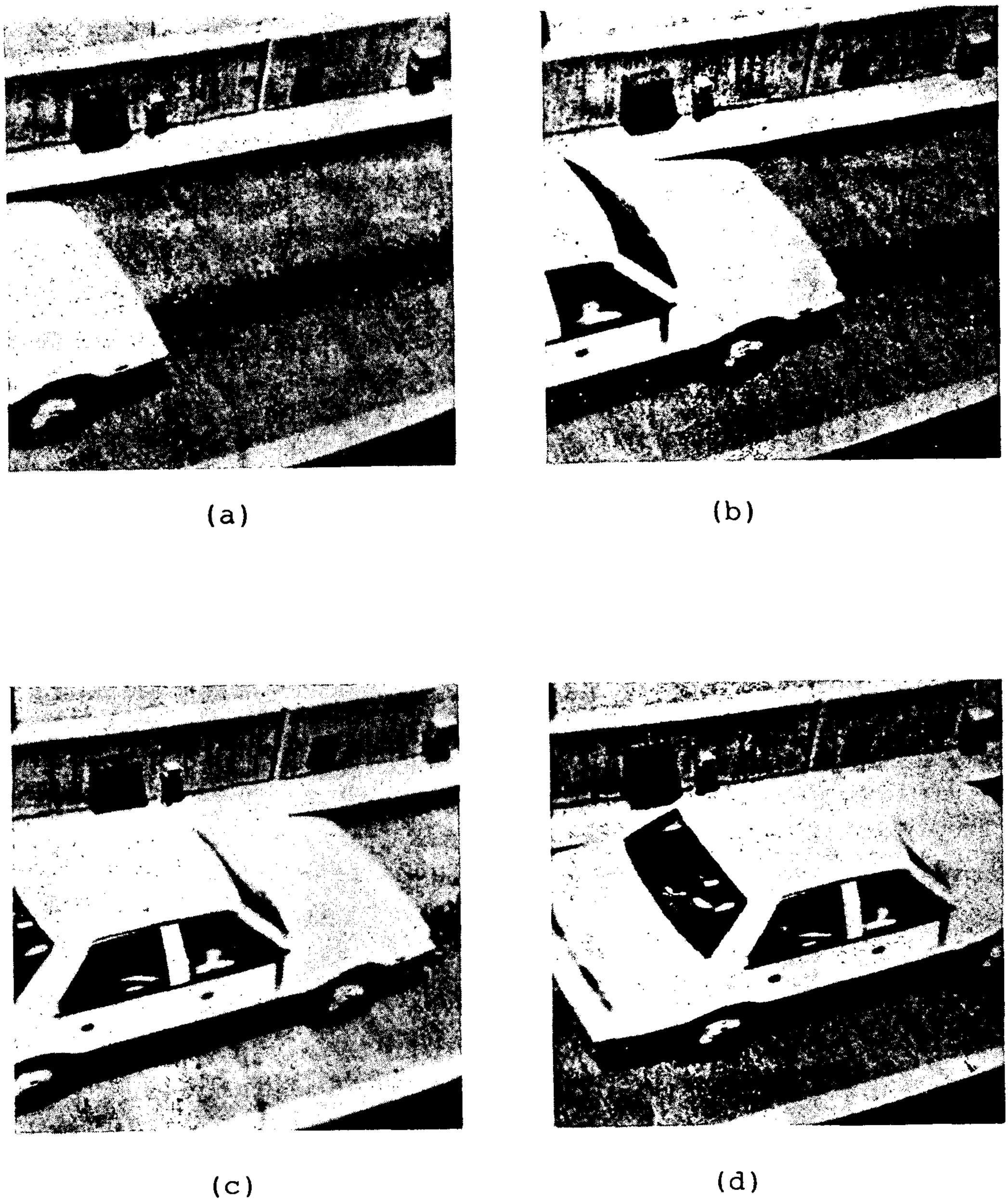


Fig. 4. Four frames of Scene A: (a) Frame 5; (b) Frame 10; (c) Frame 15; (d) Frame 20.

