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Electromagnetic Responses for a Vertical Coaxial Loop System over a Layered Earth

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Abstract: A modified expression of mutual coupling ratio for a vertical coaxial loop system over a layered earth is derived by transforming the Bessel function of order 0 to that of order 1. This expression allows us to use a single J_1 filter instead of a combination of J_0 and J_1 filters, and produces a significant reduction of computer time and core storage. In this paper, the mutual coupling ratio for a three-layered earth is obtained by means of Anderson's adaptive filter (Anderson, 1979).

INTRODUCTION

Since a digital linear filter method was introduced by Ghosh (1971a, b) in order to estimate Hankel transform integrals, many applications of it were found in geophysical problems. The evaluation of numerical convolution using predetermined filter coefficients is approximately an order of magnitude faster than the direct integration of Hankel transform. In fact, the convolution technique completely avoids Bessel function evaluations which are required in the direct integration.

Many integral transforms encountered in electromagnetic (EM) soundings are the Hankel transforms of order 0 and 1. In particular, the EM mutual coupling ratio for a vertical coaxial loop system over a layered earth has both 0th and 1st order Hankel transforms. Thus the evaluation of mutual coupling ratio needs two kinds of filter: J_0 and J_1 filters (Koefoed *et al.*, 1972; Verma, 1977; Anderson, 1979).

Any Hankel transform of order 0 can be transformed into its corresponding Hankel transform of order 1 (Das, 1984; Kim, 1985). This domain transform eliminates the need for J_0 filter in the computation of mutual coupling ratio. In this paper, a modified expression for the mutual coupling ratio, which allows us to use a single filter instead of a combination of two, is obtained by using the domain transform. A few advantages of the modified expression are shown through the numerical evaluation of mutual coupling ratio for a three-layered earth model by means of Anderson's adaptive filter (Anderson, 1979).

DOMAIN TRANSFORM

Any J_0 domain Hankel transform integral can be transformed into its corresponding J_1 domain (Das, 1984; Kim, 1985). Consider a Hankel transform integral

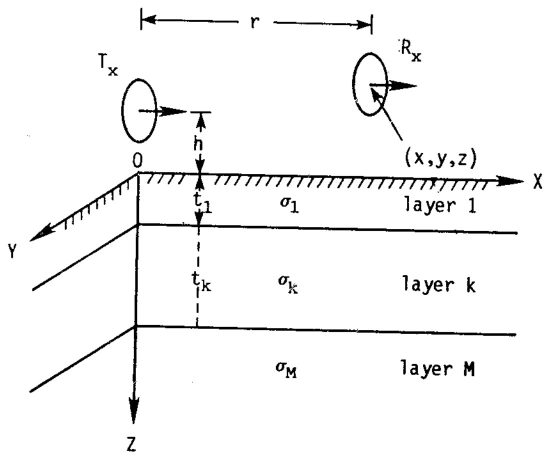

Fig. 1 Vertical coaxial loop configuration in the calculation of mutual coupling ratio.

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$$H(b)=\int_0^\infty k(g)gJ_0(bg)dg, \qquad (1)$$

where J_0 is the Bessel function of order 0, and $k(g)$ is the kernel function of Hankel transform. Applying Hankel transformation (Watson, 1962) to (1) yields

$$k(g)=\int_0^\infty H(b)bJ_0(bg)db. \qquad (2)$$

Differentiate both sides of (2) with respect to g to obtain

$$\frac{dk(g)}{dg}=-\int_0^\infty H(b)b^2J_1(bg)db, \qquad (3)$$

and apply Hankel inversion to (3) to obtain

$$H(b)=\int_0^\infty \left[-\frac{1}{b}\frac{dk(g)}{dg}\right]gJ_1(bg)dg, \quad (4)$$

where J_1 is the Bessel function of order 1. A comparison between (1) and (4) shows that the J_0 domain Hankel transform integral can be transformed into its corresponding J_1 domain Hankel integral.

