Design of the Extended PID Self-Tuner

(확장된 PID 자기 동조기의 설계)

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要 約

본 논문은 PID-B 자기 동조기[1]를 확장시켜 비최소 위상 공정을 제어할 수 있게 하였으며, 기준점이나 공정의 매개 변수가 급격히 변할 경우 그 응답 특성을 개선하였다. 제안된 Extended PID/ST는 직접 극배치 PID/ST에 Bezout 등가식을 도입하여 유도 되었으며, 그 제어 이득들은 적분 제어 이득으로 정규화 되었다. 적분 제어 이득이 1로 정규화 되었지만, 측정 벡터와 기준점을 정규화 하므로써 소위 "set point and derivative kick"을 충분히 피할 수 있다.

Abstract

In this paper the PID-B self-tuner [1] is extended to allow a less abrupt response to set point or plant parameter changes and to control a nonminimum phase plant. The proposed extended PID/ST derived from the direct pole-placement PID/ST is obtained with the Bezout identity as the underlying design method. And its control gains are normalized by the integral control gain. Although the integral control gain is normalized to 1 in our scheme, the so-called "set point and derivative kick" can be avoided sufficiently by normalizing the measurement vector and set point.

I. Introduction

Although several control algorithms have been developed, the most common algorithm used in industry is the discrete equivalent of the continuous proportional-integral-derivative (PID) controller. One reason for the popularity of the discrete PID controller is that it requires little knowledge of the dynamic characteristics of the plant under control, and moreover, the descrete PID controller has been found to give perfectly adequate performance in practice. In a discrete PIΓ controller, the sampling time, the proportional, integral, and derivative control gains need to be selected to satisfy control objectives [2]. However, in a self- tuning PID controller, the above four parameters satisfying control objectives can only be attained with adaptive algorithms [3],[4],[5]. In a recent paper Ortega and Kelly [1] presented a particularly simple PID-B self-tuner to avoid the so-called "set point and derivative kick," which includes the set point signal only in the I action, thus the selection of the initial estimation value of the integral control gain is very important for the start-up transients. However, in case the sudden changes of the plant parameters or the abrupt changes of the set point occur, its performance is poor. And also the plant under control should be minimum phase. In this paper the PID-B self-tuner is extended to

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接受日字 : 1985年 12月 11日

allow a less abrupt response to set point or plant parameter changes and to control a non-minimum phase plant with a richness condition on the control input U(k). However without a persistency of excitation assumption, only local stability may be established [6]. The main features of the proposed design technique derived from the direct pole-placement PID/ST are the followings: i) Since the integral control gain is normalized to 1, it imposes no constraints upon the selection of the initial integral control gain estimation value, thus allowing greater freedom in the design, ii) it involves insertion of the Bezout polynomial identity into the polynomial design equation which arises in arbitrary pole assignment, thus formulating a linear equation error model for estimating 3 controller parameters and 3 additional auxiliary parameters [7], iii) it also involves the normalization of the measurement vector and set point, thus allowing a less abrupt response to set point changes [8],[9], and iv) it is designed in the parameter form, thus allowing easier implementation of the adaptive algorithms. Computer simulations demonstrate the effectiveness of the proposed extended PID/ST control algorithm.

II. PID Self-Tuners

We will consider here a slightly modified PID-B structure [1], given below in the velocity form

$$S(q^{-1})\,U(k) = \alpha\, e(k) - R(q^{-1})\,Y(k) \qquad (1)$$

$$\text{with}\quad S(q^{-1}) = (1-q^{-1})(\alpha + s_1 q^{-1}) \qquad (1a)$$

$$R(q^{-1}) = (1-q^{-1})(r_1 - r_2 q^{-1}) \qquad (1b)$$

$$e(k) = Um(k) - Y(k) \qquad (1c)$$

(1) can be written as

$$S'(q^{-1})\,U(k) = e(k) - R'(q^{-1})\,Y(k) \qquad (2)$$

$$\text{with}\quad S'(q^{-1}) = (1-q^{-1})(1+s_1' q^{-1}) \qquad (2a)$$

$$R'(q^{-1}) = (1-q^{-1})(r_1' - r_2' q^{-1}) \qquad (2b)$$

where q^{-1} is the backward-shift operator, Y(k) is the plant output, U(k) the plant input, Um(k) the set point, and e(k) the tracking error. The modified PID-B is a particular version of a conventional PID structure utilized to avoid the so-called "set point and derivative kick" (sudden changes in control law (1) when the set point is changed) [10]. The plant dynamic characteristics will be approximated by a unitary-delay second-order linear time invariant model. PID/ST's are classified in this paper into two cases: direct and extended PID/ST's based on pole-placement. Let us introduce the following controller (Fig. 1) presented in [1].