4. Performance Evaluation

A real image sequence was selected for this study. The sequence consists of 30 frames of 256 x 256 samples, obtained at time-intervals of one second. Each sample was quantized to 8 bits. The scence, called "Scene A", is a sequence of images showing one moving car against a stationary background. The percentage of a frame classified as moving area varies from 40 to 83 percent. Four frames from this sequence are shown in Fig. 4.

The simulation includes a comparison of the proposed ladder algorithms and the motion compensation technique in [5]. The comparison of these two approaches is important since the motion compensation technique is currently known as the most promising interframe coder, particularly for reducing the bit-rate for data transmission. The prediction algorithms used in this comparison are given below.

Algorithm I : 2-D recursive ladder algorithm: Eq. (3.2)
Algorithm II : 3-D recursive ladder algorithm: Eq. (3.9)
Algorithm III : Adpative ladder control scheme A: Eq. (3.4)
Algorithm IV : Adpative ladder control scheme B: Eq. (3.6)
Algorithm V : Previous frame prediction: Eq. (3.3)
Algorithm VI : Motion compensated prediction (see [5])

The aim of efficient coding here is to reduce the required transmission rate for a given image quality so as to yield a reduction in transmission costs. Since any of the six coding schemes considered will produce about the same visual quality reproduction, we are primarily concerned with finding an efficient predictor which has the capability of low-bit rate coding of moving images. The choice of the best candidate among the six coders must depend on an objective performance evaluation using some statistical measures. In this simulation the following two measures of performance due to Sabri [7] are used: (1) root mean square value of the prediction error (RMSPE), and (2) entropy of the prediction error to be transmitted.

To significantly reduce bit-rates the prediction error for transmission must be quantized. Good resolution for quantization implies accurate reconstruction of the picture at the receiver. When the prediction error is quantized, the picture quality might be degraded due to the coarse quantizer. Thus the quantizer of the coding system should be designed according to the prediction scheme. For simplification in this simulation, the same 35-level quantizer described below is used for all simulations. The quantizer has the following positive representative levels: 0, 4, 10, 17, 26, 35, 44, 55, 66, 77, 89, 102, 115, 128, 141, 154, 169, 179. The decision levels are always in the middle between two succeeding levels. Other quantizers could be used, however, the relative performance of the coder does not depend heavily on the quantizer [16].

5. Simulation Results

5.1 Basic Coder Performance

Simulation results of the 2-D/3-D ladder algorithms and adaptive ladder control schemes including motion compensation and previous frame prediction are plotted in Fig. 6 and 7 for both Scene A real image sequence was selected for this study. The sequence consists of 30 frames of 256x ladder algorithms show remarkable improvement in the bit-rate reduction compared to the frame differential coder. There is no clear preference qetween the 2-D ladder and 3-D ladder algorithms. This figure also shows a substantial improvement by using the adaptive control schemes based on the 2-D/3-D ladder algorithms. There is a clear preference for adaptive ladder scheme B over scheme A. Motion compensation is comparable to the frame differential, giving 18-20 percent bit rate saving but its performance is obviously weaker than the gain in the ladder algorithm coders. A comparison of performance for all six algorithms under investigation is tabulated in Table 1 and 2. The ladder approach with adaptive control scheme B has clearly the best results, with 65-70 percent less bit rate than motion compensation.

The pictures shown in Fig. 8, photographed from a laboratory monitor, are representations of error signals produced by the six different predictors used for Frame 14 of Scene A. These error images indicate how the different predictors perform at different edge orientations. The significant pels displayed with grey or white dots represent prediction errors for the ladder predictors, moving area for the previous frame predictor, and uncompensable area for the motion compensation method, respectively. In these pictures the pel intensity is almost proportional to the bits required to code the error signal for that pel. The pels with large prediction error for the ladder predictors form a small portion of the moving area and its reduction in the white dot area compared to the non-ladder predictors is clearly seen.