MUTUAL COUPLING RATIO

The EM mutual coupling ratio for a vertical coaxial loop system, Z/Z_0, over a horizontally layered earth is given by (Frishknecht, 1967; Koefoed et al., 1972)

$$\frac{Z}{Z_0}=1+\frac{B^2}{2}\left[\int_0^\infty E(g)gJ_1(gb)dg - B\int_0^\infty E(g)g^2J_0(gB)dg\right], \qquad (5)$$

where B $(=r/\delta)$ is the coil separation normalized by skin depth (Fig. 1) and $E(g)$ is the EM kernel, i.e.,

$$E(g)=R(g)\exp(-gA), \qquad (6)$$

where $R(g)$ is a recursively defined complex function, expressed in terms of the M-layer ($M>1$) model parameters, of the form

$$R(g)=(V_1F_1-g)/(V_1F_1+g), \qquad (7)$$

with V_1 and F_1 given by backward recurrence using

$$V_k=\sqrt{g^2+2i\sigma_k/\sigma_1}, \quad k=M,\ M-1,\cdots,1 \quad (8)$$

$$F_{k-1}=[(V_{k-1}+V_kF_k)-(V_{k-1}-V_kF_k)\exp(-V_{k-1}d_{k-1})]/[(V_{k-1}+V_kF_k)+(V_{k-1}-V_kF_k)\exp(-V_{k-1}d_{k-1})], \qquad (9)$$

$$k=M,\ M-1,\cdots,2;\ F_M=1,$$

where

$i=\sqrt{-1}$

σ_k =conductivity of layer k [mho/m],

$d_k=2t_k/\delta$,

t_k =thinkness of layer k [m],

$\delta=\sqrt{2/(\omega\mu_0\sigma_1)}$; skin depth,

$\omega=2\pi f$; $f>0$ [Hz],

$\mu_0=4\pi\times10^{-7}$ [H/m],

$B=r/\delta$,

$r=\sqrt{x^2+y^2}$,

$A=(z+h)/\delta$,

(x,y,z)=receiver loop center,

$(0,0,h)$=transmitter loop center.

Using the domain transform described in the previous section, (5) is rewritten as

$$\frac{Z}{Z_0}=1+B^2\int_0^\infty Q(g)gJ_1(gB)dg, \qquad (10)$$

where

$$Q(g)=E(g)+\frac{g}{2}\frac{dE(g)}{dg}. \qquad (11)$$

with

$$\frac{dE(g)}{dg}=\left[\frac{dR(g)}{dg}-AR(g)\right]\exp(-Ag), \qquad (12)$$

$$\frac{dR(g)}{dg}=2\frac{g(V_1'F_1-V_1F_1')-V_1F_1}{(V_1F_1+g)^2} \qquad (13)$$

$$V_k'=g/V_k, \quad k=M,M-1,\cdots,1 \qquad (14)$$

$$F_{k-1}'=2\exp(-V_{k-1}d_{k-1})\{V_{k-1}[d_{k-1}(V_{k-1}+V_kF_k)(V_{k-1}-V_kF_k)-2V_kF_k]+2V_{k-1}(V_k'F_k+V_kF_k')]\}/[(V_{k-1}+V_kF_k)+(V_{k-1}-V_kF_k)\exp(-V_{k-1}d_{k-1})]^2, \qquad (15)$$

$$m=M,M-1,\cdots,2;\ F_M'=0,$$

where prime denotes the derivative with respect to g.

For the case of ground loops, $A=0$, the mutual coupling ratio is also given by (Frischknecht, 1967; Anderson, 1979)

$$\frac{Z}{Z_0}=\left(\frac{Z}{Z_0}\right)_h+\frac{B^2}{2}\left[\int_0^\infty G(g)gJ_1(gB)dg\right.$$

$$-B\int_0^\infty G(g)g^2J_0(gB)dg\Big], \quad (16)$$

where $(Z/Z_0)_h$ is the mutual coupling ratio for the homogeneous earth (Frischknecht, 1967; Wait, 1955):

$$\left(\frac{Z}{Z_0}\right)_h=[12+12kr+5k^2r^2+k^3r^3]\frac{e^{-kr}}{k^2r^2}+2-\frac{12}{k^2r^2}, \quad (17)$$

with

$$k^2r^2=i\omega\mu_0\sigma_1r^2=2iB^2, \quad (18)$$

and

$$G(g)=2gV_1(F_1-1)/[(V_1+g)(V_1F_1+g)]. \quad (19)$$

By using the domain transform, (16) is rewritten as

$$\frac{Z}{Z_0}=\left(\frac{Z}{Z_0}\right)_h+B^2\int_0^\infty H(g)gJ_1(gB)dg, \quad (20)$$

where

$$H(g)=G(g)+\frac{g}{2}\frac{dG(g)}{dg}, \quad (21)$$

and

$$\frac{dG(g)}{dg}=2[gV_1(V_1+g)^2F_1' +(F_1-1)(V_1^2F_1-g)(V_1-gV_1')] /[(V_1+g)^2(V_1F_1+g)^2]. \quad (22)$$

NUMERICAL RESULTS

In order to confirm advantages of the unifying procedure described in the previous section, which requires J_1 filter only, a three-layered earth model is tested by using Anderson's adaptive filter (Anderson, 1979). The model used is an intermediate conductive layer medel:

$$\sigma_2/\sigma_1=20,\ \sigma_3/\sigma_1=1,\ t_2/t_1=1/20.$$

Figs. 2 and 3 show the amplitude and phase of the mutual coupling ratio, respectively. These illustrations contain curves for parametric and geometric soundings. In this model, numerical differences between the results evaluated by (5) and (10) are less than 0.001%.

