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Teaching Alternative Mathematical Procedures

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Just as no mathematics textbook series serves each student equally well — explanations e satisfactory for most students, too difficult for some, and too easy for others — no one ethod of solving problems or algorithm serves all of the students in a class effectively.[1] An aid to the mathematics teacher in teaching all students effectively is the psycho- gical theory of transfer of learning. Work in this area has been done by Piaget,[2] Lesh,[3] ienes,[4] and Duncan,[5] among others. One of these others is Ellis[6] and his research- ased principles of the transfer of learning are most pertinent in assisting the mathe- atics teacher to teach every student effectively. In one of them he states, "In general, ariety of tasks, or of their stimulus components, during original learning increases the mount of positive transfer obtained." This suggests that teachers should teach a vari- y of procedures for the development of understanding and skill in mathematics algo- sms.

This application of Ellis' theory will be beneficial to two categories of students: ose who understand only one of a variety of procedures and those who comprehend all f the differing procedures for the solution of a given type of problem. If the mathemat- s teacher's explanation is clear, many students frequently will learn well using the one rocedure that the teacher has taught and only that procedure will be necessary for them. owever, there are those students that cannot learn using that particular procedure de- pite all of the mathematics teacher's efforts at step-by-step re-explanations and at us- g many concrete and semi-concrete examples. For those students, the mathematics eacher should have different procedures available.

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1) Riedesel, C. Alan. *Guiding Discovery in Elementary School Mathematics*, Second Edition, New York: Appleton-Century-Crofts, 1973, p. 108.

2) Piaget, J. *The Child's Conception of Number*, London: Humanities Press, 1952.

3) Lesh, Richard. "Applied Mathematical Problem Solving", *Educational Studies in Mathematics* 12 (May 1981) : 235~64.

4) Dienes, Zoltan P. *Modern Mathematics for Young Children*, New York: McGraw Hill Publishing Co., 1973.

5) Duncan, C. P. "Transfer after Training with Single versus Multiple Tasks", *Journal of Experimental Psychology* 55 (1958) : 63~72.

6) Ellis, Henry C. *The Transfer of Learning*, New York: Macmillan, 1965, p. 74.

For example, in teaching subtraction of mixed numbers with regrouping, the following algorithm is taught to and learned by most of the students. This is the standard algorithm and has regrouping the minuend as its main feature. Changing to equivalent fractions is the prerequisite skill.

$$\begin{array}{rrr} 3\frac{1}{6} & = 3\frac{2}{12} & = 2\frac{14}{12} \\ \underline{-1\frac{1}{4}} & = \underline{-1\frac{3}{12}} & = \underline{-1\frac{3}{12}} \\ & & 1\frac{11}{12} \end{array}$$

However, for those students who cannot learn this procedure, a different procedure can be used employing a decomposition method; the critical step involves changing the mixed numbers to improper fractions. Since the student is experiencing difficulty with the standard algorithm, it is advised that semi-concrete examples be used first for this other procedure:

ABSTRACT FORM:

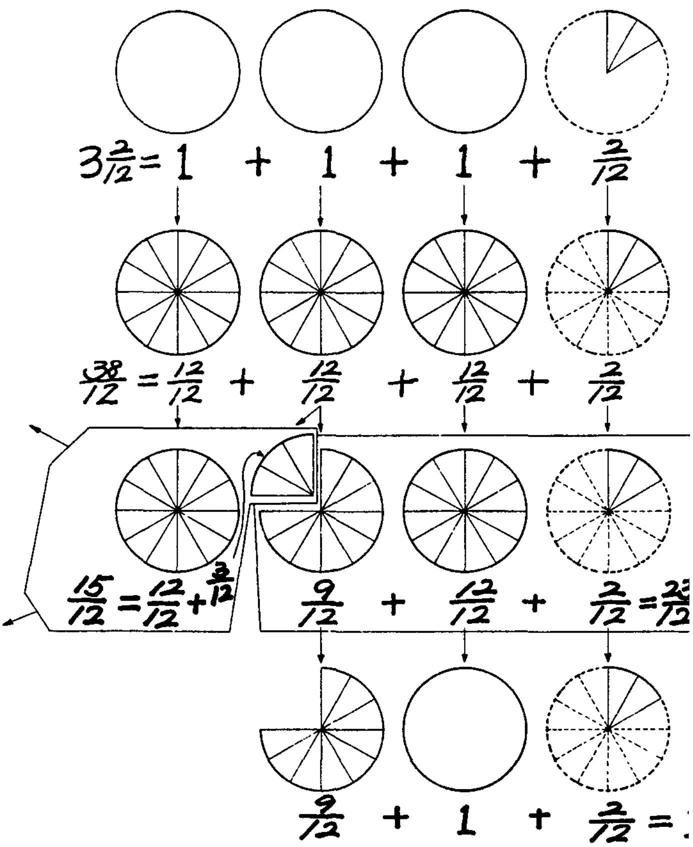
$$\begin{array}{r} 3\frac{2}{12} \\ \underline{-1\frac{3}{12}} \\ 1\frac{11}{12} \end{array}$$

