Jour. Korean Inst. Mining Geol.
Vol. 19, No. 3, p. 193-197, 1986

Time-Domain Electromagnetic Coupling in Induced Polarization Surveys on a Uniform Earth

Hee Joon Kim*

Abstract: A simple and fast solution is derived to evaluate the effects of time-domain electromagnetic coupling in induced polarization surveys on a uniform earth. The simplified solution gives an explicit statement of the dependence of time-domain electromagnetic coupling on the model parameters, and yields sufficiently accurate results for most situations encountered in practice. The co-linear dipole-dipole and Wenner arrays are used as examples in this paper, but the numerical solution can be applied to any electrode configuration.

INTRODUCTION

One of the major problems in interpretation of induced polarization (IP) data is spurious responses caused by electromagnetic (EM) coupling. For a co-linear dipole-dipole array over conductive earth, both EM coupling and polarizable material have effects to decrease the apparent resistivity as the frequency is increased.

Dey and Morrison (1973) developed a method of computing EM coupling over a layered earth in the frequency domain, and they also estimated time-domain coupling using the fast Fourier transform. However, especially in the time domain, their method requires a relatively complex procedure and a large computer time. In this paper, a fast and simple procedure is derived to estimate the time-domain EM coupling on a uniform earth.

The starting point of this study is the coupling response between parallel lines on a uniform earth given by Yost (1952). Since the Yost's solution has a problem of indeterminate in colinear arrays, it is modified to able to apply to any electrode configuration in this paper. The time-domain EM coupling is studied by computing the voltage recorded at a receiving line during the off-time of current puls applied to a transmitting line. In this paper, the co-linear dipole-dipole and Wenner arrays are used as examples, and periodic square-wave and periodic alternating square-wave currents are assumed.

* Department of Applied Geology, National Fisheries University of Pusan

EM COUPLING

The response of time-domain EM coupling for parallel lines on a uniform earth is given by Yost (1952). The coupling response is valid for the quasi-static case in which capacitive earth currents are considered to be negligible compared to conductive earth currents and the impressed current is considered to be constant along the length of line. Fig. 1 shows the line are parallel to the x-axis and two are separated by a distance y in the y-axis. For this case, a voltage $V(t)$ measured at the receiving line is

$$V(t)=V_0-\frac{\rho I}{2\pi y^2}\sum_{i=1}^{2}\sum_{j=1}^{2}(-1)^{i+j}\left[r_{ij}\mathrm{erf}(gr_{ij})-\exp(-g^2y^2)x_{ij}\mathrm{erf}(gx_{ij})\right], \quad (1)$$

where V_0 is the steady-state value of $V(t)$, I the magnitude of current, ρ the resistivity of uniform earth, $g^2=m_0/(4\rho t)$, m_0 the inductivity of free space, and t the time measured from the

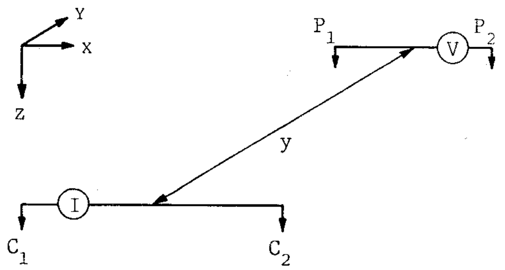

Fig. 1 Line arrangement for Yost's model (1952).

