J. Korean Math. Soc. **23** (1986), No. 2. pp. 117-125

CYCLIC DIRECTED TRIPLE SYSTEMS

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1. Introduction

In what follows an ordered pair will always be an ordered pair (x, y) where $x \neq y$. A *directed triple* is a collection of three ordered pairs of the form $\{(x, y), (x, z), (y, z)\}$ that we will always denote by $[x, y, z]$. A *directed triple system* $D_\lambda(v)$ is a pair (V, B) where V is a set of v elements and B is a collection of directed triples of elements of V(called blocks) such that every ordered pair of distinct elements of V belongs to exactly λ blocks of B. At first, Hung and Mendelsohn [5] introduced directed triple systems $D_\lambda(v)$ with $\lambda=1$. However, in the case $\lambda>1$ we call those directed triple systems as well. It is well known [5] that a $D_\lambda(v)$ with $\lambda=1$ exists if and only if $v\equiv 0$ or $1 \pmod 3$. The spectrum $D_\lambda(v)$ for every λ is $\lambda v(v-1)\equiv 0 \pmod 3$ which is determined by Seberry and Skillicorn [8]. A directed triple system $D_\lambda(v)$ is said to be *cyclic* if it admits an automorphism α consisting of a single cycle of length v, and α is called a cyclic automorphism. Colbourn and Colbourn [3] showed that a cyclic $D_\lambda(v)$ with $\lambda=1$ exists if and only if $v\equiv$ 1, 4 or 7 (mod 12).

In this paper we obtain the necessary and sufficient condition for the existence of cycle $D_\lambda(v)$ for every λ.

An (A, k)-*system* (a (B, k)-, a (C, k)-, and a (D, k)-*system*, respectively) [7] is a set of ordered pairs $\{(a_r, b_r) \mid r=1, 2, \cdots, k\}$ such that $b_r-a_r=r$ for $r=1, 2, \cdots, k$, and $\bigcup_{r=1}^k \{a_r, b_r\} = \{1, 2, \cdots, 2k\}$ ($=\{1, 2, \cdots, 2k-1, 2k+1\}$, $=\{1, 2, \cdots, k, k+2, \cdots, 2k+1\}$, and $=\{1, 2, \cdots, k, k+2, \cdots, 2k, 2k+2\}$, respectively). An (A, k)-system and a (B, k)-system are essentially the same as a Skolem k-sequence and a hooked Skolem k-sequence, respectively [6, 9]. It is well known that an (A, k)-system (a (B, k)-,

Received January 14, 1986.
This research is supported by Korea Science and Engineering Foundation Research Grand 1985.

a (C, k)-, and a (D, k)-system, respectively) exists if and only if $k \equiv 0$ or 1 (mod 4), ($k \equiv 2$ or 3 (mod 4), $k \equiv 0$ or 3 (mod 4), and $k \equiv 1$ or 2 (mod 4), $k \neq 1$, respectively) (see [6, 7, 9]). An *(E, k)-system* is a set of ordered pairs $\{(a_r, b_r) \mid r=1, 2, \cdots, k\}$ such that $b_r - a_r = r$ for $r = 1, 2, \cdots, k$. and $\cup_{r=1}^{k} \{a_r, b_r\} = \{1, 2. \cdots, (k+1)/2-1, (k+1)/2+1, \ldots, 2k+1\}$. An (E, k)-systen exists if and only if $k \equiv 1$ (mod 2) [1].

A *triple system* $S_\lambda(v)$ is a pair (V, B) where V is a set of v elements and B is a collection of subsets of three elements of V (called triples) such that every unordered pair of two distinct elements of V belongs to exactly λ triples of B. Hanani [4] determined the existence of $S_\lambda(v)$ for every λ. Cyclic $S_\lambda(v)$ for every λ were settled by Colbourn and Colbourn [2].

THEOREM 1.1 ([2]). *A cyclic $S_\lambda(v)$ exists if and only if*

(i) $\lambda \equiv 1, 5, 7, 11$ *(mod 12) and* $v \equiv 1, 3$ *(mod 6) or*

(ii) $\lambda \equiv 2, 10$ *(mod 12) and* $v \equiv 0, 1, 3, 4, 7, 9$ *(mod 12) or*

(iii) $\lambda \equiv 3, 9$ *(mod 12) and* $v \equiv 1$ *(mod 2) or*

(iv) $\lambda \equiv 4, 8$ *(mod 12) and* $v \equiv 0, 1$ *(mod 3) or*

(v) $\lambda \equiv 6$ *(mod 12) and* $v \equiv 0, 1, 3$ *(mod 4) or*

(vi) $\lambda \equiv 0$ *(mod 12) and* $v \geq 3$,

except for $\lambda = 1, 2$ *and* $v = 9$.

