Model Reference Adaptive Control Using Adaptive Observer

(적응관측기를 이용한 기준 모델 적응제어)

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要 約

본 논문에서는 미지의 시불변 단일입력 단일출력 선형 공정에 대하여 기준모델 적응제어를 하였다. 기준모델 적응제어를 수행하기 위하여 지수비중 최소자승 방법으로 유도한 적응 관측기를 이용하였다. 적응관측기는 변수 벡터와 상태 벡터의 초기치를 추정할 수 있도록 구성하였다. 공정의 출력이 안정한 기준모델의 출력에 수렴하도록 제어 입력을 결정하였다.

Abstract

In this paper, an adaptive observer based upon the exponentially weighted least-square method is implemented in the design of a model reference adaptive controller for an unknown time-invariant discrete single-input single-output linear plant. The adaptive observer estimates the parameter vectors and initial state vector. The control input is determined so that the output of the plant converges to the output of the stable model reference.

I. Introduction

The plant parameters are often unknown and the state vectors are not always directly measurable. In such cases, an adaptive observer is a method of reconstructing the state vectors of an observable unknown linear plant through parameter estimates. The adaptive observers of Kreisselmeier [1] and Suzuki [2] are derived in the parametrized form on the basis of a Luenberger observer[3].

In this paper, we derive the discrete adaptive observer which estimates the parameter vectors and initial state vector based upon the state variable filter. The obse ver of the present paper is constructed in the parametrized form, and the adaptation scheme for parameter and initial state adjustment is given by a set of recursive equations which are derived on the basis of an exponentially weighted least-square method. The initial state estimates[4] guarantee more fast convergence of all estimates.

Also this paper applies the adaptive observer to the model reference adaptive control scheme. The plant output does not converge to the model output which has a pole at z=1 and converges very slowly to the model output which has a pole near z=1 by the Suzuki's control law[2]. Therefore, in this paper, the control input is determined so that the error between the model output and the plant output is directly equal to zero. The computer simulation results demonstrate the effectiveness of the adaptive observer and control law.

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接受日字 : 1985年 7月 5日

II. Design of model reference adaptive controller

Consider a linear time-invariant nth order single-input single-output discrete minimum phase system described by

$$x(k+1) = Ax(k)+bu(k)\ ,\ x(0)=x_0 \qquad (1)$$
$$y(k) = c^T x(k)$$

where x(k) is an nxl state vector, u(k) and y(k) are the scalar input and output respectively, and A, b, and c are in the observable canonical form expressed as

$$A=\left[\begin{array}{c|c} a & I_{n-1} \\ \hline & 0 \end{array}\right] \quad , a=[a_1,a_2,\cdots\cdots,a_n]^T$$
$$b=[b_1,b_2,\text{-----},b_n]^T\ ,\ b_1\neq 0 \qquad (2)$$
$$c=[1,0,\text{-----},0]^T$$

where a_i and b_i are the unknown constant parameters.

Expressing the plant by the difference equation for y(k) yields

$$y(k) = \sum_{i=1}^{n} a_i y(k-i) + \sum_{i=1}^{n} b_i u(k-i) \qquad (3)$$

Expressing the model by the difference equation for model output $y_M(k)$ yields

$$y_M(k) = \sum_{i=1}^{n} a_{Mi} y_M(k-i)+ \sum_{i=1}^{n} b_{Mi}\, r(k-i) \qquad (4)$$

where r(k) is the bounded reference input and a_{Mi} and b_{Mi} are the constant parameters to be specified such that the reference model yields a stable and desired response to r(k). The objective of the model reference adaptive control is to adjust the control input u(k) so that the plant output y(k) follows the model output $y_M(k)$.