instant the current is stepped. The distances x_{ij} and r_{ij} are respectively

$$x_{ij}=|C_i-P_j|, \quad (2)$$

and

$$r_{ij}=(y^2+x_{ij}^2)^{1/2}, \quad (3)$$

$i=1$ and 2, and $j=1$ and 2,

where C and P denote positions of current and potential electrode, respectively, and the error function erf(z) is defined by

$$\mathrm{erf}(z)=\frac{2}{\sqrt{\pi}}\int_0^z \exp(-a^2)\ da. \quad (4)$$

In applying (1), there are no restrictions on the geometric arrangement of lines, except that the current carrying line must be parallel to voltage measuring line. However, in co-linear cases such as Wenner and dipole-dipole arrays, where $y=0$, (1) becomes indeterminate. The difficulty can be overcome by a series expansion of the error function terms in (1). The series expansion of the error function is given by (Abramowitz and Stegun, 1972, p. 297)

$$\mathrm{erf}(z)=\frac{2}{\sqrt{\pi}}\exp(-z^2)\sum_{k=0}^{\infty}\frac{2^k z^{2k+1}}{1\cdot 3\cdot\cdots\cdot(2k+1)}. \quad (5)$$

A substitution of (5) into (1) yields

$$V(t)=V_0-\frac{\rho I}{\pi^{3/2}y^2}\sum_{i=1}^{2}\sum_{j=1}^{2}(-1)^{i+j}\exp(g^2 r_2{}^{ij})$$

$$\sum_{k=0}^{\infty}\frac{2^k g^{2k+1}}{1\cdot 3\cdot\cdots\cdot(2k+1)}[r_{ij}^{2(k+1)}-x_{ij}^{2(k+1)}]$$

$$=V_0-\frac{\rho Ig}{\pi^{3/2}}\sum_{i=1}^{2}\sum_{j=1}^{2}(-1)^{i+j}\exp(-g^2 r_2{}^{ij})$$

$$[1+\frac{2}{3}g^2(2x_{ij}^2+y)$$

$$+\frac{4}{15}g^4(3x_{ij}^4+3x_{ij}^2y^2+y^4)+\cdots]. \quad (6)$$

When $y=0$, (6) is

$$V(t)=V_0-\frac{\rho Ig}{\pi^{3/2}}\sum_{i=1}^{2}\sum_{j=1}^{2}(-1)^{i+j}\exp(-g^2x_{ij}^2)$$

$$[1+\frac{4}{3}g^2x_{ij}^2+\frac{4}{5}g^4x_{ij}^4+\cdots]. \quad (7)$$

By using (7), one can evaluate the time-domain EM coupling for all co-linear arrays.

In order to obtain a more simplified solution, let's expand the exponential function term in (7):

$$V(t)=V_0-\frac{\rho Ig}{\pi^{3/2}}\sum_{i=1}^{2}\sum_{j=1}^{2}(-1)^{i+j}$$

$$[1+\frac{1}{3}g^2x_{ij}^2-\frac{1}{30}g^4x_{ij}^4+\cdots]. \quad (8)$$

Since g is small enough in most practical cases, the terms higher than g^2 are negligible, i.e.,

$$V(t)=V_0-\frac{\rho Ig}{\pi^{3/2}}\sum_{i=1}^{2}\sum_{j=1}^{2}(-1)^{i+j}[1+\frac{1}{3}g^2x_{ij}^2]. \quad (9)$$

The effect of coupling response can be shown by a relative decay voltage, which is a normalized voltage recorded at the receiving line during the on- and off-time of the current pulse applied to the transmitting line (Dey and Morrison, 1973). A ratio of transient off-time response to steady-state on-time response, $R(t)$, indicates a coupling effect of the underlying homogeneous half-space, i.e.,

$$R(t)=[V(t)-V_0]/V_0. \quad (10)$$

Fig. 2 shows typical waveforms used in the time-domain IP measurement (Sumner, 1976). For the periodic square-wave current, the relative decay voltage $V_d(t)$ is written by