2. Cyclic Directed Triple systems $D_\lambda(v)$

Throughout this paper, we will assume the set of elements of our cyclic $D_\lambda(v)$ to be $V = Z_v$, the group of residue classes of Z(the set of all integers) modulo v, and the corresponding cyclic automorphism to be $\alpha = (0 \ldots v\text{-}1)$.

For a fixed block $b = [x, y, z]$, define the set

$$C(b) = \{[x+i, y+i, z+i] \mid i = 0, 1, \ldots, v-1\},$$

where additions are taken modulo v. A collection of starter blocks for a cyclic $D_\lambda(v)$ with blocks B is a subcollection S of B for which $\{b \mid b \in C(s),\ s \in S\} = B$. A basic necessary condition for the existence of cyclic $D_\lambda(v)$ is $\lambda v(v-1) \equiv 0$ (mod 3), since this is the spectrum for $D_\lambda(v)$. Note the cyclically shifted blocks of a block $[x, y, z]$ are distinct. So, for each block b in a cyclic $D_\lambda(v)$, we have $|C(b)| = v$. It is a trivial exercise to see that if (V, B) is a $D_\lambda(v)$ then $|B| = \lambda v(v-1)/3$. Thus, a stronger necessary condition for the existence of cyclic $D_\lambda(v)$ is $\lambda(v-$

1) $\equiv 0 \pmod 3$. If we regard each directed triple $[x, y, z]$ as a set $\{x, y, z\}$ of three elements, then the existence of a $D_\lambda(v)$ implies the exist ence of a triple system $S_{2\lambda}(v)$. Finally, we have the following necessary condition for the existence of cyclic $D_\lambda(v)$ by Theorem 1. 1.

LEMMA 2. 1. *If there exists a cyc'ic $D_\lambda(v)$, then*
(i) $\lambda \equiv 1,\ 5 \pmod 6$ *and* $v \equiv 1, 4, 7 \pmod{12}$ *or*
(ii) $\lambda \equiv 2, 4 \pmod 6$ *and* $v \equiv 1 \pmod 3$ *or*
(iii) $\lambda \equiv 3 \pmod 6$ *and* $v \equiv 0, 1, 3 \pmod 4$ *or*
(iv) $\lambda \equiv 0 \pmod 6$ *and* $v \geqq 3$.

It is obvious to see that the existence of cyclic $D_\lambda(v)$ implies the existence of cyclic $D_{n\lambda}(v)$ for every positive integer n. So it is enough to construct cyclic $D_\lambda(v)$ for the following values of λ and v:

(i) $\lambda=1$ and $v \equiv 1, 4, 7 \pmod{12}$,
(ii) $\lambda=2$ and $v \equiv 10 \pmod{12}$,
(iii) $\lambda=3$ and $v \equiv 0, 3, 5, 8, 9, 11 \pmod{12}$,
(iv) $\lambda=6$ and $v \equiv 2,\ 6 \pmod{12}$.

LEMMA 2. 2. ([3]). *A cyclic $D_1(v)$ exists if and only if $v \equiv 1, 4, 7 \pmod{12}$.*

DEFINITION 2. 3. A (G, K)*-system* is a set of ordered pairs $\{(a_r, b_r) \mid r=1, 2, \dots, k\}$ such that $b_r - a_r = r$ for $r=1, 2, \dots, k$ and $\bigcup_{r=1}^{k} \{a_r, b_r\} = \{1, 2, \dots, k/2, k/2+2, \dots, 2k+1\}$.

