Direct Adaptive Control Scheme with Integral Action for Nonminimum Phase Systems

(비최소 위상 시스템에 대한 적분기를 갖는 직접 적응제어)

金 鍾 煥*, 崔 桂 根*

(Jong-Hwan Kim and Keh-Kun Choi)

要 約

본 논문은 비최소 위상 시스템에 대한 직접 적응제어 방법을 제안하였다. 제어기 파라미터들은 최소 자승법 알고리즘으로부터 추정되며, 부수적인 보조 파라미터들은 제안된 등가 다항식으로부터 얻어진다. 적응 제어기는 적분기를 포함하고 있는데 이는 단지 정상상태 오차를 줄이는 목적외에 등가 다항식이 유일한 해를 가질 조건을 만족시키기 위하여 사용되었다.

Abstract

This paper presents a direct adaptive control scheme for nonminimum phase systems of which controller parameters are estimated from the least-squares algorithm, and some additional auxiliary parameters are obtained from the proposed polynomial identity equation. Integral action is incorporated into the adaptive controller to eliminate the steady-state error, and to satisfy a condition of the unique solution for the polynomial identity as well.

I. Introduction

Recently, significant progress has been made on the problem of direct adaptive control for nonminimum phase systems. In [1], this control scheme needs the polynomial factorization or a non-linear identification procedure. With a standard linear parameter estimation Elliott [2] resolved the above problems. However, this scheme arises in arbitrary pole placement and requires the estimation of more parameters than those effectively needed for control. These extra parameters are those of a partial state predictor. Allidina and Hughes [3] resolved the problems of [1] and [2] with a self-tuning control structure. Their scheme merely requires the solution of a polynomial identity. The problem of this scheme is the introduction of unknown polynomial. And also, Praly [4] resolved the problems of [1] and [2] with a bilinear estimation. However, proposed bilinear parameter estimation problem leads to a computational burden. This paper presents a direct or implicit adaptive control structure for single-input single-output nonminimum phase systems. Here we introduce a polynomial identity which together with the Bezout identity resolves the above problems. And also, an integrator is introduced into the adaptive controller in a straightforward fashion to eliminate the steady-state error and to satisfy a condition of the unique solution for the polynomial identity as well.

*正會員, 서울大學校 電子工學科

(Dept. of Elec. Eng., Seoul Nat'l Univ.)

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To estimate controller parameters a least-squares algorithm [5], [6] is used. Hence a convergence theory in [6] can be applied equally to our scheme. Section II describes the controller structure. Section III presents a method for estimating controller parameters and we mention a convergence problem. In Section IV, a computer simulation study is presented demonstrating the feasibility of controlling unstable nonminimum phase systems discussed in [6].

II. Design of Direct Adaptive Control Structure

We consider here a causal feedback control law with the integral action:

$$S(q^{-1})\,U(k) = R\,(q^{-1})\,e(k) \qquad (1)$$

where

$$S(q^{-1}) = (1-q^{-1})\,(1+s_1q^{-1}+ \cdots + s_{ns}q^{-ns}) \qquad (1a)$$

$$R(q^{-1}) = 1+r_0+r_1q^{-1}+ \cdots + r_{nr}q^{-nr} \qquad (1b)$$

$$e(k) = Um\,(k)-Y\,(k) \qquad (1c)$$

q^{-1} is the delay operator, $Y(k)$ is the plant output, $U(k)$ the plant input, $Um(k)$ the set point, and $e(k)$ the tracking error.

Consider a single-input single-output, discrete, time-invariant plant described by

$$A\,(q^{-1})\,Y(k) = q^{-d}B\,(q^{-1})\,U(k)\ ;\ d>0 \qquad (2)$$

where $A\,(q^{-1}) = 1+a_1q^{-1}+ \cdots + a_{na}q^{-na}$ (2a)

$$B\,(q^{-1}) = b_0+b_1q^{-1}+ \cdots + b_{nb}q^{-nb} \qquad (2b)$$

We will use the following assumptions.

A1 : A and B are relatively prime polynomials.

A2 : na, nb, and d are known.