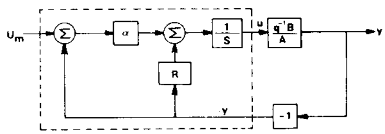

Fig. 1. Closed-loop system with PID controller.

(1) Direct Pole-Placement PID/ST

Consider a single-input single-output, discrete, time-invariant second order plant described by

$$A(q^{-1})\,Y(k) = q^{-1}B(q^{-1})\,U(k) \qquad (3)$$

$$\text{where}\quad A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} \qquad (3a)$$

$$B(q^{-1}) = b_0 + b_1 q^{-1};\ b_0 \neq 0 \qquad (3b)$$

$A(q^{-1})$ and $B(q^{-1})$ are assumed relatively prime polynomials in the delay operator q^{-1} with unknown coefficients. Equating the closed-loop transfer function with the desired one ($q^{-1}/C'(q^{-1})$), we get the polynomial identity

$$\alpha C'(q^{-1})B(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-1}B(q^{-1})\,M(q^{-1}) \qquad (4)$$

which has a unique solution in terms of α, s_1, r_1, and r_2,

where

$$M(q^{-1}) = \alpha + R(q^{-1}) \qquad (4a)$$

$$C'(q^{-1}) = 1 + c_1' q^{-1} + \cdots + c_{nc}' q^{-nc};\ nc \leqslant 3 \qquad (4b)$$

$C'(q^{-1})$ is the desired characteristic polynomial.

From (4) and (2), we get

$$\frac{B(q^{-1})}{A(q^{-1})}=\frac{(1+s_1{}'q^{-1})}{((C'(q^{-1})-q^{-1})/(1-q^{-1}))-q^{-1}(r_1{}'-r_2{}'q^{-1})} \quad (5)$$

A linear regression form may be obtained for the parameter vector $[\, s_1{}'\, r_1{}'\, r_2{}']^T$.

Note that in the control law (2), the control gains are normalized by the integral control gain α, and the coefficient α is included to tune the PID controller in terms of a pole assignment objective of (4). And also (4) then completely characterizes a fixed compensation scheme for arbitrary pole-assignment when $A(q^{-1})$ and $B(q^{-1})$ are known. In (5), $S'(q^{-1})$ contains the zero polynomial. Hence, the discrete time plant under control should be minimum phase.

(2) Extended Pole-Placement PID/ST

Let us introduce the partial state Z(k) to retain open-loop zeros, then (3) can be equivalently represented by the controllable backward shift operator representation.

$$A(q^{-1})Z(k)=U(k) \quad (6)$$
$$Y(k)=q^{-1}B(q^{-1})Z(k)$$

Application of the control law (1) results in the following closed-loop system.

$$S(q^{-1})\,A(q^{-1})\,Z(k)=S(q^{-1})\,U(k) \quad (7)$$
$$=-M(q^{-1})\,Y(k)+\alpha Um(k)$$
$$=-q^{-1}B(q^{-1})M(q^{-1})\,Z(k)+\alpha Um(k)$$

Then, (6) can be written as

$$(A(q^{-1})\,S(q^{-1})+q^{-1}B(q^{-1})M(q^{-1}))\,Z(k) \quad (8)$$
$$=\alpha Um(k)$$
$$Y(k)=q^{-1}B(q^{-1})Z(k)$$

Let $C(q^{-1})$ be a monic asymptotically stable polynomial of degree nc ($\leqslant 4$) whose zeros represent the desired closed-loop pole locations for (8). These can be assigned provided $S(q^{-1})$ and $M(q^{-1})$ satisfy

$$A(q^{-1})S(q^{-1})+q^{-1}B(q^{-1})M(q^{-1})=\alpha C(q^{-1}) \quad (9)$$

where

$$C(q^{-1})=1+c_1q^{-1}+\cdots+c_{nc}q^{-nc} \quad (9a)$$

Eq.(9) has a solution for $S(q^{-1})$ and $M(q^{-1})$ [11] When (9) holds, (8) simplifies to

$$C(q^{-1})\,Z(k)=Um(k) \quad (10)$$
$$Y(k)=q^{-1}B(q^{-1})\,Z(k)$$

Thus after cancellation the closed-loop transfer function relating Y(k) and Um(k) becomes

$$\frac{Y(k)}{Um(k)}=\frac{q^{-1}B(q^{-1})}{C(q^{-1})} \quad (11)$$

Since $A(q^{-1})$ and $B(q^{-1})$ are coprime, there exists a unique pair of polynomials

$$h(q^{-1})=h_0+h_1q^{-1} \quad (12)$$
$$k(q^{-1})=k_0+k_1q^{-1}$$

which satisfy the Bezout identity

$$q^{-1}B(q^{-1})\,h(q^{-1})+A(q^{-1})\,k(q^{-1})=1 \quad (13)$$

when (13) holds, (9) can be written as

$$A(q^{-1})\,S(q^{-1})+q^{-1}B(q^{-1})R(q^{-1})$$
$$=-\alpha q^{-1}B(q^{-1})+\alpha q^{-1}B(q^{-1})h(q^{-1})C(q^{-1})$$
$$+\alpha A(q^{-1})k(q^{-1})\,C(q^{-1}) \quad (14)$$