LEMMA 2. 4. *A (G, K)-system exists if and only if $k \equiv 0 \pmod 2$.*

Proof. Since $k/2$ is an integer, k must be even. Conversely, let $k=2t$. Then

$$\{(t+1-r, t+1+r),\ (2t+1+r,\ 4t+2-r) \mid r=1, \dots, t\}$$

is a (G, k)*-system.*

LEMMA 2. 5. *If $v \equiv 10 \pmod{12}$, then there exists a cyclic $D_2(v)$.*

Proof. Let $v=12t+10$, $t \geqq 0$ and let $\{(a_r, b_r) \mid r=1, 2, \dots, 4t+2\}$ be a $(G, 4t+2)$-system. Then

$$\{[0, 1,\ 12t+9], [0, 2,\ 6t+6]\},$$
$$\{[0, r, b_r+4t+2],\ [b_r+4t+2, r, 0] \mid r=1, 2, \dots, 4t+2\}$$

are a collection of starter blocks of a cyclic $D_2(12t+10)$.

LEMMA 2.6. *If $v\equiv 3$ or $21 (mod\ 24)$, then there exists a cyclic $D_3(v)$.*

Proof. Let $v=6k+3$, $k\equiv 0$ or 3 (mod 4), and let $\{(a_r, b_r)\mid r=1,2,\dots,k\}$ be a (C,k)-system. Then

$\{[0,\ 2k+1,\ 4k+2],\ [4k+2,\ 2k+1,\ 0]\}$,

$\{[0, r,\ b_r+k],\ [b_r+k, r, 0]\mid r=1,2,\dots,k$ taken three times$\}$

are a collection of starter blocks of a cyclic $D_3(6k+3)$ where $k\equiv 0$ or 3 (mod 4).

LEMMA 2.7. *There exists a cyclic $D_3(9)$.*

Proof. A collection of starter blocks of a cyclic $D_3(9)$ is

$[0,4,5],\ [0,7,8],\ [0,2,7],\ [0,3,6],$

$[5,4,0],\ [8,7,0],\ [7,2,0],\ [6,3,0].$

LEMMA 2.8. *If $v\equiv 9$ or $15 (mod\ 24)$, then there exists a cyclic $D_3(v)$.*

Proof. The case $v=9$ has been treated in Lemma 2.7. Let $v=6k+3$, $k\equiv 1$ or 2 (mod 4), $k\neq 1$, and let $\{(a_r, b_r)\mid r=1,2,\dots,k\}$ be a (D,k)-system. Then

$\{[0,\ 2k+1,\ 4k+2],\ [4k+2,\ 2k+1,\ 0]\}$,

$\{[0, r, b_r+k, r, 0]\mid r=1,2,\dots,k$ taken three times$\}$

are a collection of starter blocks of a cyclic $D_3(6k+3)$ where $k\equiv 1$ or (mod 4), $k\neq 1$.

LEMMA 2.9. *If $v\equiv 5$ (mod 6), then there exists a cyclic $D_3(v)$.*

Proof. Let $v\equiv 5$ (mod 6). Then

$$\{[0, r, v-r],\ [v-r, r, 0]\mid r=1,2,\dots,(v-1)/2\}$$

is a collection of starter blocks of a cyclic $D_3(v)$.

DEFINITION 2.10. A $(S, 4t-1)$-*system* is a set of ordered pairs $\{(a_r, b_r)\mid r=1,2,\dots,4t-1\}$ such that $b_r-a_r=r$ for $r=1,2,\dots,4t-1$, $a_{3t}=3t$ and $\bigcup_{r=1}^{4t-1}\{a_r, b_r\}=\{1,2,\dots,4t-1,\ 4t+1,\dots,8t-1\}$.

LEMMA 2.11. *A $(S,4t-1)$-system exists if and only if $t\geqq 1$.*

Proof. Case 1. $t\equiv 1$ (mod 2). Let

A: $((t+1)/2+1-r,\ (t+1)/2+r)$, $r=1,2,\dots,(t+1)/2$

B: $((3t+1)/2-r, (3t+1)/2+r)$, $r=1,2,\dots,(t-3)/2$ $(t>3)$

C: $(4t+r,\ 8t-r)$, $r=1,2,\dots,(3t-1)/2$

D: $((7t+1)/2-r,\ (11t-1)/2+r)$, $r=1,2,\dots,t$

E: $((3t+1)/2,\ (5t-1)/2)\ (t>1)$

F: $(2t-1+r,\ 4t-r),\ r=1,2,\dots,\ (t-1)/2\ (t>1)$.

Then $A\cup B\cup C\cup D\cup E\cup F$ is a $(S, 4t-1)$-system where $t\equiv 1 \pmod 2$.