Let us introduce the partial state $Z(k)$, then this plant can be equivalently represented by the controllable backward shift operator representation.

$$A\,(q^{-1})\,Z(k) = U(k) \qquad (3)$$
$$Y(k) = q^{-d}B(q^{-1})\,Z(k)$$

Application of the control law (1) results in the following closed-loop system.

$$\begin{aligned} S\,(q^{-1})\,A(q^{-1})\,Z(k) &= S(q^{-1})\,U(k) \qquad (4)\\ &= -R\,(q^{-1})\,Y(k)+R(q^{-1})\,Um(k)\\ &= -q^{-d}B(q^{-1})R(q^{-1})Z(k)+R(q^{-1})\,Um(k) \end{aligned}$$

Then, (3) can be written as

$$\begin{aligned} (A\,(q^{-1})\,S\,(q^{-1})+q^{-d}R(q^{-1})\,B(q^{-1}))\;Z(k) \qquad (5)\\ = R(q^{-1})\,Um(k)\\ Y(k) = q^{-d}B\,(q^{-1})\,Z(k) \end{aligned}$$

Let $C(q^{-1})$ be a monic asymptotically stable polynomial of order nc whose zeros represent the desired closed-loop pole locations for (5). These can be assigned provided $S(q^{-1})$ and $R(q^{-1})$ satisfy

$$A(q^{-1})\,S(q^{-1})+q^{-d}B(q^{-1})\,R(q^{-1}) = C(q^{-1}) \qquad (6)$$

where

$$C(q^{-1}) = 1+c_1q^{-1}+ \cdots + c_{nc}q^{-nc} \qquad (6a)$$

Since $A\,(q^{-1})$ and $B\,(q^{-1})$ are coprime, a unique solution for $S\,(q^{-1})$ and $R\,(q^{-1})$ always exists under the condition of $ns=nb+d-1$, $nr=\max\,(na,\ nc-nb-d)$ [7]. In this case, it is clear from the assumption A1 that $B\,(q^{-1})$ must have no root at $q=1$.

When (6) holds, (5) simplifies to

$$\begin{aligned} C\,(q^{-1})\,Z\,(k) &= R\,(q^{-1})\,Um(k) \qquad (7)\\ Y\,(k) &= q^{-d}B\,(q^{-1})\,Z\,(k) \end{aligned}$$

Thus after cancellation the closed-loop transfer function relating $Y(k)$ and $Um(k)$ becomes

$$\frac{Y\,(k)}{Um(k)} = \frac{q^{-d}B\,(q^{-1})\,R\,(q^{-1})}{C\,(q^{-1})} \qquad (8)$$

Since $A\,(q^{-1})$ and $B\,(q^{-1})$ are coprime, there exists a unique pair of polynomials $\bar{h}(q^{-1})$ and $\bar{k}(q^{-1})$ of orders $nh=na-1$ and $nk=nb+d-1$, respectively:

$$h(q^{-1}) = h_0 + h_1 q^{-1} + \cdots + h_{nh} q^{-nh} \quad (9)$$
$$= k_0 \bar{h}(q^{-1})$$

$$k(q^{-1}) = k_0 + k_1 q^{-1} + \cdots + k_{nk} q^{-nk} \quad (10)$$
$$= k_0 \bar{k}(q^{-1})$$

which satisfy the Bezout identity

$$q^{-d}B(q^{-1})\,h(q^{-1}) + A(q^{-1})\,k(q^{-1}) = k_0 \quad (11)$$

when (11) holds, (6) can be written as

$$A(q^{-1})\,S(q^{-1}) + q^{-d}B(q^{-1})\,R(q^{-1}) \quad (12)$$
$$= -q^{-d}B(q^{-1}) + (1/k_0)q^{-d}B(q^{-1})h(q^{-1})C(q^{-1}) + (1/k_0)\,A(q^{-1})\,k(q^{-1})\,C(q^{-1})$$

