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FLEXIBLE MALCEV-ADMISSIBLE ALGEBRAS WITH NIL CARTAN SUBALGEBRAS

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1. Flexible Malcev-admissible algebras

Let A denote an (nonassociative) algebra over a field F of characteristic $\neq 2$ with multiplication denoted by juxtaposition xy. Denote by A^- the algebra with the commutator product $[x, y]=xy-yx$ defined on the vector space A. We call A Malcev-admissible if A^- is a Malcev algebra; that is, A^- satisfies the Malcev identity

$$[[x, y], [x, z]]=[[[x, y]z], x]+[[[y, z], x], x]+[[[z, x], x], y].$$

An algebra A is said to be Lie-admissible if A^- is a Lie algebra; namely A^- satisfies the Jacobi identity

$$[[x, y], z]+[[y, z], x]+[[z, x], y]=0.$$

It is well known that any Lie-admissible algebra is Malcev-admissible but not conversely (Myung [10, 12]).

A useful identity in the study of Malcev-admissible algebras A is the well known flexible identity $(xy)x=x(yx)$, which is equivalent to the property that the adjoint map $ad_x : A\rightarrow A$ defined by $ad_x(y)=[x, y]$ is a derivation of the commutative algebra A^+ with multiplication $x \circ y=\frac{1}{2}(xy+yx)$ defined on A for all $x\in A$. Thus, A is a flexible algebra over F if and only if the identity

$$(1) \qquad [x, y \circ z]=y \circ [x, z]+[x, y] \circ z$$

holds for all $x, y, z\in A$ ([8, 10]). It can be shown that an algebra A is flexible Lie-admissible if and only if ad_x is a derivation of A, or equivalently, the identity

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$$[x, yz]=y[x, z]+[x, y]z \tag{2}$$

is satisfied for all $x, y, z \in A$ ([8]). Basic examples of flexible Malcev-admissible algebras which are not Lie-admissible are octonion algebras.

The structure of finite-dimensional flexible Malcev-admissible algebras of characteristic 0 has been studied in light of earlier investigations of flexible Lie-admissible algebras (see Myung [10, 12], for example). Reoresentation theory of Lie algebras and Cartan subalgebras play main roles for the structure of these algebras. Very little is known for the structure of flexible Malcev-admissible algebras of arbitrary dimension.

Let A be a flexible Malcev-admissible algebra over F, and suppose that A^- has a Cartan decomposition relative to a Cartan subalgebra H of A^-:

$$A=H+\sum_{\alpha\neq 0} A_\alpha, \tag{3}$$

where $A_\alpha=\{x\in A \mid (ad_h-\alpha(h)I)^n(x)=0 \text{ for some } n=n(h),\ h\in H\}$ is the root space corresponding to root α. Using the flexible identity (1), it can be shown that

$$\begin{aligned} A_\alpha \circ A_\beta &\subseteq A_{\alpha+\beta} \text{ for all roots } \alpha, \beta, \\ A_\alpha A_\beta &\subseteq A_{\alpha+\beta} \text{ for } \alpha \neq \beta, \\ A_\alpha A_\alpha &\subseteq A_{2\alpha}+A_{-\alpha}. \end{aligned} \tag{4}$$

If A is flexible Lie-admissible, then the last relation of (4) becomes $A_\alpha A_\alpha \subseteq A_{2\alpha}$. We note that the Cartan subalgebra H is a subalgebra of A. A special case of interest in this paper is when the map ad_h acts diagonally on each root space A_α for all $h\in H$ and when H is abelian; i.e., $[H, H]=0$. As is well known, the classes of finite-dimensional split semisimple Malcev algebras of characteristic 0 and of classical Lie algebras of Seligman [14] satisfy this property. An important class of infinite dimensional Lie algebas having this property is the class of Kac-Moody algebras [3]. It is the purpose of this note to investigate the structure of flexible Malcev-admissible algebras A which have a Cartan decomposition relative to an abelian Cartan subalgebra H of A^- such that all ad_h act diagonally on each root space A_α, under the assumption that each element of H is nilpotent in A.