Therefore, letting $e(k) = y_M(k) - y(k)$, we have

$$e(k) = \sum_{i=1}^{n} [a_{Mi} y_M(k-i)+b_{Mi} r(k-i) - a_i y(k-i)-b_i u(k-i)] \qquad (5)$$

Defining the estimates of a_i and b_i as $\tilde{a}_i(k)$ and $\tilde{b}_i(k)$, respectively and adjusting u(k) so that e(k) converges to zero, we have

$$u(k) = \Big\{ \sum_{i=1}^{n} [a_{Mi}\, y_M(k-i+1)+b_{Mi} r(k-i+1) - \tilde{a}_i(k)y(k-i+1)] - \sum_{i=2}^{n} \tilde{b}_i(k)\, u(k-i+1) \Big\} / \tilde{b}_1(k) \qquad (6)$$

The plant output does not converge to the model output which has a pole at z=1 and converges very slowly to the model output which has a pole near z=1 in Suzuki [2]. The control input of (6), however, guarantees that $y(k)-y_M(k)$ for all stable A_M's.

III. Description of the adaptive observer

Introducing an nxn stable matrix F defined by

$$F = \left[\begin{array}{c|c} f & I_{n-1} \\ \hline & 0 \end{array}\right] \ , f=[f_1,f_2,\cdots\cdots,f_n]^T \qquad (7)$$

the state equation of (1) can be rewritten as

$$x(k+1)=Fx(k)+\Theta_1 y(k)+\Theta_2 u(k),\ x(0)=x_0 \qquad (8)$$

where

$$\Theta_1 = a-f,\ \Theta_2=b \qquad (9)$$

Thus we can identify the unknown parameters a_i and b_i by identifying Θ_1 and Θ_2.

The purpose of an adaptive observer is to adaptively identify the unknown parameters Θ_1 and Θ_2 and to reconstruct the state of the unknown system using the input and output data only.

The z-transform of (8) becomes

$$X(z)=(zI-F)^{-1} [\Theta_1 Y(z)+\Theta_2 U(z)+zx_0] \qquad (10)$$

Now define $V_1(z)$ and $V_2(z)$ as the following equations

$$V_1(z)=(zI-F^T)^{-1} Y(z) \qquad (11)$$
$$V_2(z)=(zI-F^T)^{-1} U(z)$$

The above equations define the nxn matrices $v_1(k)$ and $v_2(k)$ as the solutions of the following difference equations

$$v_1(k+1)=F^T v_1(k)+Iy(k),\ v_1(0)=0 \quad (12)$$
$$v_2(k+1)=F^T v_2(k)+Iu(k),\ v_2(0)=0$$

Substituting (11) into (10), we have

$$X(z)=V_1(z)^T\Theta_1+V_2(z)^T\Theta_2+(zI-F)^{-1}zx_0 \quad (13)$$

Now we consider the initial state response. The initial state response decays as $k \to \infty$, but leads to a slow convergence rate. We define

$$V_3(z)=(zI-F^T)^{-1}z \quad (14)$$

which becomes

$$v_3(k+1)=F^T v_3(k),\ v_3(0)=I \quad (15)$$

Seperating the components of $v_3(k)$ and the initial state x_0 as

$$v_3(k)=[v_{31}(k),v_{32}(k)]^T \quad (16)$$

$$x_0=[x_{01},x_{02}^T]^T,\ x_{01}=y(0)$$

we have the inverse z-transform of (13) as follows

$$x(k)=v(k)^T\Theta+v_{31}(k)x_{01} \quad (17)$$

where

$$v(k)=[v_1(k)^T,v_2(k)^T,v_{32}(k)]^T$$
$$\Theta=[\Theta_1^T,\Theta_2^T,x_{02}^T]^T$$

From (17), we can construct the estimate of the state vector $\tilde{x}(k)$ as follows

$$\tilde{x}(k)=v(k)^T\tilde{\Theta}(k)+v_{31}(k)x_{01} \quad (18)$$

where $\tilde{\Theta}(k)$ is the estimates of Θ which is the parameter vectors and initial state vector.