$$V_d(t)=\sum_{k=0}^{K}[-R(t+kT)+R(t+(k+\frac{1}{2})T)], \quad (11)$$

and for the periodic alternating square-wave current,

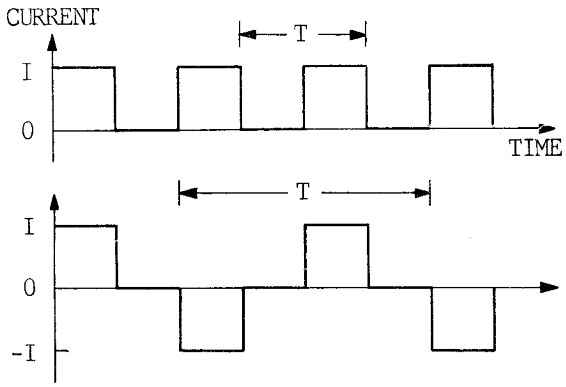

Fig. 2 Current waveforms used in time domain IP measurements: Square-wave current (upper) and alternating square-wave current (lower).

$$V_d(t)=\sum_{k=0}^{K}[-R(t+kT)+R(t+(k+\frac{1}{4})T)]$$
$$+R(t+(k+\frac{2}{4}T)-R(t+(k+\frac{3}{4}T)], \quad (12)$$

where T is the current period. In (11) and (12), $K=2$ is adequate in most practical cases.

RELATIVE DECAY VOLTAGE

In this section, the simplified solutions are derived for the co-linear dipole-dipole and Wenner arrays as shown in Fig. 3, and a few numerical results are shown.

Dipole-dipole array:

For the co-linear dipole-dipole array shown in Fig. 3, the distances between electrodes are respectively

$x_{11}=x_{22}=(n+1)a$,

$x_{12}=(n+2)a$,

and

$$x_{21}=na, \quad (13)$$

and the steady-state voltage V_0 is

$$V_0=-I\rho/[\pi an(n+1)(n+2)], \quad (14)$$

where a is the dipole length and n is the dipole separation. By using (9), (13) and (14), (10) can be written as

$$R(t)=bn(n+1)(n+2)a^3/(\rho t)^{3/2}, \quad (15)$$

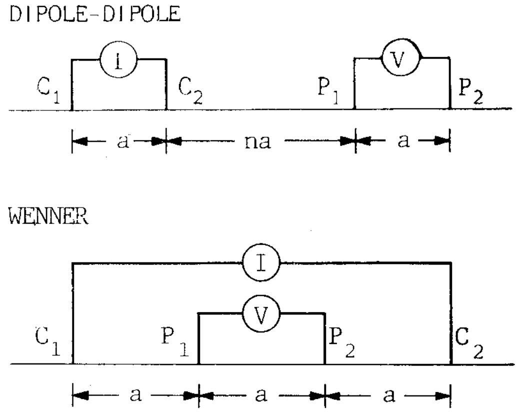

Fig. 3 Array configurations: Dipole-dipole array (upper) and Wenner array (lower).

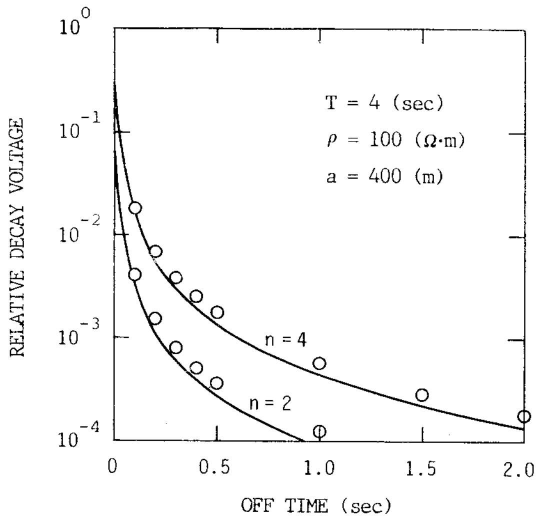

Fig. 4 Relative decay voltages for a periodic square-wave current on a uniform earth. Open circles indicate the results of Dey and Morrison (1973).

where

$$b=-2\pi/(3\sqrt{10})\times10^{-10}=-0.6623\times10^{-10}.$$

From (15), one can easily find the dependence of $R(t)$ on n, a, ρ and t for the co-linear dipole-dipole array.