Table 1. Comparison between ladder algorithms and other approaches

Process Type	RMSPE	Entropy, bits/pel
2-D Ladder	1.52	1.78
3-D Ladder	1.48	1.56
Adaptive Ladder A	0.94	1.18
Adaptive Ladder	0.74	0.78
Previous Frame	3.71	2.75
Motion Compensation	2.55	2.19

Image sequnce : Scene A
Transmitted signals: Quantized pridiction errors

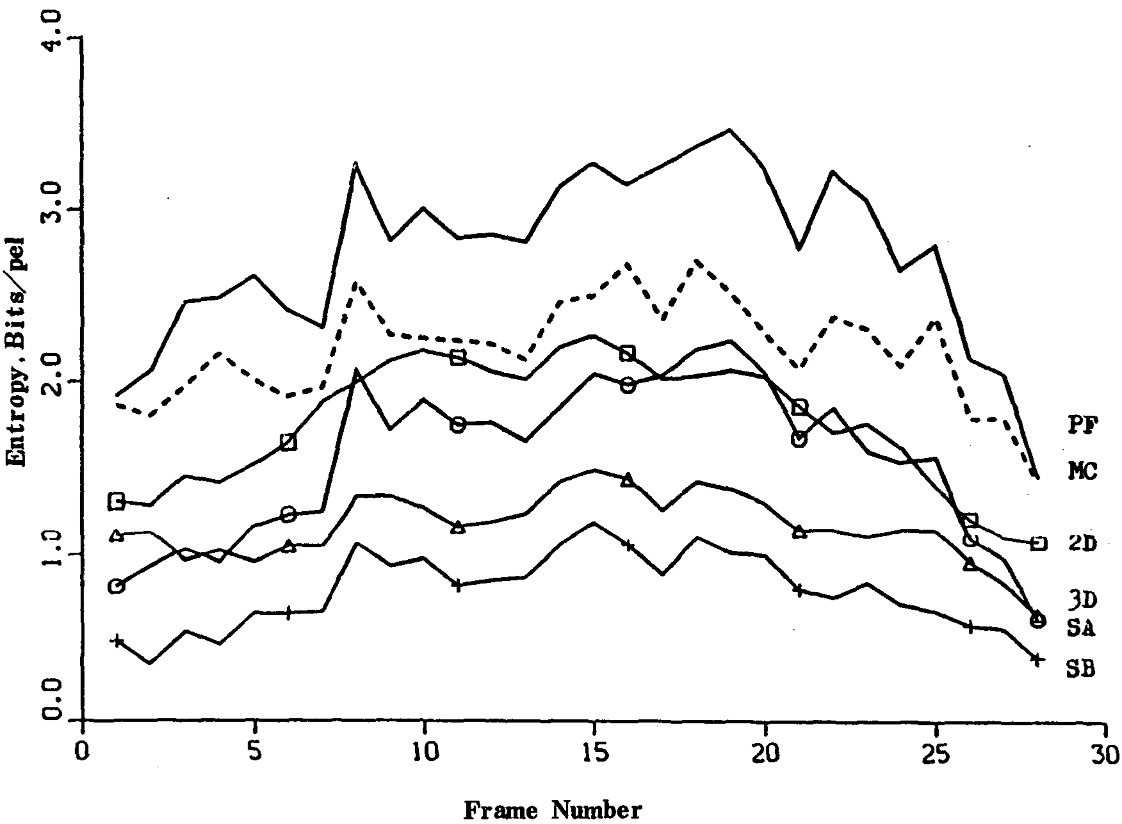


Fig. 5. Entropy for six different predictors (Scene A).
Transmitted signals: Quantized prediction errors.