where

$$R'(q^{-1}) = R(q^{-1}) - 1 \quad (12a)$$

Multiplying (12) by Z(k) and using (3) yields

$$A(q^{-1})\,S(q^{-1})\,Z(k) + q^{-d}B(q^{-1})\,R'(q^{-1})Z(k)$$
$$= -q^{-d}B(q^{-1})\,Z(k) \quad (13)$$
$$+ (1/k_0)q^{-d}B(q^{-1})h(q^{-1})C(q^{-1})Z(k)$$
$$+ (1/k_0)A(q^{-1})k(q^{-1})C(q^{-1})Z(k)$$

$$Y(k) = -S(q^{-1})\,U(k) - R'(q^{-1})\,Y(k) \quad (14)$$
$$+ (1/k_0)\,h(q^{-1})\,C(q^{-1})\,Y(k)$$
$$+ (1/k_0)\,k(q^{-1})\,C(q^{-1})\,U(k)$$

Then (14) can be written more compactly as

$$Y(k) = P^T\phi(k) \quad (15)$$

with

$$\phi(k) = [\,\phi_1^T(k)\,;\,\phi_2^T(k)\,]^T \quad (15a)$$
$$P = [\,P_1^T\,;\,P_2^T\,]^T$$

where

$$\phi_1^T(k) = [\,(q^{-1}-1)\,U(k-1) \cdots (q^{-1}-1)\,U(k-ns)$$
$$-Y(k) \cdots -Y(k-nr)\,] \quad (15c)$$

$$\phi_2^T(k) = [\,C(q^{-1})\,Y(k) \cdots C(q^{-1})Y(k-nh)$$
$$C(q^{-1})\,U(k-1) \cdots C(q^{-1})\,U(k-nk)$$
$$(\,C(q^{-1})-1+q^{-1})\,U(k)\,] \quad (15d)$$

$$P_1^T = [\,s_1 \cdots s_{ns}\,r_0 \cdots r_{nr}\,] \quad (15e)$$
$$P_2^T = [\,h_0' \cdots h_{nh}'\,k_1' \cdots k_{nk}'\,1\,] \quad (15f)$$

and

$$h_i' = \frac{h_i}{k_0} \quad ;\ 0 \leqslant i \leqslant nh\ :$$

$$k_j' = \frac{k_j}{k_0} \quad ;\ 1 \leqslant j \leqslant nk \quad (15g)$$

Then a linear regression form can be obtained for the parameter vector P. Note that in the control law (1), integral action is incorporated into the adaptive controller in a straightforward fashion. And also (6) completely characterizes a fixed compensation scheme for the arbitrary pole assignment when $A(q^{-1})$ and $B(q^{-1})$ are known. However, this control scheme requires the estimation of more parameters than those effectively needed for control. This leads to a computational burden. And the insertion of the Bezout polynomial identity into the polynomial design equation arises in arbitrary pole placement.

Now, let polynomials $h(q^{-1})$ and $k(q^{-1})$ be defined from

$$C(q^{-1})\,h(q^{-1}) = A(q^{-1}) + k_0 R(q^{-1}) \quad (16)$$

$$C(q^{-1})\,k(q^{-1}) = k_0 S(q^{-1}) - q^{-d}B(q^{-1}) \quad (17)$$

which are derived from (6) and (11) under the condition of $nc \leqslant 1$. These equations can be written as

$$S(q^{-1})\,h(q^{-1}) - R(q^{-1})\,k(q^{-1}) = 1 \quad (18)$$

By the above equation, all the open-loop zeros are retained. Then the resultant closed-loop system satisfies:

$$C(q^{-1})\begin{bmatrix} Y(k) \\ \\ U(k) \end{bmatrix} = \begin{bmatrix} q^{-d}B(q^{-1}) \\ \\ A(q^{-1}) \end{bmatrix} R(q^{-1})\,Um(k) \quad (19)$$

The auxiliary parameter vector P_2 can be determined from (18) using P_1 which is to be estimated. The proposed identity is similar to that of Allidina and Hughes [3]. However, in this scheme the identity is derived from the pole assignment equation and the Bezout identity, and the introduction of unknown polynomial can be avoided. Also, the above equation is solved relatively straightforwardly and does not impose a severe computational burden.

