J. Korean Math. Soc. 23(1986), No. 2, pp. 141-150

COMPACT TOTALLY REAL SUBMANIFOLDS WITH PARALLEL MEAN CURVATURE VECTOR IN A COMPLEX SPACE FORM

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0. Introduction

A submanifold M of a Kaehlerian manifold $\bar{M}$ is said to be *totally real* if each tangent space to M is mapped to the normal space by the complex structure of $\bar{M}$. The concept was first introduced by Chen and Ogiue [2], who studied their fundamental properties. Many subjects for totally real submanifolds were investigated from various different points of view, as one of which Chen, Houh and Lue [1] and Yachida [8, 9] obtained investigating results of m-dimensional totally real submanifolds with parallel mean curvature vector in $2m$-dimensional complex space forms. Furthermore, Urbano [7] and Ohnita [5] recently determined also manifold structures of such a submanifold of positive curvature or of non-negative curvature, respectively.

The purpose of this paper is to investigate compact totally real submanifolds with parallel mean curvature vector of a complex space form.

Manifolds, submanifolds, geometric objects and mappings discussed in this paper are assumed to be differentiable and of C^∞.

1. Totally real submanifolds of a Kaehlerian manifold

Let $(\bar{M}, \bar{g})$ be a Kaehlerian manifold of real dimension $2m$ equipped with an almost complex structure J and a Hermitian metric $\bar{g}$. Let $\bar{M}$ be covered by a system of coordinate neighborhoods $\{\bar{U}, y^A\}$, where here and in the sequel the following convention on the range of indices are used, unless otherwise stated:

$$A, B, C \ldots = 1, \ldots, n, n+1, \ldots, 2m,$$
$$h, i, j, \ldots = 1, \ldots, n,$$
$$u, v, w, \ldots = n+1, \ldots, 2m.$$

Received February 14, 1986.

The summation convention will be used with respect to those system of indices. We then have

$$(1.1)\qquad J_A{}^B J_B{}^C = -\delta_A{}^C,\quad J_B{}^C J_A{}^D \bar{g}_{CD} = \bar{g}_{BA},$$

$\delta_A{}^C$ being the Kronecker delta, $J_B{}^A$, $\bar{g}_{BA}$ the components of J and $\bar{g}$, respectively. Denoting by ∇_B the operator of covariant differentiation with respect to $\bar{g}_{AB}$, we get

$$(1.2)\qquad \nabla_B J_C{}^A = 0.$$

Let M be an n-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{U; x^h\}$ and immersed isometrically in $\overline{M}$ by the immersion $\phi : M \longrightarrow \overline{M}$. When the argument is local, M need not be distinguished from $\phi(M)$. We represent the immersion ϕ locally by $y^A = y^A(x^h)$ and put $B_j{}^A = \partial_j y^A$, $(\partial_j = \partial/\partial x^j)$, then $B_j = (B_j{}^A)$ are n-linearly independent local tangent vectors of M. We choose $2m-n$ mutually orthogonal unit normals $C_x = (C_x{}^A)$ to M. Then the induced Riemannian metric g_{ji} on M is given by

$$(1.3)\qquad g_{ji} = \bar{g}_{BC} B_j{}^B B_i{}^C.$$

Therefore, by denoting by ∇_j the operator of van der Waerden –Bortolotti covariant differentiation with respect to g_{ji}, the equations of Gauss and Weingarten for M are respectively obtained:

$$(1.4)\qquad \nabla_j B_i{}^A = h_{ji}{}^x C_x{}^A,\quad \nabla_j C_x{}^A = -h_j{}^i{}_x B_i{}^A,$$

where $h_{ij}{}^x$ are the second fundamental forms in the direction of C_x and

$$(1.5)\qquad h_j{}^h{}_x = h_{jix} g^{ih} = h_{ji}{}^y g^{ih} g_{yx},$$

$g_{yx} = \bar{g}_{BA} C_y{}^B C_x{}^A$ being the metric tensor of the normal bundle and $(g^{ji}) = (g_{ji})^{-1}$.