Case 2. $t\equiv 0 \pmod 2$. Let

A: $(2,\ 4t+1),\ (13t/2,\ 13t/2+1),\ (t/2+2, 3t/2+2),\ (1,\ t+2)$

B: $(4t+1+r,\ 8t-r),\ r=1,2,\dots,3t/2-2$

C: $(5t/2+r,\ 13t/2-r),\ r=1,2,\dots,t$

D: $(2t+r,\ 4t-r),\ r=1,2,\dots,t/2-1\ (t>2)$

E: $(2+r,\ t+2-r),\ r=1,2,\dots,\ t/2-1\ (t>2)$

F: $(t+2+r,\ 2t+1-r),\ r=1,2,\dots,\ t/2-2\ (t>4)$

G: $(3t/2+1,\ 5t/2)\ (t>2)$.

Then $A\cup B\cup C\cup D\cup E\cup F\cup G$ is a $(S,\ 4t-1)$-system where $t\equiv 0 \pmod 2$.

Lemma 2.12. *If* $v\equiv 0$ *(mod* 12*), then there exists a cyclic* $D_3(v)$.

Proof. Let $v=12t$ and let $\{(a_r, b_r)\mid r=1,2,\dots,\ 4t-1\}$ be a $(S,\ 4t-1)$-system. Then

$[0, 4t, 8t],\ [8t, 4t, 0]$,

$\{[a_r, b_r, 0]\mid r=1,2,\dots,\ 4t-1$ taken three times$\}$

are a collection of starter blocks of a cyclic $D_3(12t),\ t\geqq 1$.

Lemma 2.13. *If* $v\equiv 8$ *(mod* 12*), then there exists a cyclic* $D_3(v)$.

Proof. Let $v=12t+8,\ t\geqq 0$, and let $\{(a_r, b_r)\mid r=1,2,\dots,\ 6t+3\}$ be an $(E, 6t+3)$-system (see [1]). Then

$\{[0,\ 6t+4,\ 3t+2]\}$,

$\{[0, r, b_r],\ [b_r, r, 0]\mid r=1,2,\ \dots,\ 6t+3\}$

are a collection of starter blocks of a cyclic $D_3(12t+8),\ t\geqq 0$.

Lemma 2.14. *If* $v\equiv 6$ *(mod* 12*), then there exists a cyclic* $D_6(v)$.

Proof. Let $v\equiv 6t$ and $t=1,3,5 \pmod 6$.

Case 1. $t\equiv 3 \pmod 6$. Then

$\{[0, t, 5t],\ [0, t, 5t],\ [5t, t, 0],\ [0, 2t, 4t]\}$,

$\{[0, 3t+1-r, r],\ [r, 3t+1-r, 0]\mid r=1,2,\dots,\ t$ taken three times$\}$,

$\{[0, r, 7t-r], [7t-r, r, 0]\mid r=t+1, t+2,\dots,\ 2t-1$ taken three times$\}$

are a collection of starter blocks of a cyclic $D_6(6t)$ where $t\equiv 3 \pmod 6$.

Case 2. $t\equiv 1$ or $5 \pmod 6$. Then

$\{[0, t, 5t],\ [0, t, 5t],\ [5t, t, 0],\ [0, 2t, 4t]\}$,

$\{[0,\ 3r,\ 2t-3+6r],\ [2t-3+6r, 3r, 0] \mid r=1, 2, \ldots, t$ taken three times$\}$,

$\{[0, 3r, 6r-4t],\ [6r-4t, 3r, 0] \mid r=t+1, t+2, \ldots,\ 2t-1$ taken three times$\}$ $(t>1)$

are a collection of starter blocks of a cyclic $D_6(6t)$ where $t\equiv 1$ or 5 (mod 6).

DEFINITION 2.15. A *(H, k)-system* is a set of ordered pairs $\{(a_r, b_r) \mid r=1, 2, \ldots, k-1\}$ such that $b_r-a_r=2r+2$ for $r=1, 2, \ldots,\ k-1$ and $\bigcup_{r=1}{}^{k-1}\{a_r, b_r\} = \{2r+1 \mid r=1, 2, \ldots, (3k-3)/2,\ (3k+1)/2, \ldots, 2k-1\}$.