5.2 Variations in Predictive Coding Methods

We now modify the linear predictive coding technique used so far and evaluate the effect of the two modifications on the coder structure to insure better efficiency in coding performance. In differential coding techniques line or element differencing of the prediction errors can be used with advantage [5]. Some modifications can be made to all of the predictors considered. Here we choose element differencing of prediction errors. For instance, the ladder filtering coder would send $[e(k,j,t)-e(k,j-1,t)]$, where $e(k,j,t)=y(k,j,t)-\hat{y}(k,j,t)$ and $\hat{y}(k,j,t)$ is a one-step ahead prediction of $y(k,j,t)$ via the ladder predictor.

The second modification in coding structure is element differencing of the original sample data. The baseline of this is that the nonstationary character of the process can be represented by a model which calls for the d'th difference of the process to be stationary [17]. For simplicity, and due to the line-by-line processing scheme, the first difference of sample values along the horizontal lines was used as input for the predictors used. Line differencing was also examined but was less effective than element differencing.

The performance of these two modifications for the predictors was evaluated in terms of the RMSPE and entropy for Scene A. The results showed that element differencing of the prediction errors remarkedly improves ladder predictors in bit rate saving (while maintaining good visual quality). The performance of the second modification for the predictors indicated that element differencing of the sample data also improves ladder predictors in terms of entropy. Note that this modification resulted in slight distortion in the reconstructed pictures due to quantization noise. This problem may be overcome by the use of a properly designed quantizer.

5.3 Summary of the Simulation Results

It is informative to compare the results for the predictors according to three different coding structures. The corresponding performance in term of entropy is plotted in Fig. 7 for Scene A. Some observations can be made from this figur:,(i) For all three coder types, the ladder predictors used dominate both motion compensation and frame differential coders in terms of bit rate saving, (ii) the use of two types of coding variation significantly improves the performance of all the ladder approaches, and (iii) the bit-rate difference between two coding variations is small for the ladder predictors.

Several concluions can be drawn from the results discussed so far.

1. Substantial improvements in bit-rate saving over a motion compensation coder were obtained by the use of adpative control schemes of 2-D/3-D recursive ladder algorithms with coding