The relative decay voltage $V_d(t)$ for the periodic squarewave current, for example, is computed by substituting (15) into (11). Fig. 4 shows $V_d(t)$ for the case considered by Dey and Morrison (1973). A fairly good agreement with the Dey and Morrison's result suggests that (15) has a reasonable accuracy. In the following, three

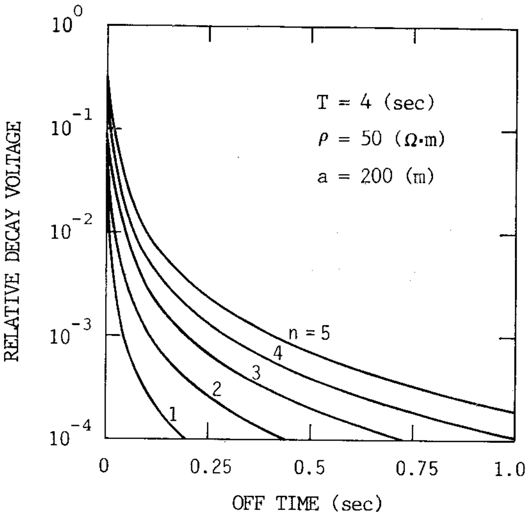

Fig. 5 Relative decay voltages for different dipole separations (n=1, 2, 3, 4 and 5) over a uniform earth.

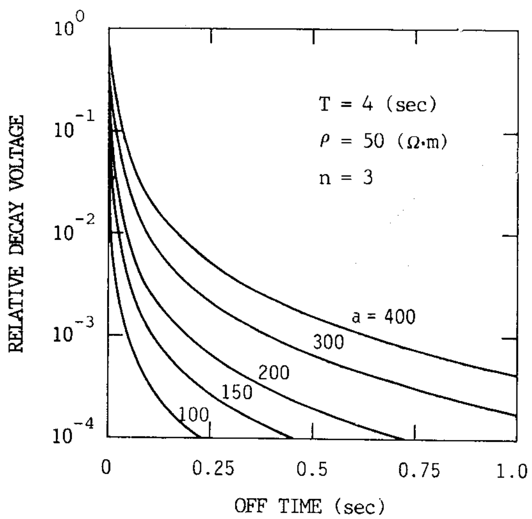

Fig. 6 Relative decay voltages for different dipole lengths (a=100, 150, 200, 300 and 400 m) over a uniform earth.

examples are shown to understand the effects of n, a and ρ to the responses of EM coupling. These results are computed for the periodic alternating square-wave current which is most usually used in time-domain IP measurements (Sumner, 1976).

Fig. 5 shows the effect of dipole separations (n=1, 2, 3, 4 and 5) to the time-domain EM coupling over a uniform earth of resistivity ρ=

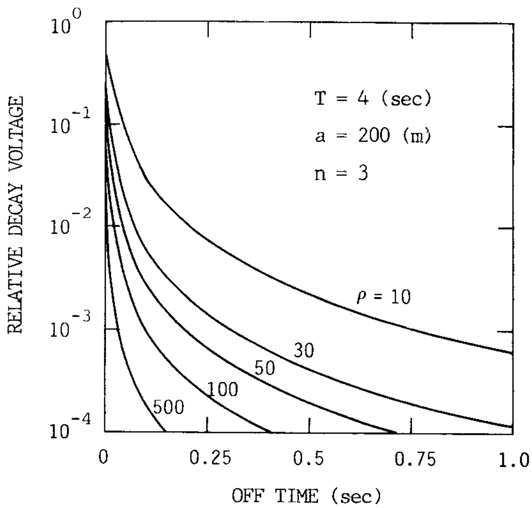

Fig. 7 Relative decay voltages for different resistivities (ρ=10, 30, 50, 100 and 500 $\Omega\cdot m$) of half-space.