LEMMA 2.16. *A (H, k)-system exists if and only if $k\equiv 1$ (mod 2), $k\neq 1$.*

Proof. Necessity is obvious. For sufficiency, let $k\equiv 1$ (mod 2) and $k\neq 1$.

A: $(1+2r,\ 2k-1-2r)$, $r=1, 2\ldots, (k-3)/2$ $(k>3)$

B: $(2k-3+2r,\ 4k+1-2r)$, $r=1, 2, \ldots,\ (k-1)/2$

C: $(k,\ 3k-2)$.

The $A\cup B\cup C$ is a (H, k)-system.

DEFINITION 2.17. An *(I, k)-system* is a set of ordered pairs $\{(a_r, b_r) \mid r=1, 2, \ldots, k\}$ such that $b_r - a_r=r$ for $r=1, 2, \ldots, k$ and $\bigcup_{r=1}{}^{k}\{a_r, b_r\} = \{1, 2, \ldots, k-1,\ k+1, \ldots,\ 2k+1\}$.

LEMMA 2.18. *If $k\equiv 2$ (mod 4), then there exists an (I, k)-system.*

Proof. $k=2$: $(4, 5), (1, 3)$.

Let $k=4t+2,\ t\geqq 1$.

A: $(r,\ 4t-r+2)$, $r=1, 2, \ldots, 2t$

B: $(4t+r+3,\ 8t-r+4)$, $r=1, 2, \ldots, t-1$ $(t>1)$

C: $(5t+r+2,\ 7t-r+3)$, $r=1, 2, \ldots, t-1$ $(t>1)$

D: $(2t+1, 6t+2)$, $(6t+3, 8t+4)$, $(4t+3, 8t+5)$, $(7t+3, 7t+4)$.

Then $A\cup B\cup C\cup D$ is an $(I,\ 4t+2)$-system, $t\geqq 1$.

DEFINITION 2.19. A *$(J, 2t+1)$-system* is a set of ordered pairs $\{(a_r, b_r) \mid r=1, 2, \ldots,\ 2t+1\}$ such that $b_r-a_r=r$ for $r=1, 2, \ldots,\ 2t+1$ and $\bigcup_{r=1}{}^{2t-1}\{a_r, b_r\} = \{1, 2, \ldots, 4t, 4t, 4t\}$.

LEMMA 2.20. *If $t\equiv 1$ (mod 2), then there exists a $(J,\ 2t+1)$-system.*

Proof. $t=1$: $(3,4)$, $(2,4)$, $(1,4)$.
Let $t\equiv 1 \pmod 2$, $t>1$.

A: $(r,\ t-1-r)$, $r=1,2,\dots,\ t-2$
B: $(2t+1+r,\ 4t+1-r)$, $r=1,2,\dots,(t-1)/2$
C: $(3t-r, 3t+r)$, $r=1,2,\dots,(t-3)/2$ $(t>3)$
D: $(t-1,3t)$, $(t,2t-1)$, $(2t,4t)$, $(2t+1,4t)$, $((7t-1)/2,\ (7t+1)/2)$.

Then $A\cup B\cup C\cup D$ is a $(J,2k+1)$-system where $t\equiv 1 \pmod 2$, $t>1$.

DEFINITION 2.21. A $(K,2t+1)$*-system* is a set of ordered pairs $\{(a_r, b_r)\mid r=1,2,\dots,2t+1\}$ such that $b_r-a_r=r$ for $r=1,3,4,\dots,\ 2t+1$, $b_2-a_2=1$, and $\bigcup_{r=1}^{2t+1}\ \{a_r,b_r\}=\{1,2,\dots,4t,4t,4t\}$.

LEMMA 2.22. *If* $t\equiv 0$ (*mod* 2), *then there exists a* $(K,2t+1)$*-system.*

Proof. $t=2$: $(1,2)$, $(6,7)$, $(5,8)$, $(4,8)$, $(3,8)$.
$t=4$: $(3,4)$, $(15,16)$, $(5,8)$, $(12,16)$, $(11,16)$, $(7,13)$, $(2,9)$, $(6,14)$, $(1,10)$.
Let $t\equiv 0 \pmod 2$, $t\geqq 6$.