In a power-associative algebra B, an element x is called nilpotent if $x^n=0$ for some positive integer n. If every element of B is nilpotent, then B is said to be nil. If there is a positive integer m such that $x^m=0$ for all $x\in B$, then the least such integer is called the nil-index of B.

Every finite-dimensional nil algebra B has the nil-index$\leq$dim$B$+1.

2. Cartan subalgebras with nil-basis

In this section we give a condition that a finite-dimensional flexible power-associative Malcev-admissible algebra A is a nil algebra in terms of a nil-basis of a Cartan subalgebra of A^-. A nil-basis of a power-associative algebra is a basis consisting of nilpotent elements. Generalizing the result in Myung [9] for Lie-admissible algebras, we have

THEOREM 1. *Let A be a finite-dimensional power-associative Malcev-admissible algebra over a field F of characteristic* 0 *or great than* 5 *and dim A. Then, A is a nil algebra if and only if A^- has a Cartan subalgebra with a nil-basis.*

Proof. The assumption on the characteristic guarantees the existence of a Cartan subalgebra (Malek [7]). Let H denote a Cartan subalgebra of A^- with a nil-basis. By a result on flexible power-associative algebras (Myung [9]), it suffices to verify that A has a nil-basis. By a scalar extension argument, we may assume that F is algebraically closed. Thus, A^- has a Cartan decomposition as in (3) relative to H. For a root α, if $A_\alpha{}^n$ denotes the linear span of Jordan products $x_1 \circ x_2 \circ \cdots \circ x_n$ of any n elements $x_1, \ldots, x_n$ of A_α in all possible associations, then it follows from the first relation of (4) that $A_\alpha{}^n \subseteq A_{n\alpha}$. Since there are only finitely many roots, it must be that $A_\alpha{}^n = 0$ for some $n > 0$. This in particular implies that A_α has a nil-basis for all roots α and hence A has a nil-basis.

There are other conditions that force flexible power-associative algebras to be nil algebras. For example, any finite-dimensional flexible power-associative algebra A of characteristic $\neq 2, 3$ such that A^- is simple must be a nil algebra (Oehmke [13]).

3. Algebras with nil Cartan subalgebras

Let A be an algebra over F of characteristic $\neq 2$ with multiplication denoted by xy. A bilinear form ϕ on A is called invariant if it satisfies the equation

$$\phi(xy, z) = \phi(x, yz)$$

for all $x, y, z \in A$. Certain invariant bilinear forms have been used for the construction of flexible Malcev-admissible algebras, and conversely those algebras satisfying certain properties arise from the construction using invariant forms (see Ko and Myung [4], Benkart [1], and Benkart and Osborn [2]). A well known invariant form is the Killing form K on a Lie or Malcev algebra of finite dimension, which is defined by

$$K(x, y) = \mathrm{tr}(ad_x \ ad_y),$$

where tr denotes the trace.

Let A be any algebra over F, and assume that $c_1, \ldots, c_n$ are linearly independent elements such that $[c_i, A]=0$ for $i=1, \ldots, n$. Let $\phi_i (i=1, \ldots, n)$ be symmetric bilinear forms on A. We define a multiplication "$*$" on the vector space A by

$$x * y = \frac{1}{2}[x, y] + \sum_{i=1}^{n} \phi_i(x, y) c_i \tag{5}$$

for $x, y \in A$, By symmetry of ϕ_i, we have $(A, *)^- = A^-$, and the Jordan product in $(A, *)$ is given by

$$x \circ y = \frac{1}{2}(x * y + y * x) = \sum_{i=1}^{n} \phi_i(x, y) c_i \tag{6}$$

for all $x, y \in A$.

LEMMA 2. *Let $(A, *)$ be the algebra with multiplication defined by (5). Then, $(A, *)$ is flexible if and only if the symmetric bilinear forms $\phi_1, \ldots, \phi_n$ are invariant on A^- or on $(A, *)^-$.*

Proof. Note first that $(A, *)$ is flexible if and only if relation (1) holds for $(A, *)$. Hence, if $(A, *)$ is flexible, then by (6)

$$\begin{aligned} 0 = [x, y \circ z] &= y \circ [x, z] + [x, y] \circ z \\ &= \sum_{i=1}^{n} [\phi_i(y, [x, z]) + \phi_i([x, y], z)] c_i, \end{aligned}$$

which implies that $\phi_i([x, y], z) = \phi_i(x, [y, z])$ for $x, y, z \in A$ and $i=1, \ldots, n$, since $c_1, \ldots, c_n$ are linearly independent. The converse is immediate.