Table 1 compares the numbers of computation of the kernel functions in (5) and (10). For

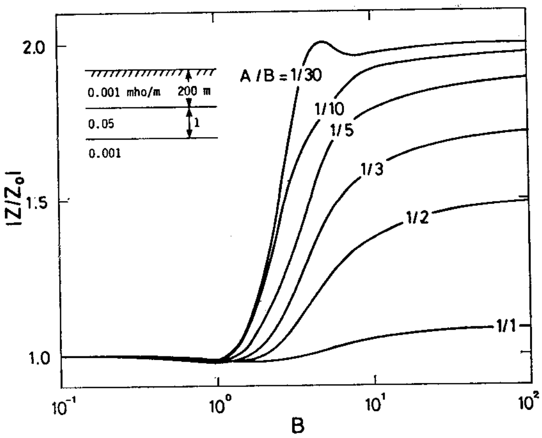

Fig. 2 Amplitudes of mutual coupling ratio for vertical coaxial loop system over the intermediate conductive layer model.

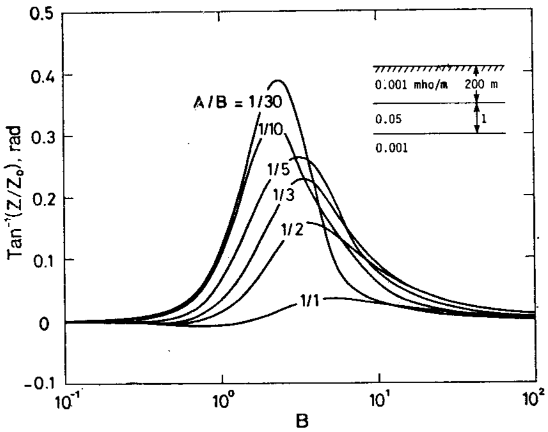

Fig. 3 Phases of mutual coupling ratio for vertical coaxial loop system over the intermediate conductive layer model.

Table 1 Numbers of the evaluations of kernel function in (5) and (10) as well as Anderson's related convolution procedure (Anderson, 1979).

Angular frequency	Eqn. (5) J_1	Eqn. (5) J_0	Eqn. (10) J_1	Related convolution J_1	Related convolution J_0
3	69	67	71	69	0
10	66	64	72	66	0
30	65	65	69	65	3
100	61	64	64	61	3
300	58	59	59	58	1
1,000	53	54	55	53	1
3,000	49	51	52	49	2
10,000	46	48	48	46	2
30,000	45	46	47	46	1
100,000	43	44	46	43	1

reference, Table 1 also contains results obtained from Anderson's related convolution procedure (Anderson, 1979).

DISCUSSION AND CONCLUSIONS

The EM mutual coupling ratio for a vertical coaxial loop system over a layered earth can unify to a single Hankel transform integral. This unification facilitates the use of J_1 filter only, instead of using of both J_0 and J_1 filters. The unification, therefore, produces reductions of computer time and core storage in estimating the mutual coupling ratio (see Table 1).

Anderson's related convolution procedure is very attractive in terms of computer time. In fact, the Anderson's procedure is slightly faster than my procedure developed in this paper. However his procedure requires about two times more storage than mine. Although the Anderson's adaptive filter was used in this paper, if we use another filter with small length of coefficients such as Ghosh's filter (Ghosh, 1971), the unifying approach makes it possible to compute the mutual coupling ratio on a programmable pocket calculator.

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층상대지에서 수직공축루프에 대한 전자응답

김 희 준

요약 : 층상대지에서 수직공축루프에 대한 상호결합비의 새로운 관계식을 방정식 중의 0차 벳셀함수를 1차로 변환함으로써 유도하였다. 이 관계식을 쓰면 J_0와 J_1 두개의 필터대신 J_1필터만이 필요하게 되며, 계산시간이나 기억용량이 크게 절감된다. 본 논문에서는 3층구조 대지에 대한 상호결합비를 앤더슨의 필터로 구하였다.