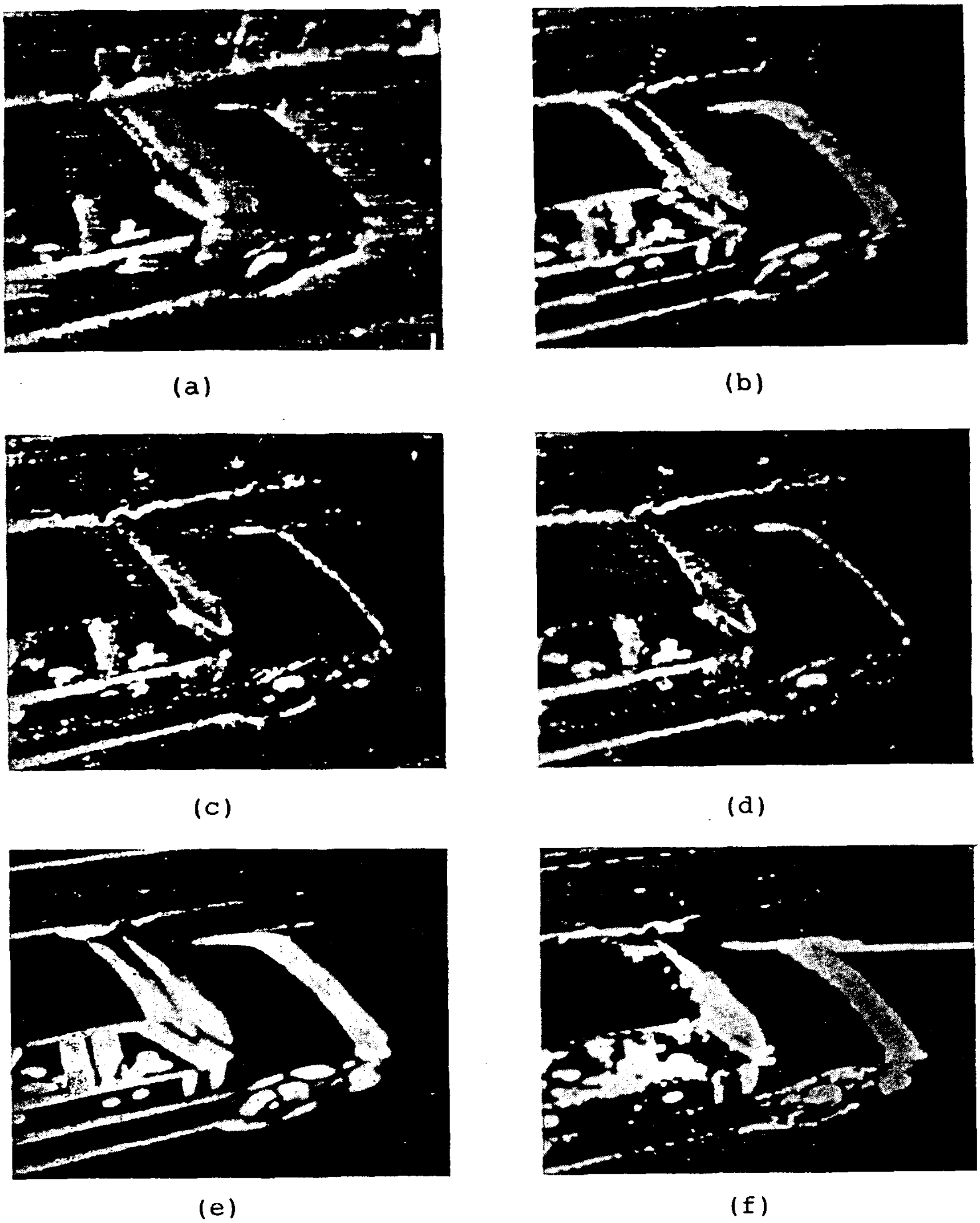
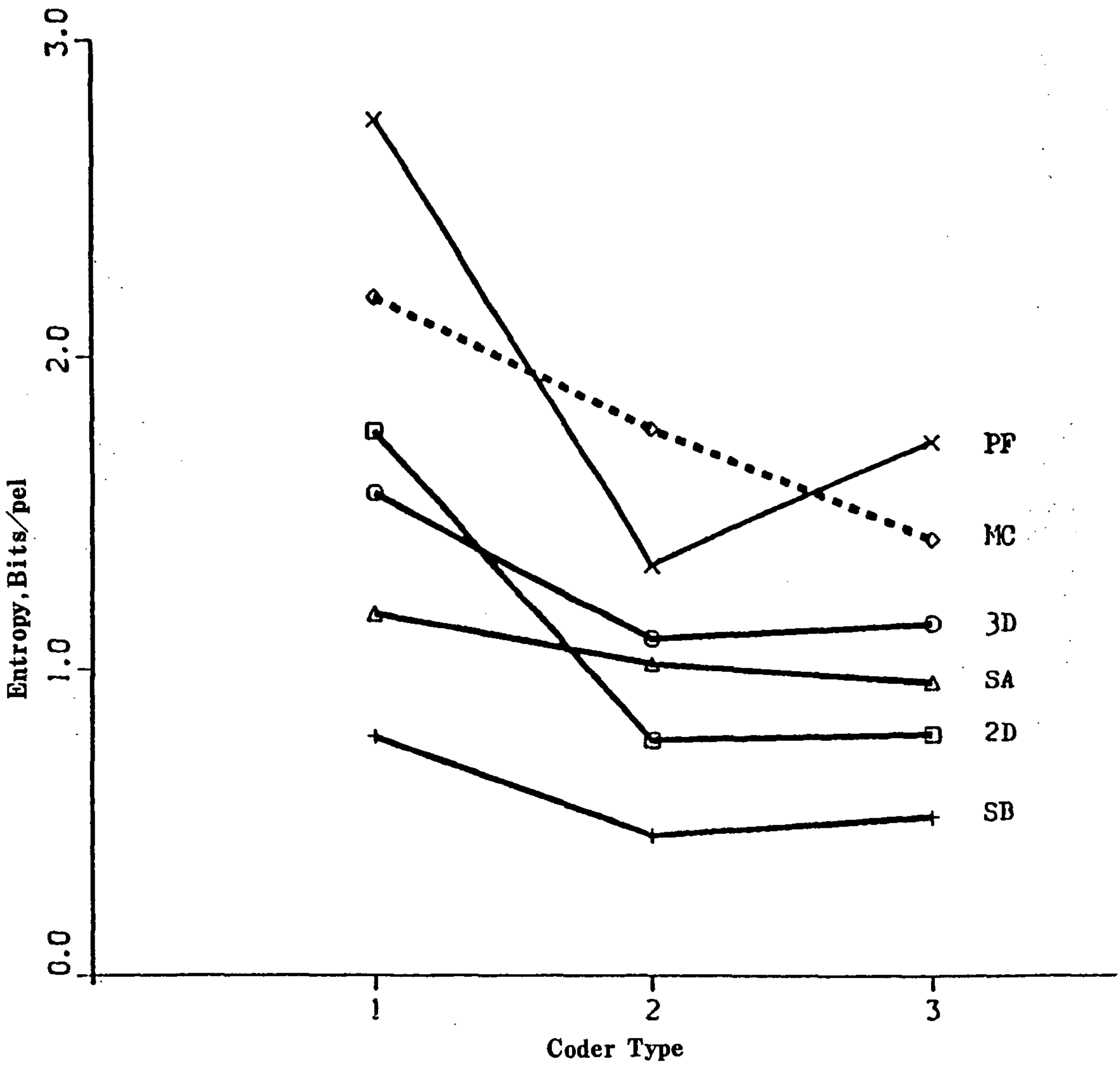