The product "$*$" of the form (5) has arisen from the classification of finite-dimensional flexible Malcev-admissible algebras A over an algebraically closed field of characteristic 0 such that A^- is reductive. In this case, the forms ϕ_i are multiples of the Killing form (Myung[10]). We extend this to a class of flexible Malcev-admissible algebras of arbitrary dimension. For this, the following results are instrumental.

LEMMA 3. *Let A be a flexible algebra over F of characteristic $\neq 2$.*

(i) *If h is a power-associative element of A and $X \in A$ is a common eigenvector of ad_h and ad_{h^2}, then $[x, h^3]=[x, h^4]=0$ imply $[x, h^2]=0$.*

(ii) *If h is a power-associative element of A and x is a common eigenvector of ad_h, ad_{h^2}, R_h, and R_h^2, then $[x, h^4]=[x, h^5]=0$ imply $[x, h^3]=0$. Here, R_h denotes the right multiplication in A by h.*

LEMMA 4. *Let A be a flexible Malcev-admissible algebra with multiplication denoted by xy (not necessarily finite-dimensional) over a field F of characteristic $\neq 2$, and let H be an abelian Cartan subalgebra of A^-. Assume that A^- has a Cartan decomposition relative to H such that each $ad_h (h \in H)$ diagonally acts on root space A_α for all roots α; i.e., $[h, x]=\alpha(h)x$ for all $x \in A_\alpha$ and $h \in H$.*

(i) *If $h \in H$ and $x \in A_\alpha$ for $\alpha \neq 0$, then hx and xh are multiples of x.*

(ii) *If $x \in A_\alpha$, $y \in A_\beta$ and $\alpha \neq -\beta$ for $\alpha, \beta \neq 0$, then xy is a multiple of $[x, y]$.*

(iii) *If $HH=0$, then $xy=\frac{1}{2}[x, y]$ for all $x, y \in A$, and hence A is a Malcev algebra isomorphic to A^-.*

(iv) *If the center of A^- is zero and H is a nil algebra under the product xy with bounded nil index, then $HH=0$ and hence A is a Malcev algebra.*

A proof of Lemma 3 may be found in Myung [8]. Several different versions of Lemma 4 have been proved. Lemma 4 was first proved by Myung [8] when A is finite-dimensional and $\dim A_\alpha=1$ for $\alpha \neq 0$. Benkart [1] proved the present form for Lie-admissible algebras. A proof of Lemma 4 for the finite-dimensional case has been given by Malek [6], and the present form of the lemma is due to Ko and Myung [5].

For the case of characteristic $p>0$, two well known classes of algebras satisfying the conditions of Lemma 4 are the classical Lie algebras (Seligman [14]) and generalized Witt algebras. Among infinite-dimensional Lie algebras of characteristic 0 which satisfy the hypotheses of Lemma 4 are the Virasoro algebra which arises in relativistic string dual model theory, and the Kac-Moody algebras which are best understood infinite-dimensional Lie algebras (Kac [3]). Examples are given in Myung [8] and Malek [6] to show that the hypotheses that the center of A^- is 0 and the Cartan subalgebra H is nil in A are necessary in Lemma 4 (iv). We note that Kac-Moody algebras have in general nonzero center, but there exist Kac-Moody algebras without center; for example, certain affine Kac-Moody algebras. Our principal result in this section is to determine flexible Malcev-admissible algebras A satisfying the conditions of Lemma 4, when the Cartan subalgebra H is a nil subalgebra of A of nil-index ≤ 4.

THEOREM 5. *Let A be a flexible Malcev-admissible algebra over F of characteristic $\neq 2$, which satisfies the hypotheses of Lemma 4. Assume that the center Z of A^- is finite-dimensional and $\{c_1, \ldots, c_n\}$ is a basis of Z. If the Cartan subalgebra H of A^- is nil in A and of nil-index ≤ 4, then the multiplication "$*$" in A is given by relation* (5) *for some symmetric invariant forms $\phi_1, \ldots, \phi_n$ on A^- satisfying the property*

$$(7) \qquad \phi_i(h^3, h) = \phi_i(h^2, h^2) = 0, \quad i = 1, \ldots, n$$

for all $h \in H$.