An n-dimensional Riemannian manifold M immersed isometrically in $\overline{M}$ is called a *totally real* submanifold of $\overline{M}$ if $JM_p \subset M_p{}^{\perp}$ for each point p of M, where M_p denotes the tangent space of M at p and $M_p{}^{\perp}$ the normal space to M at p. In this case, JX is a normal vector to M, provided that X is a tangent vector on M. Thus it follows that the dimensions satisfy $m \geqq n$. Let $N(M_p)$ be an orthogonal complement of JM_p in $M_p{}^{\perp}$. Then the decomposition is obtained: $M_p{}^{\perp} = JM_p \oplus N(M_p)$. Hence, it follows that the space $N(M_p)$ is invariant under the action of J. Accordingly we can put in each coordinate neighborhood of M,

$$(1.6)\qquad J_B{}^A B_j{}^B = J_j{}^x C_x{}^A,$$

$$(1.7)\qquad J_B{}^A C_x{}^B = -J_x{}^i B_i{}^A + f_x{}^y C_y{}^A,$$

where we put $J_{jx}=\bar{g}(JB_j, C_x)$, $J_{xj}=-\bar{g}(JC_x, B_j)$ and $f_{xy}=\bar{g}(JC_x, C_y)$. From these definitions we see that

$$(1.8) \qquad f_{xy}+f_{yx}=0, \quad J_{jx}=J_{xj}.$$

By taking account of (1.1) and (1.3), it follows from (1.6) and (1.7) that

$$(1.9) \qquad \begin{cases} J_j{}^x J_x{}^h=\delta_j{}^h, \quad J_j{}^x f_x{}^y=0, \\ f_x{}^z f_z{}^y=-\delta_x{}^y+J_x{}^i J_i{}^y, \end{cases}$$

where $J_j{}^x=J_{jy}g^{yx}$, $f_y{}^x=f_{yz}g^{zx}$ and g^{yx} is the contravariant component of g_{yx}. These show that $f^3+f=0$. f being of constant rank, it defines the so-called *f-structure* in the normal bundle [10].

If we apply the operator ∇_j of the covariant differentiation to (1.6) and (1.7) and make use of (1.1), (1.2), (1.4) and these equations, we get respectively

$$(1.10) \qquad h_{ji}{}^x J_{xh}=h_{jh}{}^x J_{xi},$$

$$(1.11) \qquad \nabla_j J_i{}^x=h_{ji}{}^z f_z{}^x,$$

$$(1.12) \qquad \nabla_j f_y{}^x=h_{jiy}J^{ix}-h_{ji}{}^x J_y{}^i.$$

In the sequel, we assume that the ambient Kaehlerian Manifold $\overline{M}$ is of constant holomorphic sectional curvature $4c$ and of real dimension $2m$, which is called a *complex space form* and denoted by $\overline{M}^{2m}(c)$. Then the curvature tensor $\bar{R}$ of $\overline{M}^{2m}(c)$ is given by

$$\bar{R}_{DCBA}=c(\bar{g}_{DA}\bar{g}_{CB}-\bar{g}_{CA}\bar{g}_{DB}+J_{DA}J_{CB}-J_{CA}J_{DB}-2J_{DC}J_{BA}).$$

Since the submanifold M is totally real, it follows from equations (1.6) $\sim$(1.9) that equations of Gauss, Codazzi and Ricci for M are respectively obtained:

$$(1.13) \qquad R_{kjih}=c(g_{kh}g_{ji}-g_{jh}g_{ki}) +h_{kh}{}^x h_{jix}-h_{jh}{}^x h_{kix},$$

$$(1.14) \qquad \nabla_k h_{ji}{}^x-\nabla_j h_{ki}{}^x=0,$$

$$(1.15) \qquad R_{jiyx}=c(J_{jx}J_{iy}-J_{ix}J_{jy})+h_{jrx}h_i{}^r{}_y-h_{irx}h_j{}^r{}_y,$$

where R_{kjih} and R_{jiyx} are the Riemannian curvature tensor of M and that with respect to the connection induced in the normal bundle of M, respectively. We see from (1.13) that the Ricci tensor R_{ji} of M can be expressed as follows:

$$(1.16) \qquad R_{ji}=c(n-1)g_{ji}+h^x h_{jix}-h_{jr}{}^x h_i{}^r{}_x, \quad (h_x=g^{ji}h_{ji}x).$$