Fig. 6. Error images for six different predictors: Frame 14 of Scene A.
(a) 2-D ladder; (b) 3-D ladder; (c) Adaptive ladder scheme A; (d) Adaptive ladder scheme B; (e) Previous frame; (f) Motion compensation.



Coder type 1 : Basic coding method
Coder type 2 : Element-differencing of prediction errors
Coder type 3 : Element-differencing of sample data

Fig. 7. Comparison of entropy for three coder types with six different predictors (Scene A).

variations.

2. Regarding coding variations, element differencing of the prediction errors is preferable to element differencing of the sample data because of the higher quality of reconstructed images.

3. Adaptive ladder predictor-control scheme B using element differencing of the prediction errors gave the best results.

4. Taking algorithm complexity into consideration, the 2-D ladder algorithm using the first coding variation is the most attractive one among the coders investigated.

6. Concluding Remarks

In this paper an extension of recursive ladder filters to three dimensions was made and its computational performace was verified via an application of low-bit-rate coding of time-varying images. Two types of adpative predictor-control schemes were also proposed for improving an image encoding system with a quantizer. The simulation results using a real sequence indicated that, compared to motion compensation techniques, the performance of all ladder predictors demonstrates a significant improvement in bit rate while maintaining good visual quality. Two modifications of the basic coding structure provided a more substantial reduction in entropy due to ladder predictors.

Although the effect of the proposed ladder approaches was confirmed through simulations in this paper, more efforts must be made to bring them into actual application. There are several areas for further investigation which may prove both novel and significant: adaptive quantizer design, variable length encoding, variations of the filtering structure in both the 2-D and 3-D ladder schemes, reduction of the ladder algorithm complexity, and design of algorithms with the capability of reducing noise in moving images.

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