Proof. By the assumption, Lemma 4 (i) and Lemma 3 (ii) we note that h^3 lies in the center Z for all $h \in H$, since $h^4 = h^5 = 0$. In view of Lemma 3 (i), this in turn implies that $h^2 \in Z$ for all $h \in H$. Since H is abelian, we have $h_1 * h_2 = \frac{1}{2}[(h_1 + h_2)^2 - h_1^2 - h_2^2]$ and hence $H * H \subseteq Z$. Thus, we can let

$$h_1 * h_2 = \phi_1(h_1, h_2)c_1 + \cdots + \phi_n(h_1, h_2)c_n$$

for some symmetric bilinear forms $\phi_1, \ldots, \phi_n$ on H. Let $x \in A_\alpha$ for any nonzero root α. If $h, h' \in H$, then since $H * H \subseteq Z$, by (1)

$$\begin{aligned} 0 = [h \circ h', x] &= [h, x] \circ h' + h \circ [h', x] \\ &= \alpha(h) x \circ h' + \alpha(h') h \circ x, \end{aligned}$$

which shows that $A_\alpha \circ H = 0$, and hence $h * x = -x * h = \frac{1}{2}\alpha(h)x = \frac{1}{2}[h, x]$ for $h \in H$, $x \in A_\alpha$, $\alpha \neq 0$. In particular, we have $A_\alpha * Z = Z * A_\alpha = 0$ for $\alpha \neq 0$.

Let α, β be nonzero roots. For $x \in A_\alpha$ and $y \in A_\beta$, it follows that

$$(8) \qquad 0 = [h \circ x, y] = \beta(h) x \circ y + h \circ [x, y]$$

for all $h \in H$. If $\alpha+\beta \neq 0$, then (8) gives $x \circ y=0$, since $\beta \neq 0$ and $H \circ A_\gamma=0$ for $\gamma \neq 0$. Hence,

$$x * y=-y * x=\frac{1}{2}[x, y],$$

if $x \in A_\alpha$ and $y \in A_\beta$ for roots α, β with $\alpha+\beta \neq 0$. Assume then that $x \in A_\alpha$ and $y \in A_{-\alpha}$. Since $\alpha \neq 0$ and $h \circ [x, y] \in Z$, by (8) we have $x \circ y \in Z$. Thus, we can write $x \circ y=\phi_1(x, y) c_1+\cdots+\phi_n(x, y) c_n$ where $\phi_i(x, y)=\phi_i(y, x) \in F$ $(i=1, \ldots, n)$ is uniquely determined by $x \in A_\alpha$ and $y \in A_{-\alpha}$. Extending $\phi_1, \ldots, \phi_n$ bilinearly to A by defining $\phi_i(A_\alpha, A_\beta)=0$ for all roots α, β with $\alpha+\beta \neq 0$, we have the multiplication "$*$" given by relation (5), since $x * y=\frac{1}{2}[x, y]+x \circ y$. Since A is flexible, the invariance of the ϕ_i follows from Lemma 2. Since $h^2 h^2=h^3 h=0$ for all $h \in H$ and $H * H \subseteq Z$, relation (7) is immediate from (5).

4. Application to affine Kac-Moody algebras

In this section, we give an application of Theorem 5 to the algebras A when A^- has a one-dimensional center. There are some algebras of interest where A^- has a one-dimensional center; for examples, matrix algebras, quadratic algebras including quaternion and octonion algebras, and reductive Lie or Malcev algebras with one-dimensional center.