2. Parallel mean curvature vector

Let M be an n-dimensional totally real submanifold in a complex space form $\bar{M}^{2m}(c)$ of constant holomorphic curvature $4c$. A normal vector field $\xi=(\xi^x)$ is called a *parallel section* in the normal bundle if it satisfies $\nabla_j \xi^x=0$, and furthermore a tensor field F on M is said to be *parallel* in the normal bundle if $\nabla_j F$ vanishes identically. In this section, the f-structure in the normal bundle is assumed to be parallel. In this case, the equation (1.12) is reduced to

$$h_{jry} J^{rx} = h_{jr}{}^x J_y{}^r. \tag{2.1}$$

Multiplying $h^{jiy} J_y{}^h$ to (1.10) and summing up for j, i and h and making use of (2.1), we find

$$h^{jiy} h_{jix} J_{yh} J^{xh} = h^{jix} h_{jhx} J_{yi} J^{yh},$$

which together with (1.9) gives

$$h^{jiy} h_{jiz} (\delta_y{}^x + f_y{}^z f_z{}^x) = h^{jix} h_{jix}.$$

Thus it follows that

$$h_{ji}{}^z f_z{}^x = 0, \quad \text{i.e.,} \quad \nabla_j J_i{}^x = 0 \tag{2.2}$$

for any index x, where we have used (1.11).

REMARK. We notice from (1.9) that $f_y{}^x$ vanishes identically if $m=n$. Thus, an n-dimensional totally real submanifold of a real $2n$-dimensional Kaehlerian manifold has always a trivial f-structure in the normal bundle.

Applying $J_y{}^h$ to (1.10) and summing up for h, we obtain $h_{jiy} = h_{jr}{}^x J_y{}^r J_{xi}$ with the aid of (1.9) and (2.2), from which we get, taking the skew-symmetric part of this with respect to indices j and i,

$$(h_{jr}{}^x J_y{}^r) J_{ix} - (h_{ir}{}^x J_y{}^r) J_{jx} = 0.$$

Therefore we see, by a direct consequence of (1.9) and (2.2), that

$$h_{jr}{}^x J_y{}^r = P_{yz}{}^x J_j{}^z,$$

where $P_{yz}{}^x$ is defined by $P_{yz}{}^x = h_{ji}{}^x J_y{}^j J_z{}^i$ and hence it satisfies

$$P_{yz}{}^x f_x{}^w = 0. \tag{2.3}$$

Denoting $P_{xyz} = g_{zw} P_{xy}{}^w$, we see that P_{xyz} is symmetric with respect to all indices, because of (2.1). It follows from (2.3) that

$$h_{ji}{}^x = P_{yz}{}^x J_j{}^y J_i{}^z, \tag{2.4}$$

which together with (1.9) and (2.3) gives $P_{xyz} P^{xyz} = h_{ji}{}^x h^{ji}{}_x$ and

$$h^x = P^x, \tag{2.5}$$

where $P^x = P_y{}^{yx}$.

From now on we denote the index $n+1$ by $*$. When $x=n+1$ in

(2.4), we have

$$h_{ji}{}^{*}=P_{yz}{}^{*}J_j{}^{y}J_i{}^{z}. \tag{2.6}$$

From this equation and (2.4) it is easily seen that

$$h_{jr}{}^{x}h_i{}^{r*}=P_{zu}{}^{x}P_y{}^{u*}J_j{}^{z}J_i{}^{y}. \tag{2.7}$$

Let $\mathfrak{J}$ be a mean curvature vector field of the submanifold. Namely, it is defined by

$$\mathfrak{J}=g^{ji}h_{ji}{}^{x}C_x/n=h^x C_x/n,$$

which is independent of the choice of the local field of orthonormal frames $\{C_x\}$. Since the fact that the mean curvature vector is parallel in the normal bundle is assumed, we may choose a local field $\{e_x\}$ in such a way that $\mathfrak{J}=aC_{n+1}$, where $a=\|\mathfrak{J}\|$ is constant. Because of the choice of the local field, the parallelism of $\mathfrak{J}$ yields

$$\begin{cases} h^x=0, & x\geqq n+2, \\ h^*=na. \end{cases} \tag{2.8}$$

$\mathfrak{J}$ being a normal vector field on M, the curvature tensor R_{jiyx} of the connection in the normal bundle shows that $R_{ji*x}=0$ for any index x. Thus the Ricci equation (1.15) gives

$$h_{jr}{}^{x}h_i{}^{r*}-h_{ir}{}^{x}h_j{}^{r*}=c(J_{j*}J_i{}^{x}-J_{i*}J_j{}^{x}). \tag{2.9}$$

By the way, we notice from the first equation of (2.2) that

$$f_*{}^{x}=0, \tag{2.10}$$

because of the fact that $\mathfrak{J}$ is non trivial. For a normal vector field ξ, let A_ξ be a shape operator of the tangent space M_p at p in the direction of ξ, which is defined by $g(A_\xi X, Y)=\bar{g}(\sigma(X, Y), \xi)$ for any tangent vectors X and Y of M_p, where σ denotes the second fundamental form on the submanifold. In particular, the shape operator in the direction of C_{n+1} is denoted by A^*. The following property is then obtained.