Multiplying (14) by Z(k) and using (6) yields

$$A(q^{-1})S(q^{-1})Z(k)+q^{-1}B(q^{-1})R(q^{-1})Z(k)$$
$$=-\alpha q^{-1}B(q^{-1})\,Z(k)+\alpha q^{-1}B(q^{-1})h(q^{-1})C(q^{-1})Z(k)$$
$$+\alpha A(q^{-1})k(q^{-1})C(q^{-1})Z(k) \quad (15)$$

$$\alpha Y(k)=-S(q^{-1})\,U(k)-R(q^{-1})Y(k)$$
$$+\alpha h(q^{-1})C(q^{-1})Y(k)$$
$$+\alpha k(q^{-1})C(q^{-1})U(k) \quad (16)$$

The constant k_0 in (12) can be taken as 1 because the Bezout identity must be satisfied. Then (16) can be written more compactly as

$$Y(k) = P^T \phi(k) \quad (17)$$

with
$$\phi(k) = [\ (q^{-1}-1)\ U(k-1)\ \ (q^{-1}-1)Y(k)\ \ (1-q^{-1})Y(k-1)\ \ C(q^{-1})Y(k)\ \ C(q^{-1})Y(k-1)\ \ C(q^{-1})U(k-1)\ \ (C(q^{-1})-1+q^{-1})\ U(k)\ ]^T \quad (17a)$$

$$P = [\ s_1'\ r_1'\ r_2'\ h_0\ h_1\ k_1\ \ 1\ ]^T \quad (17b)$$

Define the normalized system :

$$Y^n(k) = P^T X(k) \quad (18)$$

where

$$Y^n(k) = \frac{Y(k)}{N(k)}\ ; \quad X(k) = \frac{\phi(k)}{N(k)} \quad (18a)$$

and

$$N(k) = \mathrm{Max}\ (\ 1,\ \|\phi(k)\|\ ) \quad (18b)$$

Note that $\|X(k)\| \leqslant 1$ (19)

The normalized tracking error $e^n(k)$ is then determined as follows :

$$e^n(k) = U^n_m(k) - P^T X(k) \quad (20)$$

where

$$U^n_m(k) = \frac{Um(k)}{N(k)} \quad (20a)$$

III. Derivation of the Adaptive Law

Since the plant parameter a_i and b_i are unknown, it is natural to replace the vector P by the adjustable vector $\hat{P}(k)$ which will be updated by the adaptation mechanism. To evaluate the deviation between the plant output and set point, we introduce the following criterion function :

$$J(k) = \frac{1}{2} \sum_{j=1}^{k} [\ U^n_m(j) - X(j)^T \hat{P}(k)\ ]^2 \quad (21)$$

The estimate $\hat{P}(k)$ is determined so that the criterion function J(k) becomes minimum at each k. Letting the gradient of J(k) with respect to $\hat{P}(k)$ be zero and employing the matrix inversion lemma yields the following recursive equations.

$$\hat{P}(k) = \hat{P}(k-1) + L(k)\ [\ U^n_m(k) - X(k)^T \hat{P}(k-1)] \quad (22)$$

$$F(k) = [\ I - L(k)X(k)^T\ ]\ \ F(k-1) \quad (23)$$

$$L(k) = F(k-1)X(k)\ /\ [1 + X(k)^T F(k-1)X(k)] \quad (24)$$

We can avoid the unnecessary storages and improve both accuracy and computational efficiency by applying UDU^T factorization method [12], [13].

IV. Computer Simulations

The following two examples illustrate some features of the self-tuning PID controller.

Consider the following plant :

$$A(q^{-1}) = 1 - 0.857q^{-1} + 0.548q^{-2} \quad (25)$$
$$B(q^{-1}) = 0.381 + 0.310q^{-1}$$

The desired characteristic equation is

$$C(q^{-1}) = 1 + 0.7500q^{-1} + 0.2466q^{-2} \quad (26)$$

UDU^T factorization method was used throughout for the estimation of the parameters. In the PID-B/ST, $\alpha(0) = 0.1$ and all the remaining initial conditions were taken equal to zero, in the direct PID/ST, the initial conditions were taken as $[\ 0.6438\ 2.9640\ 1.7490\ ]^T$, and in the extended PID/ST, all the initial conditions were taken equal to 0.6.