There is an important class of infinite-dimensional Kac-Moody algebras with one-dimensional center, called (non-twisted) affine Kac-Moody algebras. For convenience, we give a realization of these algebras. Let $L=F[t, t^{-1}]$ be the commutative associative algebra of Laurent polynomials (or equivalently, the group algebra on an infinite cyclic group). For any Laurent polynomial P, the residue $\operatorname{Res}(P)$ of P is defined as the coefficient of t^{-1} in P. Thus, $\operatorname{Res}(P)$ is a linear form on L defined by the relations

$$\operatorname{Res}(t^{-1})=1, \quad \operatorname{Res}\left(\frac{dP}{dt}\right)=0.$$

Define the bilinear form $(\ ,\ )$ on L by

$$(P, Q)=\operatorname{Res}\left(\frac{dP}{dt} Q\right). \tag{9}$$

Then, it is readily seen that the form $(\ ,\ )$ satisfies

$$(P,Q)=-(Q,P),$$
(10) $$(PQ,\ R)+(QR,\ P)+(RP,\ Q)=0$$

for all $P, Q, R\in L$. In particular, we have $(t^m, t^n)=m\delta_{m,-n}$.

Let G be a finite-dimensional Lie algebra over an algebraically closed field F of characteristic 0, and let $K(\ ,\ )$ be the Killing form on G. Consider the tensor product $L(G)=L\otimes_F G$ and define a Lie algebra product on $L(G)$ by

$$[P\otimes x,\ Q\otimes y]=PQ\otimes[x,y]$$

for P, $Q\in L$ and $x, y\in G$. Then, $L(G)$ is an infinite dimensional Lie algebra ove F, called the Loop algebra (see Kac [3, p. 73]). Define a bilinear form $\phi_0(\ ,\ )$ on $L(G)$ by

$$\phi_0(P\otimes x,\ Q\otimes y)=(P,Q)K(x,y)$$

for $P, Q\in L$ and $x, y\in G$, where $(\ ,\ )$ is the bilinear form defined by (9). It easily follows from (10) that ϕ_0 is skew symmetric and is an F-valued 2-cocycle of the Lie algebra $L(G)$ in the sense that the identity

(11) $$\phi_0([a,b],c)+\phi_0([b,c],a)+\phi_0([c,a],b)=0$$

holds for all $a, b, c\in L(G)$. We now make a one-dimensional central extension of $L(G)$ to the algebra $\tilde{L}(G)=L(G)\oplus Fc$ with multiplication defined by

(12) $$[a+\lambda c,\ b+\mu c]=[a,b]+\phi_0(a,b)c$$

for $a, b\in L(G)$ and $\lambda, \mu\in F$. Then, $\tilde{L}(G)$ becomes an infinite dimensional Lie algebra with one-dimensional center Fc called an (non-twisted) affine Kac-Moody algebra, which satisfies the conditions of Lemma 4 holding for A^-.

THEOREM 6. *Let A be a flexible Malcev-admissible algebra over F of characteristic $\neq 2$ for which A^- satisfies the hypotheses of Lemma 4. Assume that A^- has a one-dimensional center Fc. If the Cartan subalgebra H of A^- is a nil subalgebra of A of nil-index ≤ 4, then the multiplication "$*$" in A is given by*

(13) $$x*y=\frac{1}{2}[x,y]+\phi(x,y)c$$

for $x, y\in A$, where ϕ is a symmetric invariant from on A^- satisfying *the property*

(14) $$\phi(c, A)=\phi(A_\alpha, A_\beta)=0$$

for all roots α, β *with* $\alpha+\beta\neq 0$. *In this case,* A *is a nil algebra of nil-index* ≤ 3, *and* A *is a Malcev algebra if and only if* $\phi=0$. *Conversely, for any prescribed Malcev algebra product* $[\ ,\ ]$ *on* A^- *and a symmetric invariant form* ϕ *on* A^-, *if* c *is a fixed element of the center of* A^-, *then the product "$*$" defined by* (13) *on* A *is flexible Malcev-admissible.*

Proof. The proof is similar to that of Theorem 5, in which we have shown that $H*H\subseteq Fc$ and $h_1*h_2=\phi(h_1, h_2)c$ for some symmetric bilinear form on H. It follows from (1) that the center Fc is a subalgebra of A and hence $c^2=0$. Thus, for each $h\in H$, $Fh+Fc$ is a nil subalgebra of A of nil-index ≤ 3, and so is associative. This proves that $H*c=0$ and hence by the proof of Theorem 5 $A*c=c*A=0$. It follows from this and the proof of Theorem 5 that (13) and (14) hold for A. Thus, for $x\in A$, $x*x=\phi(x, x)c$ and $0=(x*x)*x=x*(x*x)$ The remainder of the proof is immediate.