The z-transform of (8) for Y(z) becomes

$$Y(z)=c^T(zI-F)^{-1}[\Theta_1Y(z)+\Theta_2U(z)+zx_0] \quad (19)$$

Now define $W_1(z)$ and $W_2(z)$ as the following equations

$$W_1(z)=(zI-F^T)^{-1}cY(z)\cdot \quad (20)$$
$$W_2(z)=(zI-F^T)^{-1}cU(z)$$

which represents the state variable filter. The above equations define the nx1 state vectors $w_1(k)$ and $w_2(k)$ as the solutions of the following difference equations

$$w_1(k+1)=F^T w_1(k)+cy(k),\ w_1(0)=0 \quad (21)$$
$$w_2(k+1)=F^T w_2(k)+cu(k),\ w_2(0)=0$$

Substituting (20) into (19), we have

$$Y(z)=W_1(z)^T\Theta_1+W_2(z)^T\Theta_2+c^T(zI-F)^{-1}zx_0 \quad (22)$$

Now we consider the initial state response. We have

$$W_3(z)=(zI-F^T)^{-1}zc \quad (23)$$

which becomes

$$w_3(k+1)=F^T w_3(k),\ w_3(0)=[1,0,\cdots,0]^T=c \quad (24)$$

Seperating the components of $w_3(k)$ as

$$w_3(k)=[w_{31}(k),w_{32}(k)^T]^T \quad (25)$$

we have the inverse z-transform of (22) as follows

$$y(k)=w(k)^T\Theta+w_{31}(k)y(0) \quad (26)$$

where

$$w(k)=[w_1(k)^T,w_2(k)^T,w_{32}(k)^T]^T$$

From (26), we can construct the output of the observer as follows

$$\tilde{y}(k)=w(k)^T\tilde{\Theta}(k)+w_{31}(k)y(0) \quad (27)$$

The adaptive observe adjusts $\tilde{\Theta}(k)$ so that $\tilde{y}(k)$ converges to $y(k)$. As a result, $\tilde{\Theta}(k)$ converges to Θ.

Ⅳ. Derivation of the adaptation scheme

The adaptation scheme for the estimates of the parameter vectors and initial state vector based upon the exponentially weighted least-square method [2] will be described.

Now we introduce the criterion function J(k) as follows and the estimate $\tilde{\Theta}(k)$ is determined so that J(k) becomes minimum at each k.

$$J(k) = \sum_{j=1}^{k} [\lambda^{k-j}(y(j)-w(j)^T\tilde{\Theta}(k)-w_{31}(j)y(0))]^2 \tag{28}$$

where λ is a weighting coefficient such as $0<\lambda<1$. Letting the gradient of J(k) with respect to $\tilde{\Theta}(k)$ be zero yields

$$\sum_{j=1}^{k} \lambda^{2(k-j)} w(j)(y(j)-w(j)^T\tilde{\Theta}(k)-w_{31}(j)y(0))=0 \tag{29}$$

Equation (29) can be represented by

$$W(\lambda, k)W(\lambda, k)^T\tilde{\Theta}(k)=W(\lambda, k)Y(\lambda, k) \tag{30}$$

where

$$W(\lambda, k)=[\lambda^{k-1} w(1), \lambda^{k-2} w(2), \cdots, w(k)] \tag{31}$$

$$Y(\lambda, k)=(\lambda^{k-1}(y(1)-w_{31}(1)y(0)), \lambda^{k-2}(y(2)-w_{31}(2)y(0)), \cdots, y(k)-w_{31}(k)y(0)]^T \tag{32}$$

If $W(\lambda, k)W(\lambda, k)^T$ is invertible, $\tilde{\Theta}(k)$ is given by

$$\tilde{\Theta}(k)=\Gamma(\lambda, k)W(\lambda, k)Y(\lambda, k) \tag{33}$$

where

$$\Gamma(\lambda, k)=[W(\lambda, k)W(\lambda, k)^T]^{-1} \tag{34}$$

Using the inversion lemma, equations (33) and (34) is described as follows

$$\tilde{\Theta}(k)=\tilde{\Theta}(k-1)+\Gamma(\lambda, k)w(k)(y(k)-w_{31}(k)y(0)-w(k)^T\tilde{\Theta}(k-1)) \tag{35}$$

$$\Gamma(\lambda,k)=\frac{\Gamma(\lambda,k-1)}{\lambda^2}-\frac{\frac{\Gamma(\lambda,k-1)}{\lambda^2}w(k)w(k)^T\frac{\Gamma(\lambda,k-1)}{\lambda^2}}{1+w(k)^T\frac{\Gamma(\lambda,k-1)}{\lambda^2}w(k)} \tag{36}$$