1. Change $3\frac{2}{12}$ to an improper fraction: $3\frac{2}{12} = \frac{38}{12}$.

2. $1\frac{3}{12}$ becomes $\frac{15}{12}$. Subtract $\frac{15}{12}$ from $\frac{38}{12}$. $\frac{38}{12} - \frac{15}{12} = \frac{23}{12}$.

3. $\frac{23}{12}$ is changed back to a mixed number, $1\frac{11}{12}$.

SEMI-CONCRETE FORM:



Then only the abstract form of this procedure is introduced.

$$\begin{array}{rcrcr} 3\frac{1}{6} & = & 3\frac{2}{12} & = & \frac{38}{12} \\ -1\frac{1}{4} & = & -1\frac{3}{12} & = & -\frac{15}{12} \\ \hline & & & & \frac{23}{12} = 1\frac{11}{12} \end{array}$$

This procedure may aid those students who had difficulty with the standard algorithm vith-regrouping. An advantage is that the student eventually makes use of the same hange-mixed-number-to-improper-fraction step in both the procedures of subtraction of ıixed numbers and multiplication of mixed numbers.

Even this second procedure may not be mastered by a few students. Thus, a third ·ocedure should be available. In this classroom-tested procedure, which involves re- :ructuring the original problem by using an equal additions approach, the amount to be dded to both the subtrahend and minuend is the same amount that is necessary to be dded to the subtrahend in order to rename it as a whole number.

$$\begin{array}{rcr|rcrcr} 3\frac{1}{6} & = & 3\frac{2}{12} & 3\frac{2}{12} + \frac{9}{12} & = & 3\frac{11}{12} & = & 3\frac{11}{12} \\ -1\frac{1}{4} & = & -1\frac{3}{12} & -1\frac{3}{12} + \frac{9}{12} & = & -1\frac{12}{12} & = & -2 \\ \hline & & & & & & & 1\frac{11}{12} \end{array}$$

For an understanding of this procedure, it is essential that the student recognize that ıe same amount that is added to the minuend also will be subtracted from it when in- luded in the subtrahend, thus resulting in a canceling effect. Again, semi-concrete exam- les may be advisable before the abstract form of this algorithm is introduced.

These procedures were informally tested in a pilot study in the elementary classrooms f several Iowa schools. The results indicated that the average of the subtraction-of- ıixed-number classes that used these alternative procedures was significantly higher han the average of those classes using the standard algorithm only. Because random se- ection and placement of students was not possible, the interpretation of these results is mited.

DISCUSSION

Those students learning all three procedures will benefit the most from the application f Ellis' variety-of-tasks theory; it will now be easier for them to solve other subtraction of-mixed-number problems compared to an equal amount of practice on only the stand- rd algorithm. This is because varied training with different kinds of procedures pro- ·ides experience with different stimulus situations, thus making new learning easier.[7] 'or example, understanding of a mathematics topic can be improved not so much by

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Ellis, Henry C. *Fundamentals of Human Learning, Memory and Cognition*, Dubuque, Iowa: William C. Brown Publishing Company, 1978, p. 268.

repeated re-reading of the same material but by obtaining a different slant on that mathematics topic or looking at another approach to the topic or seeing the same idea presented in a somewhat new context.

Because the use of alternative procedures for remediation is a viable teaching strategy, alternative procedures to standard algorithms should be developed for all standard algorithms used in the solution of mathematics problems.

In addition, teachers could encourage creativity in gifted students by challenging them to devise alternative procedures of their own for standard algorithms.

The use of several alternative procedures in the event that an original procedure could not be learned by certain students would result in: more effective teaching, students learning more mathematics, and in the teaching-learning experience being less frustrating and more rewarding for both teachers and students.

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