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On Quasi Connected Spaces

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. Introduction

n [1], S. G. Yim was introduced quasi open sets which is strictly weaker than open and investigated the characterizations of quasi open sets. In this paper, we introduce ısi connected spaces which are strong form of connected. We give several characteri- ions of such spaces.

hroughout the present paper X means a topological space on which no separation oms are assumed unless explicitly stated.

. Definitions and Characterizations

efinition 1. A subset Q in a space X is a *quasi open* set if and only if $Q\subset Int(Cl(Q))$, ere "*Int*" and "*Cl*" denote the interior and closure of Q, respectively. The comple- nt of a quasi open set is called *quasi closed.*

y Definition 1, an open set is a quasi open set. But the converse is false [1].

efinition 2. A *quasi separation* of a space X is a pair U and V of disjoint nonempty ısi open subsets of X whose union is X. The space X is said to be *quasi connected* there does not exist a quasi separation of X.

learly, every quasi connected space is connected. But the converse of this statement ıot necessarily true as is shown by the following example.

xample. Let $X=\{a, b, c\}$ and $T=\{\phi, \{a\}, X\}$, then (X, T) is connected. Since all subsets of X are quasi open sets, there exists a separation of X. Hence X is not ısi connected.

heorem 3. *For a space X, the following are equilvalent:*

X is quasi connected.

the only subsets of X which are both quasi open and quasi closed are the empty set and X itself.

X is not the union of two disjoint nonempty quasi open (quasi closed) sets.

Proof. (1)$\Longrightarrow$(2) : If A is a nonempty proper subset of X which is both quasi open and si closed in X, then the sets $U=A$ and $V=X-A$ constitute a quasi separation of for they are quasi open, disjoint, and nonempty, and their union is X.

) $\Longrightarrow$ (3) : Clear.

) $\Longrightarrow$ (1) : This is an immediate consequence of Definition 2.

Definition 4. A subset A of a space X is said to be a *quasi connected* set if the subspace A of X is quasi connected.

For A to be a quasi connected subset of X it is necessary and sufficient that, for each covering of A by two quasi open(or quasi closed) subsets B, C of X such that $A\cap B$ and $A\cap C$ are nonempty, we have $A\cap B\cap C\neq\phi$.

Example. In any space, the empty set and every set consisting of a single point are quasi connected.

Lemma 5. *If the sets C and D form a quasi separation of X, and if Y is a quasi connected subset of X, then Y lies entirely within either C or D.*

Proof. Since C and D are both quasi open in X, the sets $C\cap Y$ and $D\cap Y$ are quasi open in Y [1, Theorem 2.1]. These two sets are disjoint and their union is Y; if they were both nonempty, they would constitute a quasi separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or D.

Theorem 6. *The union of a collection of quasi connected sets that have a point in common is quasi connected.*

Proof. Let $\{A_i\}$ be a collection of quasi connected subsets of a space X; let e be a point of $\cap A_i$. We prove that the set $Y=\cup A_i$ is quasi connected. Suppose that $Y=U\cup V$ is a quasi separation of Y. The point e is in one of the sets U or V; suppose $e\in U$. Since the set A_i is quasi connected, it must lie entirely in either U or V, and it cannot lie in V because it contains the point e of U. Hence $A_i\subset U$ for every i: whence $\cup A_i\subset U$ contradicting the fact that V is nonempty.

Definition 7. A function $f: X\longrightarrow Y$ is said to be *strongly quasi continuous* (*quasi continuous* [1]) if for each quasi open(resp. open) set Q in Y, the set $f^{-1}(Q)$ is quasi open in X.

It is obvious that strongly quasi continuity implies quasi continuity. However, the converse is not true, as the following examples show:

Example. Let $X=\{a,b,c\}$ and $\sigma=\{\phi,\{a\},\{a,b\},\{a,c\},X\}$. Let $Y=\{x,y,z\}$ and $\tau=\{\phi,\{x\},\{x,y\},Y\}$. Define a function $f:(X,\sigma)\longrightarrow(Y,\tau)$ as follows: $f(a)=y$ and $f(b)=f(c)=z$. Then, f is quasi continuous, but not strongly quasi continuous.

The concepts of continuity and strongly quasi continuity are independent of each other, as the following examples show.

Example. Let $X=\{a,b,c\}$ and let T and T^* be the indiscrete and the discrete topologies, respectively. Let $f:(X,T)\longrightarrow(X,T^*)$ be the identity function. Then f is strongly quasi continuous, but not continuous.

Example. Let $X=\{a,b,c\}$ and let $T^{**}=\{\phi,\{a\},X\}$, T be the indiscrete topology. Let $f:(X,T^{**})\longrightarrow(X,T)$ be the identity function. Then f is continuous, but not strongly quasi continuous.

Theorem 8. *The image of a quasi connected space under a strongly quasi continuous function is quasi connected.*

Proof. Let $f: X\longrightarrow Y$ be a strongly quasi continuous function; let X be quasi connected. We wish to prove the image $Z=f(X)$ is quasi connected. Since the function obtained from f by restricting its range to the space Z is also strongly quasi continuous, it suffices to consider the case of a strongly quasi continuous surjective function $g: X \longrightarrow Z$. Suppose that $Z = A \cup B$ is a quasi separation of Z into two disjoint nonempty quasi open sets in Z. Then $g^{-1}(A)$ and $g^{-1}(B)$ are disjoint sets whose union is X; they are quasi open in X because g is surjective. Therefore, they form a quasi separation of X, contradicting the assumption that X is quasi connected. This shows that Z is quasi connected.

Theorem 9. *The image of a quasi connected space under a quasi continuous function is connected.*

Proof. The proof is analogous to that of Theorem 8.

References

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