LEMMA 2.1. *Let M be a totally real submanifold with parallel f-structure in the normal bundle in a complex space form $\overline{M}^{2m}(c)$. If the mean curvature vector is non trivial and parallel, and if A^* has no simple roots, then $c=0$.*

Proof. Since the shape operator $A^*=(h_j{}^{i*})$ is diagonalizable, a local field $\{e_i\}$ of orthonormal frames in M can be chosen in such a way that $h_j{}^{i*}=\lambda_j\delta_j{}^{i}$. Namely, $\lambda_1, \ldots, \lambda_n$ are eigenvalues of A^*. The eqaution (2.9) is then reduced to

$$(\lambda_i-\lambda_j)h_{ij}{}^x=c(J_j{}^*J_i{}^x-J_i{}^*J_j{}^x).$$

We put $[i]=\{j:\lambda_i=\lambda_j\}$. For any integer i the assumption implies that there is an integer j in $[i]$ different from i, and hence $\lambda_i=\lambda_j$. It yields $cJ_i{}^*=0$, because of (1.9), and hence $c=0$ by means of (2.10). This concludes the proof.

REMARK. Let M be an n-dimensional totally real submanifold in $\overline{M}^{2n}(c)$ $(c\neq 0)$. It is shown that if the nontrivial mean curvature vector is parallel in the normal bundle, then the shape operator A^* has simple roots.

Now, the equation (2.9) together with (2.7) yields

$$(P_{zu}{}^xP_y{}^{u*}-P_{yu}{}^xP_z{}^{u*})J_j{}^zJ_i{}^y=c(J_{j*}J_i{}^x-J_{i*}J_j{}^x).$$

Hence it follows that

$$P_{zu}{}^xP_y{}^{u*}-P_{yu}{}^xP_z{}^{u*}=c(\delta_{z*}J_y{}^iJ_i{}^x-J_z{}^jJ_j{}^x\delta_{y*}) \tag{2.11}$$

by means of (1.9), (2.3) and (2.10). Contracting x and y in (2.11) and making use of (1.9), (2.5) and (2.8), we find

$$P_{zux}P^{xu*}-h^*P_z{}^{**}=c(n-1)\delta_{z*}, \tag{2.12}$$

and hence

$$P_{yx*}P^{yx*}=h^*P_*{}^{**}+c(n-1). \tag{2.13}$$

By multiplying h^z to (2.11) and summing up for z, it is easily seen that

$$h^*P_{*u}{}^xP_y{}^{u*}-h^*P_{yu}{}^xP_*{}^{u*}=c(h^*J_y{}^iJ_i{}^x-h^x\delta_{y*})$$

by means of (2.3), (2.5), (2.8) and (2.10). From the fact that P_{xyz} is symmetric for all indices it follows that

$$P_{ux*}P_y{}^{u*}P^{xy*}=P_{yx}{}^uP^{yx*}P_u{}^{**}+c(h^*-P_*{}^{**}),$$

because $\mathfrak{H}$ is non trivial, where we use (2.5) and (2.8).

Substituting (2.12) into the last equation and making use of (2.3), we obtain

$$P_{ux*}P_y{}^{u*}P^{xy*}=h^*P_{z*}{}^*P_*{}^{z*}+c(n-2)P_{**}{}^*+ch^*. \tag{2.14}$$

LEMMA 2.2. *Let M be a totally real submanifold with parallel f-structure in the normal bundle in $\overline{M}^{2m}(c)$. If the non trivial mean curvature vector is parallel, then*

$$\Delta(h_{ji}{}^*h^{ji*})=2\|\nabla_kh_{ji}{}^*\|^2, \tag{2.15}$$

where Δ is the operator of Laplacian.