III. Derivation of the Adaptive Law

Since the plant parameter a_i and b_i are unknown, it is natural to replace the vector P by adjustable parameters vector $\hat{P}(k)$ which will be updated by the adaptation mechanism and the identity equation (18). To evaluate the deviation between the plant output and set point, we introduce the following criterion function:

$$J(k) = \frac{1}{2} \sum_{j=1}^{k} [\, Y(j) - \phi^T(j)\,\hat{P}(k)\,]^2 \qquad (20)$$

The estimate $\hat{P}(k)$ is determined so that the criterion function J (k) becomes minimum at each k. Letting the gradient of J (k) with respect to $\hat{P}(k)$ be zero and employing the matrix inversion lemma yields the following recursive equations.

$$\hat{P}_1(k)=\hat{P}_1(k-1)+L(k)\,[\bar{Y}(k)-\phi_1(k)^T\hat{P}_1(k-1)] \qquad (21)$$

$$F(k) = [\, I - L(k)\phi_1(k)^T\,]\, F(k-1) \qquad (22)$$

$$L(k)=F(k-1)\phi_1(k)\, /\, [1+\phi_1(k)^T F(k-1)\phi_1(k)] \qquad (23)$$

where

$$\bar{Y}(k)=Y(k)-\phi_2^T(k)\,\hat{P}_2(k-1) \qquad (23a)$$

The auxilliary parameter vector $\hat{P}_2$ (k) is to be found from (18) using $\hat{P}_1$ (k) estimated from the above least-squares algorithm. For general, linear, discrete, time-invariant, deterministic systems, the convergence properties of a least-squares algorithm for indirect adaptive control are presented in [6]. Although our scheme is designed to estimate the controller parameters directly, it is important to note that their convergence theorem can be applied equally to our method.

IV. Computer Simulations

The following two examples illustrate some features of the algorithm. UDU^T factorization method [8], [9] was used throughout for the estimation of the controller parameters. The following systems were discussed in [6].

Ex. 1 : The following system is considered:

$$A(q^{-1}) = 1 - 2.0q^{-1} + 0.99q^{-2} \qquad (24)$$

$$q^{-d}B(q^{-1}) = q^{-1}(0.5+1.0q^{-1}) \qquad (25)$$

The following conditions were used

$$C(q^{-1}) = 1.0 \qquad (26)$$

$$\begin{aligned} Um(k) &= 1.0 \qquad 0 \leqslant k < 30 \qquad (27)\\ &= -1.0 \qquad 30 \leqslant k \leqslant 60 \end{aligned}$$

Initial condition of $\hat{P}_1$ was taken as $[1.2\ 0.9\ -2.6\ 1.0]^T$, and initial condition of $\hat{P}_2$ was given from (18).

Ex. 2 : Consider the following system:

$$A(q^{-1}) = 1.0 - 1.2q^{-1} \qquad (28)$$

$$q^{-d}B(q^{-1}) = q^{-1}(1.0-3.1q^{-1}+2.2q^{-2}) \qquad (29)$$

The following conditions were used

$$C(q^{-1}) = 1.0 \qquad (30)$$

$$Um(k) = 1.0 \qquad 0 \leqslant k \leqslant 60 \qquad (31)$$

Initial condition of $\hat{P}_1$ was taken as $[152.9\ -293.1\ -152.1\ 159.9]^T$, and initial condition of $\hat{P}_2$ was given from [18].

We have considered very difficult examples presented in [6]. Fig. 1 shows the output and input signals and the estimated controller parameters for Ex. 1. And Fig. 2 shows the output and input signals for Ex.2. From the computer simulation studies we can see that proposed controller can be also used in the control of nonminimum phase systems.