The initial value of $\Gamma(\lambda, k)$ should be set as

$$\Gamma(\lambda, 0)=d^2 I \quad , \quad d>>1 \tag{37}$$

so that (36) is applicable for all k. From (6), if the reference input r(k) is sufficiently rich [2], so is the input u(k). Thus $W(\lambda, k)W(\lambda, k)^T$ is non-singular when $k \geq 3n-1$ and the plant is completely controllable.

We now discuss the stability of the overall system. When $\Gamma(\lambda, k)$ converges to its true value, $\tilde{\Theta}(k)$ converges to

$$[W(\lambda, k)W(\lambda, k)^T]^{-1}[W(\lambda, k)Y(\lambda, k)] \tag{38}$$

From equations (26) and (32), $Y(\lambda, k)$ becomes

$$Y(\lambda, k)=W(\lambda, k)^T\Theta \tag{39}$$

Therefore, $\tilde{\Theta}(k)$ converges to Θ and $\tilde{y}(k)$ converges to y(k). When $\tilde{\Theta}(k)$ converges to Θ, $\tilde{a}_i(k)$ and $\tilde{b}_i(k)$ of (6) converge to a_i and b_i, respectively. Thus u(k) of (6) is a control input which makes $e(k)=y_M(k)-y(k)$ converge to zero. Consequently u(k) guarantees that $y(k) \rightarrow y_M(k)$.

The schematic diagram of the model reference adaptive controller is shown in Fig. 1.

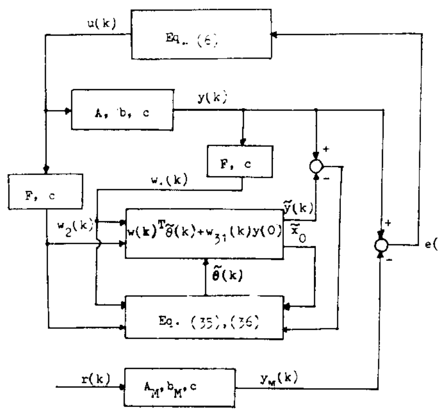


Fig. 1. Schematic diagram of the model reference adaptive controller.

V. Computer simulation studies

We set the plant to be controlled as follows.

$$x(k+1)=\begin{bmatrix}1.52 & 1\\ -0.6 & 0\end{bmatrix} x(k)+\begin{bmatrix}0.43\\ -0.35\end{bmatrix} u(k)$$

$$y(k)=[1,0]\, x(k) \tag{40}$$

The weighting coefficient λ, f_1, f_2, and $\Gamma(\lambda, 0)$ are given by $\sqrt{0.5}$, 1.49, -0.55, and 100I, respectively. The initial state is given by $x_0=[5, -5]$. All initial estimates of the parameter vectors are equal to zero except that $\tilde{b}_1(0) = 0.2$.

In Fig. 2 (a) and Fig. 2 (b), $x_1(k) - \tilde{x}_1(k)$ and $x_2(k) - \tilde{x}_2(k)$ are plotted, respectively. Also, the estimates of the parameter vectors and initial state vector are shown in Fig. 3. The observer of the present paper can reconstruct the state vectors of an observable unknown plant through parameter estimates.