Proof. The mean curvature vector being parallel in the normal bundle, the Laplacian of $h_{ji}{}^{*}$ is given, using the Ricci formula for $h_{ji}{}^{*}$, by

$$\Delta h_{ji}{}^{*}=R_{jr}h_{i}{}^{r*}-R_{kjih}h^{kh*}. \tag{2.16}$$

On the other hand, it follows from (1.16) and (2.8) that

$$R_{ji}=c(n-1)g_{ji}+h^{*}h_{ji}{}^{*}-h_{jr}{}^{x}h_{i}{}^{r}{}_{x}.$$

If we substitute this and (1.13) into (2.16), we obtain

$$\Delta h_{ji}{}^{*}=cnh_{ji}{}^{*}-ch^{*}g_{ji}+h^{*}h_{jr}{}^{*}h_{i}{}^{r*}-h_{kh}{}^{y}h^{kh*}h_{jiy} +h_{ki}{}^{y}h^{kh*}h_{jhy}-h_{j}{}^{hy}h_{rhy}h_{i}{}^{r*}.$$

By means of (2.10), it turns out to be

$$\Delta h_{ji}{}^{*}=cnh_{ji}{}^{*}-ch^{*}g_{ji}+h^{*}h_{jr}{}^{*}h_{i}{}^{r*}-h_{kh}{}^{y}h^{kh*}h_{jiy} -ch_{jr}{}^{y}(J_{*}{}^{r}J_{iy}-J_{i*}J_{y}{}^{r}),$$

or, taking account of (1.9), (2.3), (2.4) and (2.7), we have

$$\Delta h_{ji}{}^{*}=cnh_{ji}{}^{*}-ch^{*}g_{ji}+h^{*}P_{zu}{}^{*}P_{y}{}^{u*}J_{j}{}^{z}J_{i}{}^{y} -P_{zx}{}^{y}P^{zx*}h_{jiy}-c(P_{yz}{}^{*}J_{j}{}^{z}J_{i}{}^{y}-P^{y}J_{jy}J_{i*}).$$

Thus it follows from (2.5), (2.6) and (2.8) that

$$\Delta h_{ji}{}^{*}=c(n-1)h_{ji}{}^{*}-ch^{*}(g_{ji}-J_{j*}J_{i*}) +h^{*}P_{zu}{}^{*}P_{y}{}^{u*}J_{j}{}^{z}J_{i}{}^{y}-P_{zx}{}^{y}P^{zx*}h_{jiy}.$$

Consequently it follows from the last equation that

$$h^{ji*}\Delta h_{ji}{}^{*}=c(n-1)P_{xy*}P^{xy*}-ch^{*2}+ch^{*}P_{**}{}^{*} +h^{*}P_{yz}{}^{*}P_{u}{}^{z*}P^{yu*}-(P_{zx}{}^{y}P^{zx*})(P_{uvy}P^{uv*}),$$

where we have used (1.9), (2.3), (2.6), (2.7) and (2.8). Substituting (2.12)$\sim$(2.14) into the above equation, we obtain $h^{ii*}\ \Delta h_{ji*}=0$. This completes the proof.

COROLLARY 2.3. *Let M be an m-dimensional totally real submanifold in $\overline{M}^{2m}(c)$. If the nontrivial mean curvature is parallel, then* (2.15) *is valid.*

3. Characterization of submanifolds

This section is devoted to investigating the manifold structure of compact totally real submaniolds in a complex space form $\overline{M}^{2m}(c)$. Let M be an n-dimensional compact totally real submanifold of $\overline{M}^{2m}(c)$ such that the f-structure in the normal bundle is parallel. If the non trivial mean curvature vector $\mathfrak{H}$ on M is parallel, then Lemma 2.2 says the second fundamental form $h_{ji}{}^{*}$ in the direction of $\mathfrak{H}$ is parallel, that is, $\nabla_{k}h_{ji}{}^{*}=0$