Example 1 : PID-B/ST, Direct and Extended PID/ST Performance Comparison.

Fig. 2 and Fig. 3 show the output and the reference signal when the controller structure PID-B/ST and direct PID/ST, respectively, were used. In Fig. 2, before the estimation parameters have converged to the proper values, it does not show a good performance. And in Fig. 3, with the well selected initial estimation values, it looks like having a good characteristic.

However, when sudden changes of the plant parameters occur, its performance is very poor. And the above controllers have a problem of the selection of the initial estimation values. These drawbacks are overcome with the extended PID/ST, whose behavior is shown in Fig. 4.

Example 2 : PID-B/ST and Extended PID/ST Performance to Plant Parameter Changes.

Consider the sudden changes of the plant parameters as follows :

Equation (25) $0 \leqslant k < 100$

$A(q^{-1}) = 1 - 0.3q^{-1} + 0.1q^{-2}$, $B(q^{-1}) = 0.35 + 0.1q^{-1}$ $100 \leqslant k < 200$

Equation (25) $200 \leqslant K \leqslant 350$

The behavior of the PID-B/ST and the extended PID/ST are shown in Fig.5 and Fig. 6, respectively. Notice that the performance of the extended PID/ST is superior to that of the PID-B/ST in the sense of the classical figures of merit, such as overshoots, settling time, and rise time.

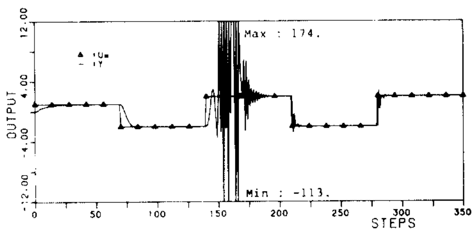

Fig. 2. Output Y(k) and Set point Um(k) (PID-B/ST).

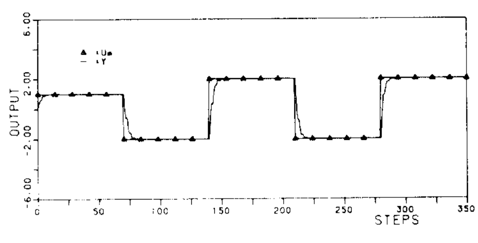

Fig. 3. Output Y(k) and Set point Um(k) (Direct PID/ST).

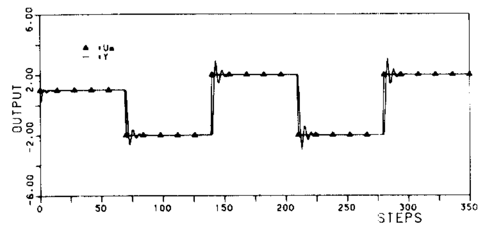

Fig. 4. Output Y(k) and Set point Um(k) (Extended PID/ST).

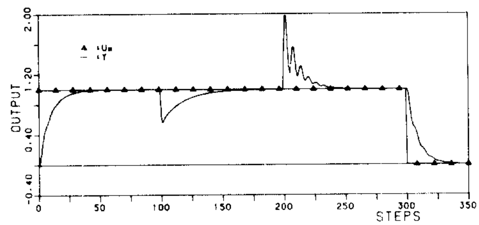

Fig. 5. Output Y(k) and Set point Um(k) (PID-B/ST).

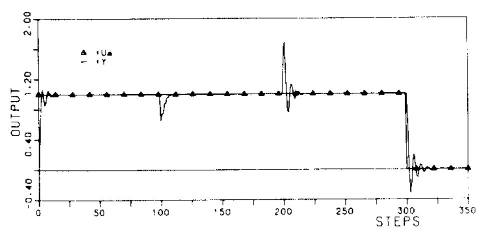

Fig. 6. Output Y(k) and Set point Um(k) (Extended PID/ST).

V. Conclusions

A direct and an extended PID/ST's for single-input single-output linear second-order plant have been designed. The direct PID/ST which has a problem of the selection of the initial estimation values is required to estimate only 3 controller parameters. An extended PID/ST is obtained with the Bezout identity as underlying design method. And this controller can be applied to nonminimum phase systems, and has better performance than the others in the sense of the classical figures of merit, such as overshoots, settling time, and rise time. The effectiveness of the proposed self-tuning PID control algorithms has been demonstrated by computer simulations.

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