The multiplication defined by (13) has some relation with the construction of certain Lie-admissible algebras in terms of 2-cocycles in Lie algebras. For a Malcev algebra M, a skew symmetric bilinear form ϕ_0 on M is called a 2-cocycle of M if ϕ_0 satisfies

(15) $$\phi_0([x, y], [x, z])=\phi_0([[x, y], z], x)+\phi_0([[y, z], x], x)+\phi_0([[z, x], x], y)$$

for all $x, y, z\in M$. Assume that A is a Malcev-admissible algebra, and let ϕ be a bilinear form on A. We define a skew symmetric and symmetric bilinear forms ϕ^- and ϕ^+ by

$$\phi^-(x, y)=\phi(x, y)-\phi(y, x),$$
$$\phi^+(x, y)=\frac{1}{2}[\phi(x, y)+\phi(y, x)]$$

for $x, y\in A$. Then, $\phi=\frac{1}{2}\phi^-+\phi^+$. A bilinear form ϕ on A is called a 2-cocycle of A if ϕ^- is a 2-cocycle of A^-; that is, ϕ^- satisfies relation (15) in A^-.

Suppose now that A denotes a flexible Malcev-admissible algebra described in Theorem 6, and let ϕ be a symmetric invariant form on A^- in relation (13). Let H_0 be a subspace of H complementary to Fc, and denote $A_0=H_0+\sum_{\alpha\neq 0} A_\alpha$. Then, we have the vector space direct sum $A=A_0 \oplus Fc$. For $x, y \in A_0$, denote by $x \circledast y$ and $\phi_0(x, y)c$ the projections of $x*y$ onto A_0 and Fc, respectively. It is readily see that "$\circledast$" is an anticommutative product on A_0 and $x \circledast y=\frac{1}{2}[x, y]_0$ for $x, y \in A_0$, where $[x, y]_0$ denotes the projection of $[x, y]$ onto A_0. Since for $x, y \in A_0$,

$$x * y = x \circledast y + \phi_0(x, y)c, \tag{16}$$

we have $x \circledast y=\frac{1}{2}[x, y]-\frac{1}{2}\phi_0^-(x, y)c$. Since A^- is a Malcev algebra, we see that $(A_0, \circledast)=(A_0, \frac{1}{2}[\ ,\ ]_0)$ is a Malcev algebra and ϕ_0^- is a 2-cocycle of $(A_0, \circledast)$. It also follows from (13) and (16) that $\phi_0^+=\phi$ on A_0 and hence ϕ_0^+ is a symmetric invariant form $(A_0, \circledast)$. Thus, the multiplication "$*$" in (13) can be reformulated as

$$\begin{aligned}(x+\lambda c)*(y+\mu c) &= \frac{1}{2}[x, y]_0+\phi_0(x, y)c \\ &= x \circledast y+\phi_0(x, y)c\end{aligned} \tag{17}$$

for $x, y \in A_0$ and $\lambda, \mu \in F$. The converse of these remarks is

COROLLARY 7. *Let A_0 be a Malcev algebra with multiplication denoted by "$\circledast$" over a fiele F of characteristic $\neq 2$, and let Fc be a one-dimensional space over F. Assume that ϕ_0 is a bilinear form on A_0 such that ϕ_0^- is a 2-cocycle of $(A_0, \circledast)$ and ϕ_0^+ is an invariant form on $(A_0, \circledast)$. Then, the vector space direct sum $A=A_0 \oplus Fc$ with multiplication "$*$" defined by* (17) *is a flexible Malcev-admissible nil algebra of nil-index* ≤ 3.

The proof of Corollary 7 is straightforward. When Theorem 6 is applied to a Kac-Moody algebra with one-dimensional center, we have

COROLLARY 8. *Let A be a flexible Lie-admissible algebra such that A^- is isomorphic to a Kac-Moody algebra with one-dimensional center and with Cartan subalgebra H. If H is nil in A and of nil-index ≤ 4, then the multiplication in A is determined by* (13) *or* (17).

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