50 $\Omega\cdot m$ for a dipole length a=200m. As one normally expects, the coupling responses are higier for larger dipole separations. Especially in short off-times, the shorter the dipole separation, the sharper the decay rate.

Fig. 6 shows the effect of dipole lengths (a= 100, 150, 200, 300 and 400 m) to the time-domain EM coupling over a uniform earth of resistivity ρ=50 $\Omega\cdot m$ for a dipole separation n= 3. The increase in the coupling effect is significant as the dipole length is increased from 100 m to 400 m. The selection of a proper value of a is of prime importance in IP survey, because the larger the a, the greater the depth of exploration. However, Fig. 6 indicates that for a normally resistive ground, the coupling response with large a at long off-times overshadow the normal polarization decay of a mineralized inhomogeneity.

Fig. 7 shows the effect of resistivities (ρ=10, 30, 50, 100 and 500 $\Omega\cdot m$) of the uniform half-space to the EM coupling for a dipole length a=200 m and a dipole separation n=3. The relative decay voltage decreases as the resistivity of half-space is increased. While the relative sharpness of the decay in short off-time (shorter than 0.5 sec) increases with an increase in the resistivity of half-space.

Wenner array:

For the Wenner array shown in Fig. 3, the steady-state voltage V_0 is

$$V_0 = I\rho/(2\pi a). \quad (16)$$

Substitution of (9) and (16) into (10) yield

$$R(t) = ca^3/(\rho t)^{3/2}, \quad (17)$$

where

$$c = 4\pi/\sqrt{10}\times 10^{-10} = 3.9738\times 10^{-10}.$$

It should be noted that (17) gives an explicit statement of the dependence of $R(t)$ on a, ρ and t, and it is remarkably simple. Since (15) and (17) are very similar to each other, the time-domain EM response for the Wenner array can easily be deduced by that for dipole-dipole array. Hence the EM response for the Wenner array is not shown in this paper.

CONCLUSIONS

A simple and fast solution to estimate the time-domain EM coupling between parallel lines on a uniform earth has been developed by modifying the Yost's solution (1952). The Yost's solution is inadequate for the co-linear array, but the solution presented here can be used for any array. In addition, the simplified solutions (15) and (17) gives an explicit statement of the dependence of EM coupling on model parameters, and yield sufficiently accurate results for most situations encountered in practice. The results for the periodic square-wave current are in good agreement with the results of Dey and Morrison (1973).

ACKNOWLEDGEMENTS

I wish to thank to Dr. Y.Q. Kang for his careful reading of the manuscript. A part of this study was carried out while I was visiting the University of Hawaii under sponsorship by the Korea Science and Engineering Foundation.

REFERENCES

Abramowitz, M. and Stegun, I.A. (1972) Handbook of Mathematical Functions. Dover, 1046pp.

Dey, A. and Morrison, H.F. (1973) Electromagnetic coupling in freguency and time-domain induced-polarization surveys over a multilayered earth. Geophysics, v.38, p. 380-405.

Sumner, J.S. (1976) Principles of Induced Polarization for Geophysical Exploration. Elsevier, 277pp.

Yost, W.J. (1952) The interpretation of electromagnetic reflection data in geophysical exploration: Part I, general theory. Geophysics, v. 17, p. 89-106.

균질대지에 대한 시간영역 유도분극법의 전자기결합

김 희 준

요약 : 균질대지에서 시간영역 유도분극탐사를 실시할 때 생기는 전자기결합의 영향을 간단히 계산할 수 있는 해를 구하였다. 여기서 유도한 근사해는 시간영역의 전자기결합에 모델변수가 얼마 만큼영향을 미치는지를 쉽게 이해할 수 있으며, 또한 실제 상황에서 충분히 사용할 수 있는 정확도를 가지고 있다. 본논문에서는 쌍극자 및 Wenner 배치만을 다루웠지만, 이 근사해는 모든 전극배치에 대하여 유용하다.