A: $(2t+4+r, 4t+1-r)$, $r=1,2,\dots,\ t-3$
B: $(3+r, 2t-1-r)$, $r=1,2,\dots,\ t/2-3$ $(t>6)$
C: $(t+1-r, t+3+r)$, $r=1,2,\dots,\ t/2-2$
D: $(1,t+1)$, $(2,2t+2)$, $(3,2t+1)$, $(2t-1,2t)$, $(t+2,3t+3)$, $(t+3,3t+2)$, $(2t+3,4t)$, $(2t+4,4t)$, $(t/2+1,t/2+2)$.

Then $A\cup B\cup C\cup D$ is a $(K,2t+1)$-system where $t\equiv 0 \pmod 2$, $t\geqq 6$.

LEMMA 2.23. *If* $v\equiv 2$ (*mod* 12), *then there exists a cyclic* $D_6(v)$.

Proof. Let $v=12t+2$, $t\geqq 1$.
Case 1. $t\equiv 1 \pmod 2$.
A collection of starter blocks: $B=B_1\cup B_2\cup B_3\cup B_4\cup B_5\cup B_6$
where

B_1: $\{[0,2,6t+2],\ [0,2,12t],\ [0,9t+2,3t]\}$
B_2: $\{[0,2b_r,2a_r]\mid r=1,2,\dots,t-1\}$ $(t>1)$

where $\{(a_r,b_r)\mid r=1,2,\dots,t-1\}$ is a (H,t)-system,

B_3: $\{[0,r,b_r]\mid r=1,2,\dots,6t\}$

where $\{(a_r,b_r)\mid r=1,2,\dots,6t\}$ is an $(I,6t)$-system,

B_4: $\{[0,r,b_r],[b_r,r,0]\mid r=1,2,\dots,6t\}$

where $\{(a_r, b_r) \mid r=1, 2, \ldots, 6t\}$ is a $(B, 6t)$-system,

B_5: $\{[0, r, 12t+2-r] \mid r=1, 3, \ldots, 3t-2, 3t+2, \ldots, 6t-1, r \text{ is odd}\}$

B_6: $\{[2t+1+b_r, r, 0]=1, 2, \ldots, 2t+1\}$

where $\{(a_r, b_r) \mid r=1, 2, \ldots, 2t+1\}$ is a $(J, 2t+1)$-system.

Case 2. $t=0 \pmod 2$.

A collection of starter blocks: $B=B_1 \cup B_2 \cup B_3 \cup B_4 \cup B_5 \cup B_6$
where

B_1: $\{[2, 0, 1]\}$

B_2: $\{[0, r, b_r], [0, r, b_r], [b_r, r, 0] \mid r=1, 2, \ldots, 6t\}$

where $\{(a_r, b_r) \mid r=1, 2, \ldots, 6t\}$ is an $(A, 6t)$-system,

B_3: $\{[b_r+2t+1, r, 0], [b_r+2t+1, 1, 0] \mid r=1, 3, 4, \ldots, 2t+1\}$

where $\{(a_r, b_r) \mid r=1, 2, \ldots, 2t+1\}$ is a $(K, 2t+1)$-system,

B_4: $\{[0, 2r, 2(b_r+t)] \mid r=1, 2, \ldots, t\}$

where $\{(a_r, b_r) \mid t=1, 2, \ldots, t\}$ is an (A, t)-system or a (B, t)-system depending on $t\equiv 0 \pmod 4$ or $t\equiv 2 \pmod 4$,

B_5: $\{[0, r, 12t+2-r] \mid r=1, 3, \ldots, 3t+1, r \text{ is odd}\}$

B_6: $\{[0, 12t+2-r, r] \mid r=3t+3, 3t+5, \ldots, 6t-1, r \text{ is odd}\}$.

In each case, B is a collection of starter blocks of a cyclic $D_6(12t+2)$.

Summarizing, we have the following theorem.

THEOREM 2.24. *A cyclic $D_\lambda(v)$ exists if and only if*

(i) $\lambda\equiv 1, 5 \pmod 6$ *and* $v\equiv 1, 4, 7 \pmod{12}$ *or*

(ii) $\lambda\equiv 2, 4 \pmod 6$ *and* $v\equiv 1 \pmod 3$ *or*

(iii) $\lambda\equiv 3 \pmod 6$ *and* $v\equiv 0, 1, 3 \pmod 4$ *or*

(iv) $\lambda\equiv 0 \pmod 6$ *and* $v\geqq 3$.

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