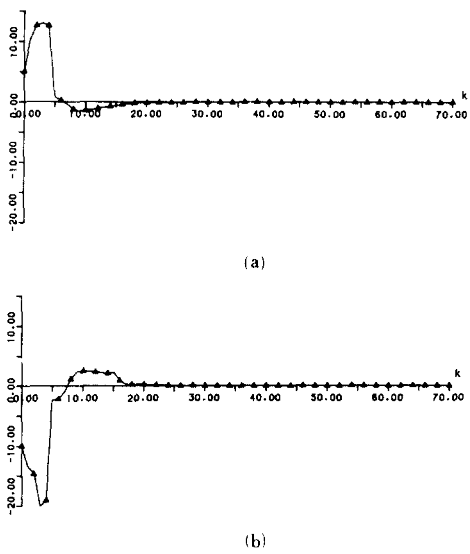
We set the parameters of the model which has a pole at z=1 as follows.

$$a_{M1}=1.5,\ a_{M2}=-0.5,\ b_{M1}=1.0,\ b_{M2}=0.0 \tag{41}$$

The reference input r(k) is given by

$$r(k)=\sin(0.2k)+\sin(0.5k) \tag{42}$$

Fig. 4 (a) shows that with this model the plant output does not converge to the model output by the Suzuki's control law. The plant outputs without and with estimates of the initial state are shown in Fig. 4 (b) and (c), respectively. The convergence of the plant



(a)

(b)

Fig. 2. State errors.
(a) $x_1(k)-\tilde{x}_1(k)$.
(b) $x_2(k)-\tilde{x}_2(k)$.

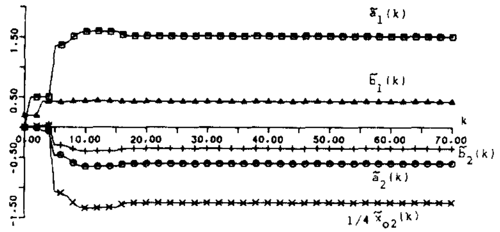
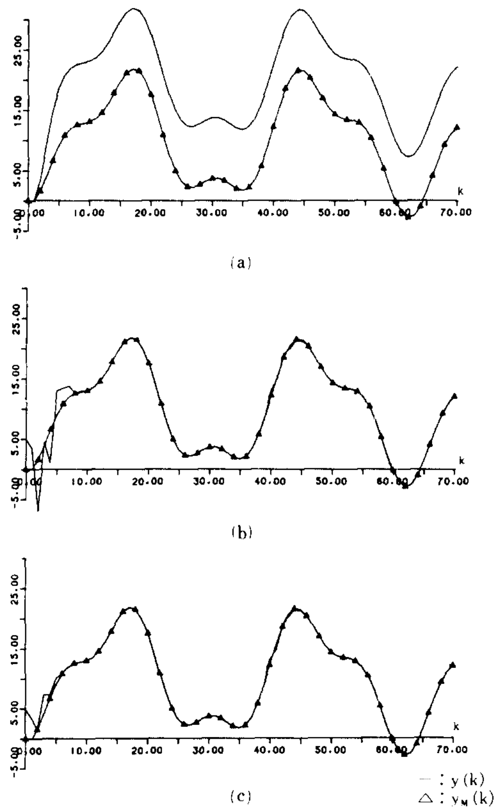


Fig. 3. Estimates of the parameter vectors and initial state vector.



(a)

(b)

(c)

Fig. 4. Plant output and model output.
(a) by the Suzuki's control law.
(b) by the control law of eq. (6) without estimates of the initial state.
(c) with estimates of the initial state.

output to the model output is more fast when the estimates of the initial state are included. When k=40 the parameters of (40) are changed into

$$a_1=1.70,\ a_2=-0.74,\ b_1=0.47,\ b_2=-0.39 \quad (43)$$

Fig. 4 (b) and (c) show this effect on the plant output.

VI. Conclusion

The model reference adaptive controller is implemented on the basis of a discrete adaptive observer. The observer estimates the parameter vectors and initial state vector in the parametrized form. Consequently the observer is easy to implement.

The control input u(k) is synthesized so that the error between the model output and plant output is directly equal to zero. Therefore, the plant output has a fast rate of convergence and converges to the model output for all stable A_M's.

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