on M. When a function h_m for any integer $m \geqq 1$ is given by

$$h_m = h_{i_1}{}^{i_2*} h_{i_2}{}^{i_3*} \dots h_{i_m}{}^{i_1*},$$

it is easily seen that h_m is constant on M for any integer m, because $h_{ji}{}^*$ is parallel. This implies that each eigenvalue λ_j of the shape operator A^* is constant on M. By $\mu_1, \cdots, \mu_\alpha$ mutually distinct eigenvalues of A^* are denoted. Let $n_1, \dots, n_\alpha$ be their multiplicities. Since distinct eigenvalues μ_a $(a=1, \dots, \alpha)$ is constant, the smooth distribution T_a which consists of all eigenspaces associated with the eigenvalue can be defiend, and they are then mutually orthogonal. Furthermore, A^* being parallel, these distributions T_a are parallel and hence completely integrable. Thus, by means of the de Rham decomposition theorem [3], the submanifold M is a product of Riemannian manifolds $M_1 \times \dots \times M_\alpha$, where the tangent bundle of M_a corresponds to T_a. First of all, we shall prove

THEOREM 3.1. *Let M be an n-dimensional compact totally real submanifold imbedded in a $2m$-dimensional complex Euclidean space $\boldsymbol{C}_m$. If an f-structure in the normal bundle is parallel and if the mean curvature vector is parallel, then M is a product submanifold $M_1 \times \dots \times M_\alpha$, where M_a is a compact n_a-dimensional totally real submanifold imbedded in $\boldsymbol{C}_{m_a}$ and M_a is contained in a hypersphere in $\boldsymbol{C}_{m_a}$.*

Since the proof is accomplished by the quite same discussion as that in [1] and [6], it is only sketched. Since the ambient space is complex Euclidean, it can not admit compact minimal submanifolds. So, the mean curvature vector $\mathfrak{g}$ is not trivial. Furthermore, since $\mathfrak{g}$ is parallel in the normal bundle, each shape operator A_y satisfies $[A^*, A_y]=0$, which implies $A_y T_a \subset T_a$ for any indices y and a. By means of Moore's Theorem [4], $M = M_1 \times \dots \times M_\alpha$ is a product submanifold imbedded in $\boldsymbol{C}_m = \boldsymbol{C}_{m_1} \times \dots \times \boldsymbol{C}_{m_\alpha}$. Moreover, M_a is a totally real submanifold imbedded in some $\boldsymbol{C}_{m_a}$, because we can choose an orthonormal basis $e_{1*}, \dots, e_{m*}$ for JM_p and an orthonormal basis $e_{n+1}, \dots, e_m, e_{n+1*}, \dots, e_{m*}$ for $N(M_p)$ in such a way that

$$h_{ij}{}^k = h_{jk}{}^i = h_{ki}{}^j, \quad h_{ij}{}^\lambda = 0 \text{ for } \lambda = n+1, \dots, m^*.$$

Let $\pi_a(\mathfrak{g})$ be the component of $\mathfrak{g}$ in the subspace $\boldsymbol{C}_{m_a}$. Then $\pi_a(\mathfrak{g})$ is a parallel mean curvature of M_a in $\boldsymbol{C}_{m_a}$, and M_a is umbilical with respect to $\pi_a(\mathfrak{g})$. Therefore it follows that M_a lies in a small hypersphere in C_{m_a} which is orthogonal to $\pi_a(\mathfrak{J})$, and hence it is a compact minimal submanifold in the hypersphere. This completes the proof.

As a direct consequence of Lemma 2.1 and Theorem 3.1, we have

THEOREM 3.2. *Let M be an n-dimensional compact totally real submanifold with parallel f-structure in the normal bundle imbedded in a complex space form $\bar{M}^{2m}(c)$. If the non trivial mean curvature vector is parallel and if the shape operator A^* has no simple roots, then $c=0$. In particular, if $\bar{M}^{2m}(c)=C_m$, then M is a product submanifold $M_1\times\ldots\times M_\alpha$.*

THEOREM 3.3. *Let M be an n-dimensional compact totally real submanifold with parallel f-structure in the normal bundle in a complex space form $\bar{M}^{2m}(c)$. If the non trivial mean curvature vector is parallel and if $\bar{M}$ has no zero sectional curvature, then $c=0$. In particular, if $\bar{M}^{2m}(c)=C_m$, then M must be minimally contained in a hypersphere of positive curvature in C_m.*

THEOREM 3.4. *Let M be a compact totally real submanifold with parallel f-structure in the normal bundle in a complex space form $\bar{M}^{2m}(c)$. If the non trivial mean curvature vector is parallel and if the shape operator A^* has mutually distinct eigenvalues, then M is flat and moreover the second fundamental form is parallel.*

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