V. Conclusions

In this paper we have presented a direct

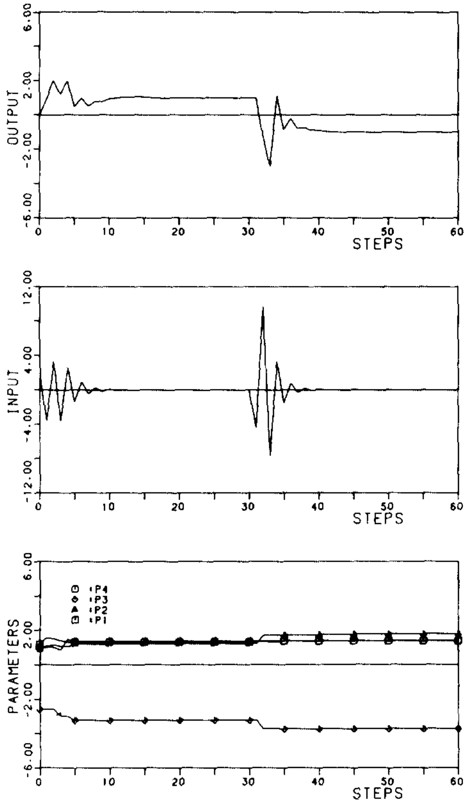

Fig. 1. (a) Output for Ex. 1.
(b) Input for Ex. 1.
(c) Estimated controller parameters for Ex. 1.

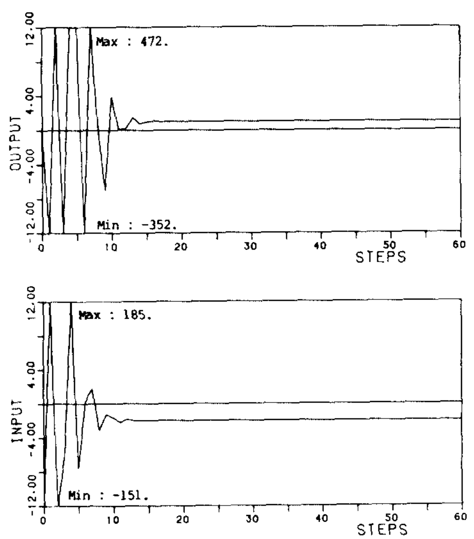

Fig. 2. (a) Output for Ex. 2.
(b) Input for Ex. 2.

scheme for adaptively controlling linear time-invariant discrete-time single-input single-output systems. This controller is applicable to either minimum or nonminimum phase systems. In this scheme, a polynomial identity equation has been derived from the pole placement equation and the Bezout identity, and an integrator has been introduced into the controller to eliminate the steady-state error and to satisfy a condition of the unique solution for the polynomial identity equation as well. As a result, computational burden can be reduced. Examples illustrating the performance of the algorithm have also been given.

References

[1] K.J. Åström, "Direct methods for nonminimum phase systems," in Proc. IEEE 19th Conf. on Decision Contr., Dec. 1980.

[2] H. Elliotts, "Direct adaptive pole placement with application to nonminimum phase system," *IEEE Trans. on Automat. Contr.*, vol. AC-27, June 1982.

[3] A.Y. Allidina and F.M. Hughes, "Self-tuning controllers for deterministic systems," Int. J. Contr., vol. 37, no. 4, 1983.

[4] L. Praly, "Towards a globally stable direct adaptive control scheme for not necessarily minimum phase systems," *IEEE Trans. on Automat. Contr.*, vol. AC-29, no. 10, Oct. 1984.

[5] G.C. Goodwin and K.S. Sin, Adaptive Filtering Prediction and Control. New York: Prentice Hall, 1984.

[6] G.C. Goodwin and K.S. Sin, "Adaptive control of nonminimum phase systems," *IEEE Trans. on Automat. Contr.*, vol. AC-26, no. 2, April 1981.

[7] I.D. Landau and R. Lozano L., "Unification of Discrete Time Explicit Model Reference Adaptive Control Designs, "Automatica, vol. 17, no. 4, 1981.

[8] G.J. Bierman, Factorization Method for Discrete Sequential Estimation. Academic Press, New York, 1977.

[9] Jong-Hwan Kim and Keh-Kun Choi, "Model Reference Adaptive Pole-Placement Controller of Nonminimum Phase Systems," J. of K.I.E.E., vol. 22, no. 6, Nov. 1985. *